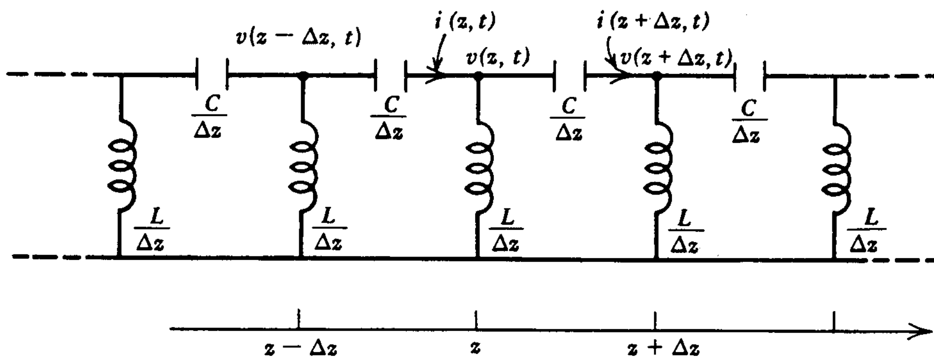
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications

Problem Set #7 | Issued: 10/25/05
Fall Term 2005 | Due: 11/2/05

Suggested Reading Assignment: Sections 5.2, 10.6.4

Problem 7.1

An unusual type of distributed system is formed by series capacitors and shunt inductors.



(a) What are the governing partial differential equations relating the voltage and current? **Hint:** Review Lecture 10, pp. 2-3 (Section I.C.)

(b) What is the dispersion relation between ω and k for signals of the form $e^{j(\omega t-kz)}$?

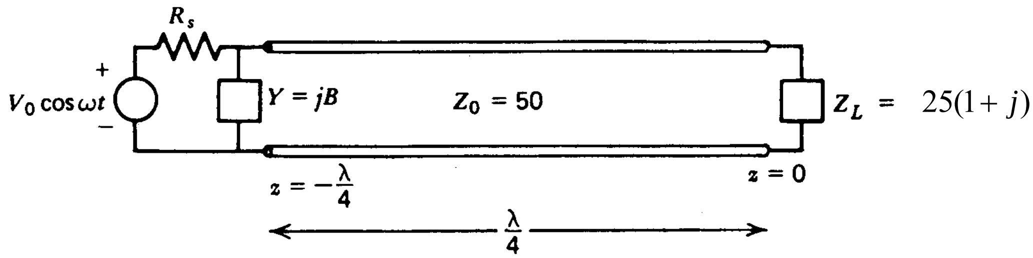
(c) What are the group $(d\omega/dk)$ and phase velocities (ω/k) of the waves? Why are such systems called "backward wave"?

(d) A voltage $V_0 \cos\omega t$ is applied at $z = -l$ with the $z = 0$ end short circuited. What are the voltage and current distributions along the line?

(e) What are the resonant frequencies of the system?

Problem 7.2

For the transmission line shown, the length of the line is ¼ wavelength $(\lambda/4)$ at the driving frequency ω of the voltage source.



(a) Find the values of lumped reactive admittance $Y = jB$ and non-zero source resistance R_s that maximizes the power delivered by the source. **(Hint:** Do not use the Smith chart.)

(b) If the lumped reactive admittance $Y = jB$ is made from a short circuited transmission line of length l and characteristic impedance $Z_0 = 50\Omega$, what is l in terms of wavelength λ, i.e., $l = a\lambda$, what is a?

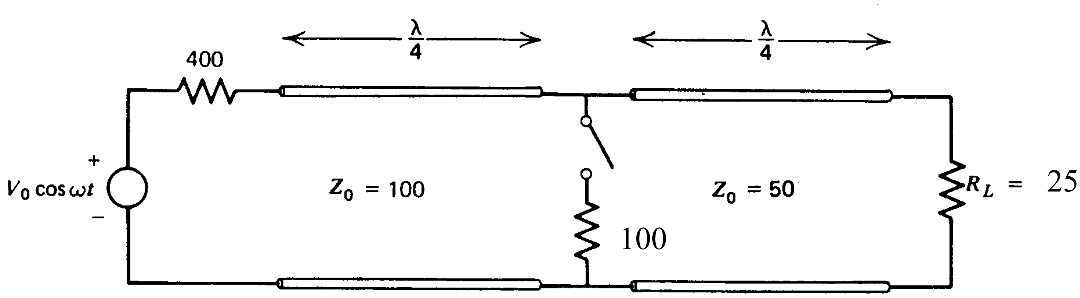
(c) What is the time-average power dissipated in the load?

(d) The driving frequency of the voltage source is now doubled. What is the transmission line length in terms of wavelengths λ at the frequency 2ω? Repeat (a) to (c).

Adapted from Problem 8.19 in *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, 1987. Used with permission.

Problem 7.3

(a) Find the time-average power delivered by the source for the transmission line system shown below when the switch is open or closed. **(Hint:** Do not use the Smith chart.)

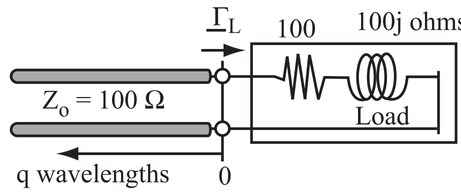


(b) For each switch position, what is the time average power dissipated in the load resistor R_L?

Adapted from Problem 8.20 in *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, 1987. Used with permission.

Problem 7.4

A 100-ohm TEM transmission line operating at frequency f is terminated with a load consisting of a 100-ohm resistor in series with an inductor having a reactance of 100j, as illustrated. Additional details and values are shown in the figures.



a) In terms of the complex reflection coefficient $\underline{\Gamma}_L$ of the load, what fraction A of the power incident upon the load is reflected?

b) For this load what is the numerical value of the complex reflection coefficient $\underline{\Gamma}_L = a + jb$?

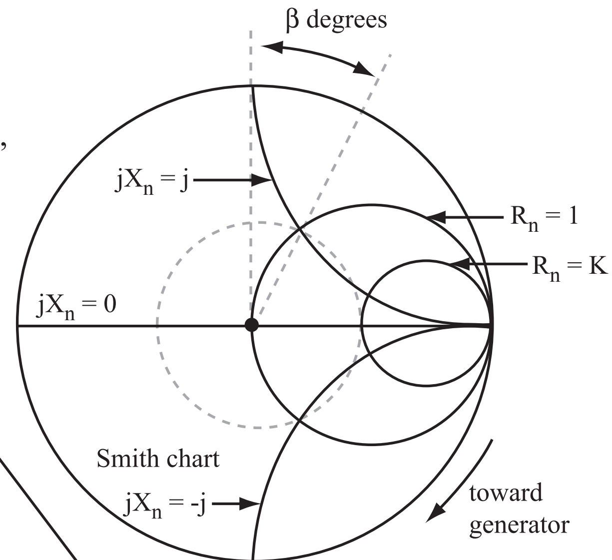
c) At what fraction of a wavelength $q = D/\lambda$ (and in terms of β, see figure), is the distance D from the load of the first point where $\underline{Z}(z)$ is purely real?*

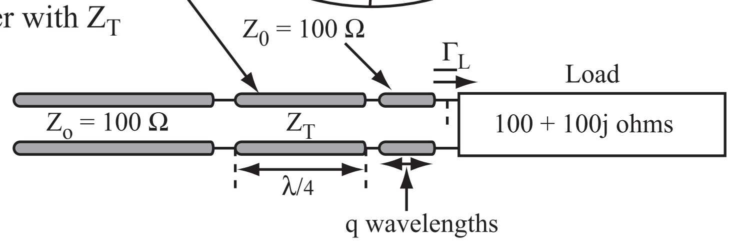
d) To match this load a quarter-wave transformer is inserted at the first point where $\underline{Z}(z)$ is purely real. In terms of K, what should be the characteristic impedance Z_T of the quarter-wave transformer (see figure)?

e) What is K?

f) Find another set of values for q, K and Z_T that allow this load to be matched with a quarter-wave transformer with Z_T real.

***Hint** for part (c): Use Smith Chart given below.





The Complete Smith Chart

Black Magic Design