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6.013/ESD.013J Electromagnetics and Applications, Fall 2005

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6.013 Quiz 1 Solutions, 10/20/2005

Problem 1

a) Apply Integral form of Ampere's Law:

$$2\pi r H_\phi = \int_{\phi=0}^{2\pi} \int_{r=0}^r J_z(r) r dr d\phi \rightarrow \mathbf{H} = \frac{J_0 r^2}{3R_1} \mathbf{i}_\phi$$

b) The total current going up the inside conductor must return going down in the outside conductor:

$$2\pi \int_{r=0}^{R_1} J_z(r) r dr = 2\pi R_2 |\mathbf{K}| \rightarrow \mathbf{K} = -\frac{J_0 R_1^2}{3R_2} \mathbf{i}_z$$

Problem 1 Alternate Method:

$$R_1 < r < R_2$$

$$2\pi r H_\phi = \int_{\phi=0}^{2\pi} \int_{r=0}^{R_1} \frac{J_0 r}{R_1} r dr d\phi \rightarrow H_\phi = \frac{J_0 R_1^2}{3r}$$

$$\mathbf{K}_z = -H_\phi(r=R_2) = -\frac{J_0 R_1^2}{3R_2}$$

Problem 2

a) The boundary conditions at $\phi=0$ and $\phi=\alpha$ determine the unknowns A and B

$$\Phi(\phi=0) = 0 \rightarrow \mathbf{B} = 0$$

$$\Phi(\phi=\alpha) = V_0 \rightarrow A = \frac{V_0}{\alpha}$$

b) The electric field is the negative gradient of the potential. Problem is best done in cylindrical coordinates:

$$\mathbf{E} = -\nabla\Phi = -\frac{1}{r} \frac{\partial\Phi(\phi)}{\partial\phi} \mathbf{i}_\phi = -\frac{V_0}{\alpha r} \mathbf{i}_\phi$$

c) Use the boundary condition on the normal D field at the upper perfect conductor:

$$\sigma_{sf} = \mathbf{i}_\phi \cdot [\epsilon_0 \mathbf{E}(\phi = \alpha^+) - \epsilon \mathbf{E}(\phi = \alpha^-)] = \frac{\epsilon V_0}{\alpha r}$$

d) Since $C=Q/V_0$ and the total charge is given by

$$Q = d \int_{r=R_1}^{R_2} \sigma_{sf}(r) dr = \frac{\epsilon d}{\alpha} V_0 \ln\left(\frac{R_2}{R_1}\right) \rightarrow C = \frac{\epsilon d}{\alpha} \ln\left(\frac{R_2}{R_1}\right)$$

Problem 3

a) By inspection $\omega = 2\pi f = 2\pi \times 10^8 \rightarrow f = 10^8 \text{ Hz}$

b) By inspection $|\mathbf{k}| = \sqrt{k_x^2 + k_z^2} = \pi\sqrt{1+3} = 2\pi \rightarrow \lambda = \frac{2\pi}{|\mathbf{k}|} = 1\text{m}$

c) $c = f\lambda = 1 \times 10^8 \text{ m/s}$

d) From trigonometry $k_x = \pi$, $k_z = \pi\sqrt{3}$, $\tan(\theta_i) = \frac{k_x}{k_z} = \frac{1}{\sqrt{3}} \rightarrow \theta_i = \frac{\pi}{6}$ radians,
equal to 30 degrees.