| **6.013/ESD.013J — Electromagnetics and Applications** | Fall 2005 |
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# Problem Set 8 - Solutions

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## Problem 8.1

### A

At $t = 0$,

$$\left.\begin{aligned} v &= v_+ + v_- = V_0 \\ i &= Y_0(v_+ - v_-) = 0 \end{aligned}\right\} \implies v_+ = v_- = \frac{V_0}{2}$$

### B

After $t = 0$, $\Gamma_S = 1$ and $\Gamma_L = 0$.

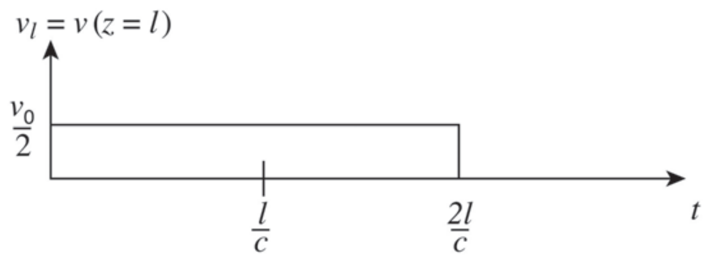


Figure 1: Time dependence of voltage load. (Image by MIT OpenCourseWare.)

## Problem 8.2

### A

At $t = 0$,

$$\left.\begin{aligned} v &= v_+ + v_- = V_0 \\ i &= Y_0(v_+ - v_-) = 0 \end{aligned}\right\} \implies v_+ = v_- = \frac{V_0}{2}$$

### B

After $t = 0^+$, $\Gamma_L = -1$ and $\Gamma_S = 0$.

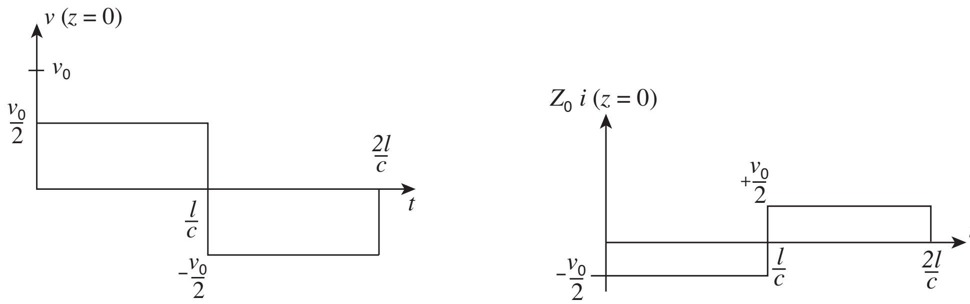


Figure 2: Voltage and current at $z = 0$. (Image by MIT OpenCourseWare.)

## C

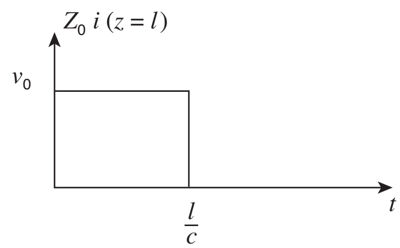


Figure 3: Current at $z = l$. (Image by MIT OpenCourseWare.)

## D

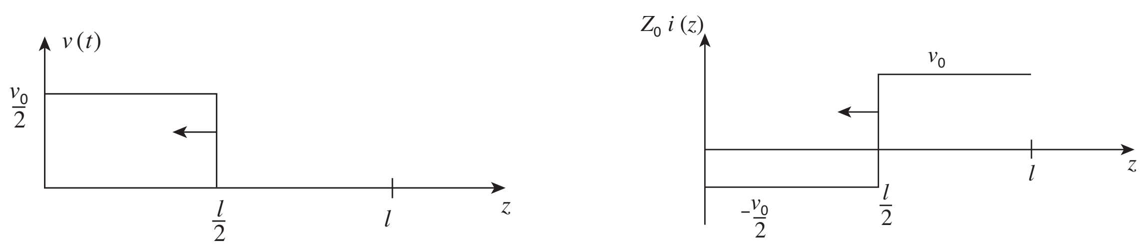


Figure 4: Voltage and current for $0 < z < l$ at time $t = T/2$. (Image by MIT OpenCourseWare.)

# Problem 8.3

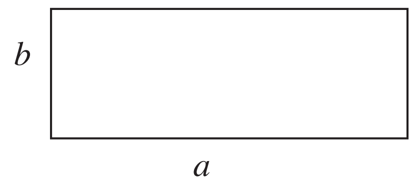

Figure 5: Rectangular waveguide. (Image by MIT OpenCourseWare.)

## A

The mode that has the lowest cutoff is the 1, 0 mode for that mode a corresponds to a half wavelength.

$$\frac{\lambda}{2} = a \implies (2a)f = c \implies f = \frac{c}{2a}$$

## B

The next modes with a higher cutoff frequency would be the 0, 2 and 1, 0 modes.

$$f = \frac{c}{2b} = \frac{c}{4a}$$

The range of frequencies where only one mode propagates are

$$7.5 \times 10^9 \text{ Hz} < f < 15 \times 10^9 \text{ Hz}.$$

# Problem 8.4

## A

$$\sigma_S = \hat{\mathbf{n}} \cdot [\mathbf{D}_1 - \mathbf{D}_2]$$

$$\text{TE}_n : \text{tangential } E \text{ only} \implies \sigma_S = 0$$

$$\text{TM}_n : \sigma_S = +\varepsilon k_x \cos(n\pi)\cos(\omega t - k_x x) = +\varepsilon k_x (-1)^n \cos(\omega t - k_x x) \qquad \text{on top plate}$$

$$\sigma_S = -\varepsilon k_x \cos(\omega t - k_x x) \qquad \text{on the bottom plate}$$

## B

$$\mathbf{k} = \hat{\mathbf{n}} \times [\mathbf{H}_1 - \mathbf{H}_2]$$

$$\begin{aligned} \text{TE}_n : \ \mathbf{K}_{\text{top}} &= \frac{E_0}{\eta k}[k_z \cos(n\pi)\cos(\omega t - k_x x)]\mathbf{i}_y \\ &= \frac{E_0}{\eta k}(-1)^n k_z \cos(\omega t - k_x x)\mathbf{i}_y \\ \mathbf{K}_{\text{bottom}} &= -\frac{E_0}{\eta k}[k_z \cos(\omega t - k_x x)]\mathbf{i}_y \end{aligned}$$

$$\text{TM}_n: \ \mathbf{K}_{\text{top}} = -\frac{E_0}{\eta}(-1)^n \cos(\omega t - k_x x)\mathbf{i}_x \qquad \text{top plate}$$

$$\mathbf{K}_{\text{bottom}} = \frac{E_0}{\eta}\cos(\omega t - k_x x)\mathbf{i}_x \qquad \text{bottom plate}$$

## C

$$\frac{dz}{dx} = +\frac{k_x}{k_z}\tan\left(\frac{n\pi}{d}z\right)\tan(k_x x)$$

$$\begin{aligned}
&\Longrightarrow \int k_x \tan(k_x x)\ dx = \int k_z \cot(k_z z)\ dz \\
&\Longrightarrow +\ln[\cos(k_x x)] = -\ln[C\sin(k_z z)] \\
&\Longrightarrow \frac{1}{\cos(k_x x)} = C\sin(k_z z)
\end{aligned}$$

$$C = \frac{1}{\cos(k_x x_0)\sin(k_z z_0)} \Longrightarrow \frac{\sin(k_z z)\cos(k_x x)}{\sin(k_z z_0)\cos(k_x x_0)} = 1$$

## D

$$\frac{dz}{dx} = \frac{E_z}{E_x} = +\frac{k_x}{k_z}\cot(k_z z)\cot(k_x x)$$

$$\begin{aligned}
&\Longrightarrow \int k_z \tan(k_z z)\ dz = \int k_x \cot(k_x x)\ dx \\
&\Longrightarrow -\ln(\cos(k_z z)) = \ln(C\sin(k_x x)) \\
&\Longrightarrow \frac{1}{\cos(k_z z)} = C\sin(k_x x)
\end{aligned}$$

$$\Longrightarrow \frac{\sin(k_x x)\cos(k_z z)}{\sin(k_x x_0)\cos(k_z z_0)} = 1$$

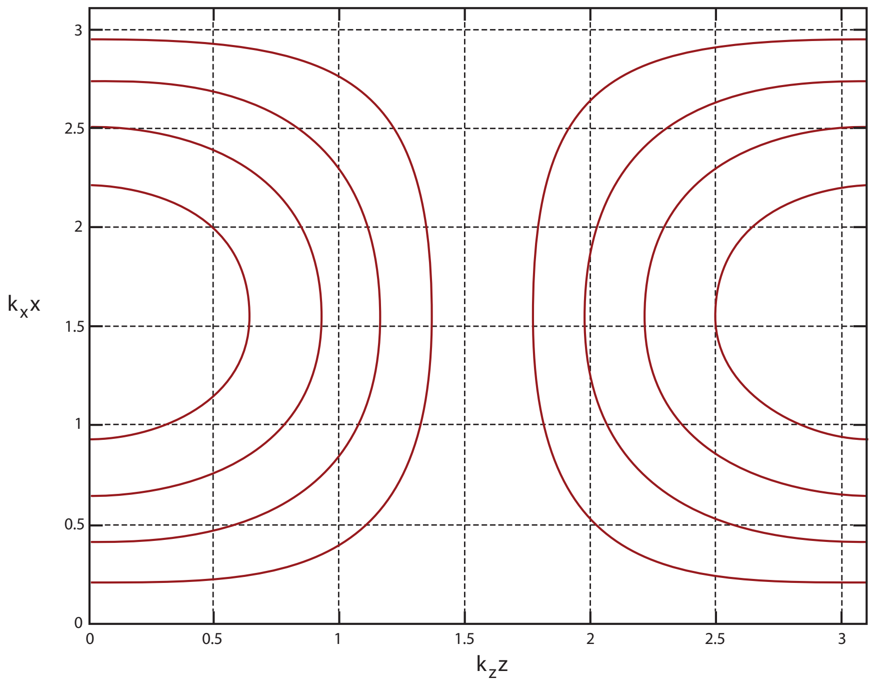


Figure 6: Magnetic field lines for $TE_1$. (Image by MIT OpenCourseWare.)

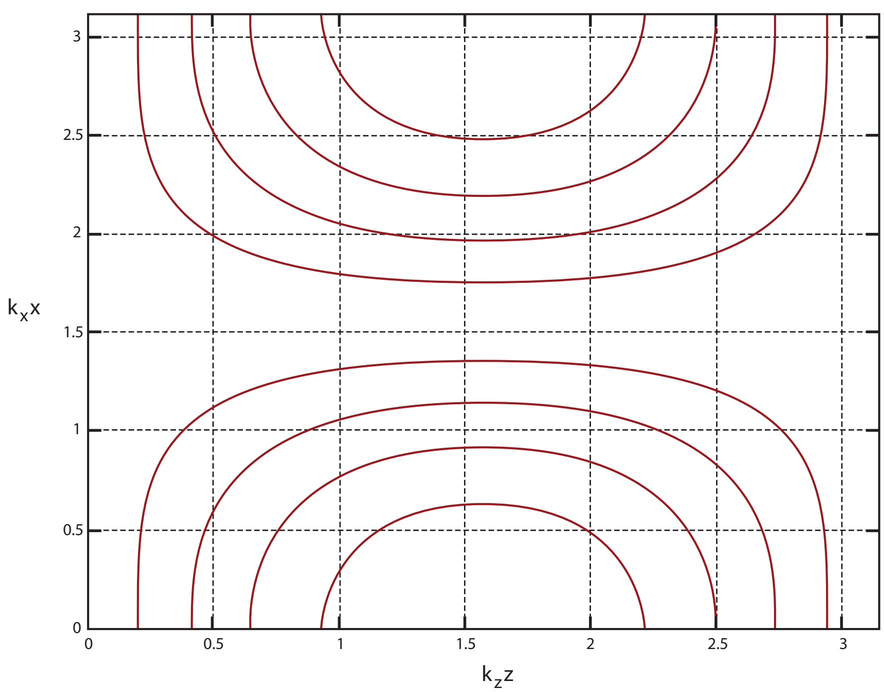


Figure 7: Electric field lines for $TM_1$. (Image by MIT OpenCourseWare.)