**6.013 Quiz 1 Solutions, 10/20/2005**

Problem 1

a) Apply Integral form of Ampere's Law:

$$2\pi r H_\phi = \int_{\phi=0}^{2\pi}\int_{r=0}^{r} J_z(r) r dr d\phi \rightarrow \boxed{\mathbf{H} = \frac{J_0 r^2}{3R_1}\mathbf{i}_\phi}$$

b) The total current going up the inside conductor must return going down in the outside conductor:

$$2\pi\int_{r=0}^{R_1} J_z(r) r dr = 2\pi R_2 |\mathbf{K}| \rightarrow \boxed{\mathbf{K} = -\frac{J_0 R_1^2}{3R_2}\mathbf{i_z}}$$

Problem 1 Alternate Method:

$$R_1 < r < R_2$$

$$2\pi r H_\phi = \int_{\phi=0}^{2\pi}\int_{r=0}^{R_1} \frac{J_0 r}{R_1} r dr d\phi \rightarrow H_\phi = \frac{J_0 R_1^2}{3r}$$

$$\boxed{\mathrm{K_z} = -H_\phi(r = R_2) = -\frac{J_0 R_1^2}{3R_2}}$$

Problem 2

a) The boundary conditions at $\phi = 0$ and $\phi = \alpha$ determine the unknowns A and B

$$\Phi(\phi = 0) = 0 \rightarrow \boxed{B = 0}$$

$$\Phi(\phi = \alpha) = V_0 \rightarrow \boxed{A = \frac{V_0}{\alpha}}$$

b) The electric field is the negative gradient of the potential. Problem is best done in cylindrical coordinates:

$$\mathbf{E} = -\nabla\Phi = -\frac{1}{r}\frac{\partial\Phi(\phi)}{\partial\phi}\mathbf{i}_\phi = \boxed{-\frac{V_0}{\alpha r}\mathbf{i}_\phi}$$

c) Use the boundary condition on the normal D field at the upper perfect conductor:

$$\sigma_{sf} = \mathbf{i}_\phi \bullet [\varepsilon_0 \mathbf{E}(\phi = \alpha^+) - \varepsilon \mathbf{E}(\phi = \alpha^-)] = \boxed{\frac{\varepsilon V_0}{\alpha r}}$$

d) Since C=Q/V$_0$ and the total charge is given by

$$Q = d\int_{r=R_1}^{R_2} \sigma_{sf}(r) dr = \frac{\varepsilon d}{\alpha} V_0 \ln\left(\frac{R_2}{R_1}\right) \rightarrow \boxed{C = \frac{\varepsilon d}{\alpha}\ln\left(\frac{R_2}{R_1}\right)}$$

Problem 3

a) By inspection $\omega = 2\pi f = 2\pi \times 10^8 \rightarrow \boxed{f = 10^8 \text{ Hz}}$

b) By inspection $|\mathbf{k}| = \sqrt{k_x^2 + k_z^2} = \pi\sqrt{1+3} = 2\pi \rightarrow \boxed{\lambda = \frac{2\pi}{|\mathbf{k}|} = 1\text{m}}$

c) $\boxed{c = f\lambda = 1\times 10^8 \text{ m/s}}$

d) From trigonometry $k_x = \pi, \quad k_z = \pi\sqrt{3}, \quad \tan(\theta_i) = \frac{k_x}{k_z} = \frac{1}{\sqrt{3}} \rightarrow \boxed{\theta_i = \frac{\pi}{6}}$ radians,

equal to 30 degrees.