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6.013/ESD.013J Electromagnetics and Applications, Fall 2005

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Problem Set 8 - Solutions

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Problem 8.1**A**At $t = 0$,

$$\left. \begin{array}{l} v = v_+ + v_- = V_0 \\ i = Y_0(v_+ - v_-) = 0 \end{array} \right\} \implies v_+ = v_- = \frac{V_0}{2}$$

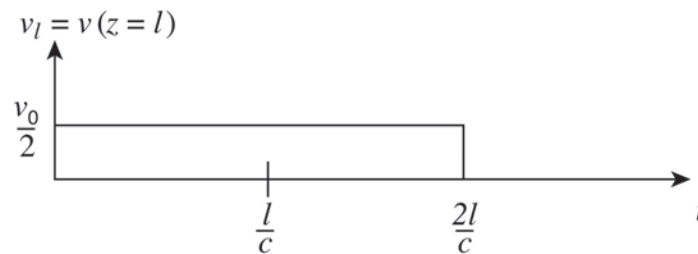
BAfter $t = 0$, $\Gamma_S = 1$ and $\Gamma_L = 0$.

Figure 1: Time dependence of voltage load. (Image by MIT OpenCourseWare.)

Problem 8.2**A**At $t = 0$,

$$\left. \begin{array}{l} v = v_+ + v_- = V_0 \\ i = Y_0(v_+ - v_-) = 0 \end{array} \right\} \implies v_+ = v_- = \frac{V_0}{2}$$

BAfter $t = 0^+$, $\Gamma_L = -1$ and $\Gamma_S = 0$.

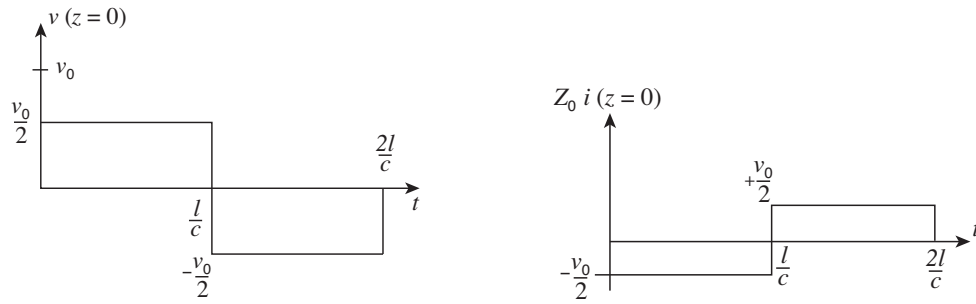


Figure 2: Voltage and current at $z = 0$. (Image by MIT OpenCourseWare.)

C

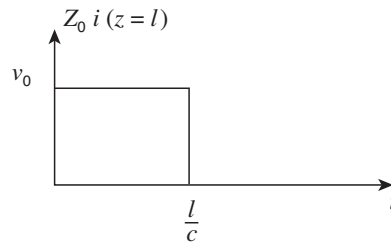


Figure 3: Current at $z = l$. (Image by MIT OpenCourseWare.)

D

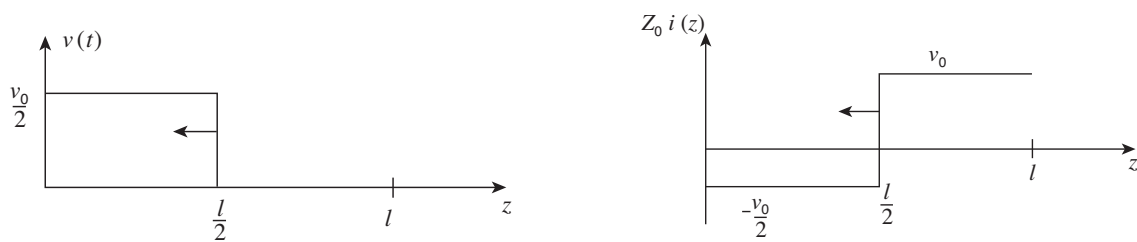


Figure 4: Voltage and current for $0 < z < l$ at time $t = T/2$. (Image by MIT OpenCourseWare.)

Problem 8.3

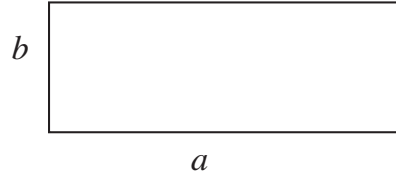


Figure 5: Rectangular waveguide. (Image by MIT OpenCourseWare.)

A

The mode that has the lowest cutoff is the 1, 0 mode for that mode a corresponds to a half wavelength.

$$\frac{\lambda}{2} = a \implies (2a)f = c \implies f = \frac{c}{2a}$$

B

The next modes with a higher cutoff frequency would be the 0, 2 and 1, 0 modes.

$$f = \frac{c}{2b} = \frac{c}{4a}$$

The range of frequencies where only one mode propagates are

$$7.5 \times 10^9 \text{ Hz} < f < 15 \times 10^9 \text{ Hz}.$$

Problem 8.4

A

$$\sigma_S = \hat{\mathbf{n}} \cdot [\mathbf{D}_1 - \mathbf{D}_2]$$

$$\text{TE}_n : \text{tangential } E \text{ only} \implies \sigma_S = 0$$

$$\begin{aligned} \text{TM}_n : \sigma_S &= +\varepsilon k_x \cos(n\pi) \cos(\omega t - k_x x) = +\varepsilon k_x (-1)^n \cos(\omega t - k_x x) && \text{on top plate} \\ \sigma_S &= -\varepsilon k_x \cos(\omega t - k_x x) && \text{on the bottom plate} \end{aligned}$$

B

$$\mathbf{k} = \hat{\mathbf{n}} \times [\mathbf{H}_1 - \mathbf{H}_2]$$

$$\begin{aligned} \text{TE}_n : \mathbf{K}_{\text{top}} &= \frac{E_0}{\eta k} [k_z \cos(n\pi) \cos(\omega t - k_x x)] \mathbf{i}_y \\ &= \frac{E_0}{\eta k} (-1)^n k_z \cos(\omega t - k_x x) \mathbf{i}_y \end{aligned}$$

$$\mathbf{K}_{\text{bottom}} = -\frac{E_0}{\eta k} [k_z \cos(\omega t - k_x x)] \mathbf{i}_y$$

$$\begin{aligned} \text{TM}_n : \mathbf{K}_{\text{top}} &= -\frac{E_0}{\eta} (-1)^n \cos(\omega t - k_x x) \mathbf{i}_x && \text{top plate} \\ \mathbf{K}_{\text{bottom}} &= \frac{E_0}{\eta} \cos(\omega t - k_x x) \mathbf{i}_x && \text{bottom plate} \end{aligned}$$

C

$$\begin{aligned} \frac{dz}{dx} &= +\frac{k_x}{k_z} \tan\left(\frac{n\pi}{d}z\right) \tan(k_x x) \\ \implies \int k_x \tan(k_x x) dx &= \int k_z \cot(k_z z) dz \\ \implies +\ln[\cos(k_x x)] &= -\ln[C \sin(k_z z)] \\ \implies \frac{1}{\cos(k_x x)} &= C \sin(k_z z) \\ C &= \frac{1}{\cos(k_x x_0) \sin(k_z z_0)} \implies \frac{\sin(k_z z) \cos(k_x x)}{\sin(k_z z_0) \cos(k_x x_0)} = 1 \end{aligned}$$

D

$$\begin{aligned} \frac{dz}{dx} &= \frac{E_z}{E_x} = +\frac{k_x}{k_z} \cot(k_z z) \cot(k_x x) \\ \implies \int k_z \tan(k_z z) dz &= \int k_x \cot(k_x x) dx \\ \implies -\ln(\cos(k_z z)) &= \ln(C \sin(k_x x)) \\ \implies \frac{1}{\cos(k_z z)} &= C \sin(k_x x) \\ \implies \frac{\sin(k_x x) \cos(k_z z)}{\sin(k_x x_0) \cos(k_z z_0)} &= 1 \end{aligned}$$

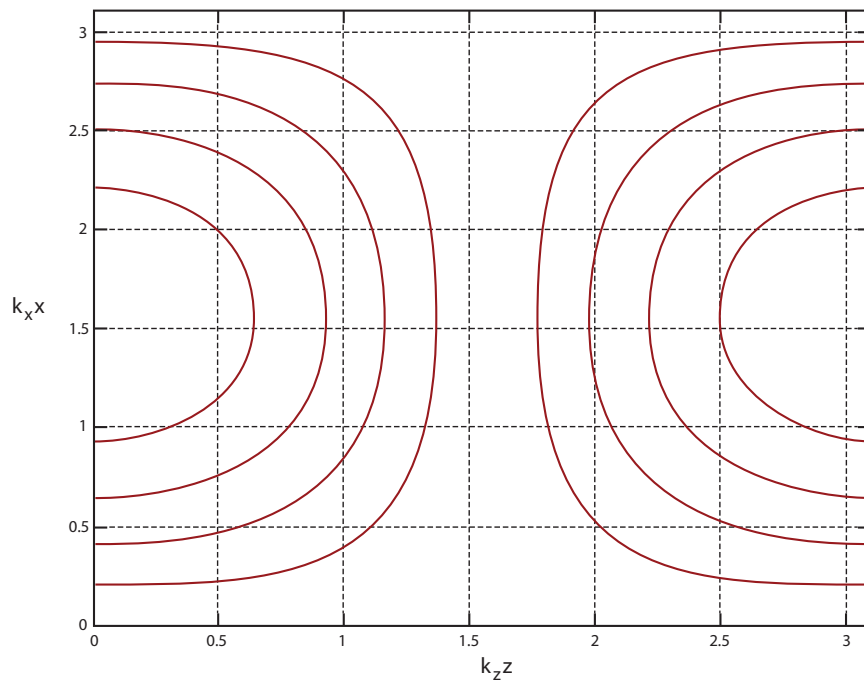


Figure 6: Magnetic field lines for TE₁. (Image by MIT OpenCourseWare.)

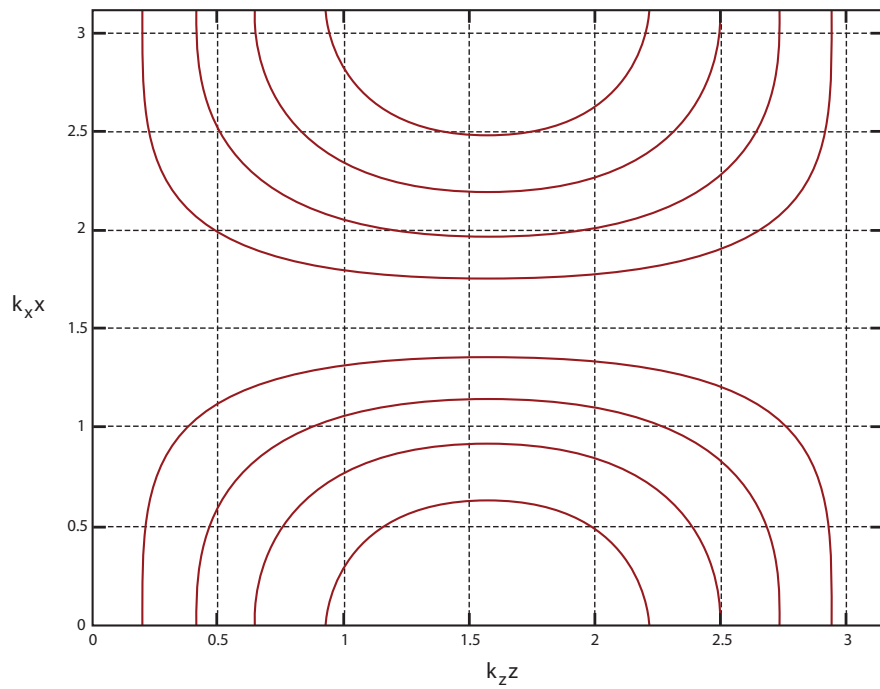


Figure 7: Electric field lines for TM₁. (Image by MIT OpenCourseWare.)