Vector picture for first- and second-order statistics; MMSE and LMMSE estimation

6.011, Spring 2018

Lec 13

Covariance and correlation

Covariance: $\sigma_{X,Y} = E[(X - \mu_X)(Y - \mu_Y)]$

$$= \underbrace{E[XY]}_{\text{Correlation } r_{X,Y}} - \mu_X \mu_Y$$

Shorthand notation: σ_{XY}, r_{XY}

Correlation coefficient

Effect of shifting and scaling: If $V = \alpha(X - \beta)$

then $\mu_V = \alpha(\mu_X - \beta)$, ${}_V = \alpha\sigma_X$

If $W = \gamma(Y - \delta)$ then $\sigma_{VW} = \alpha\gamma\,\sigma_{XY}$

For a shift- and scale-invariant measure:

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \rho_{VW}$$

A geometric picture

Think of X and Y as vectors, with inner product $E[XY]$

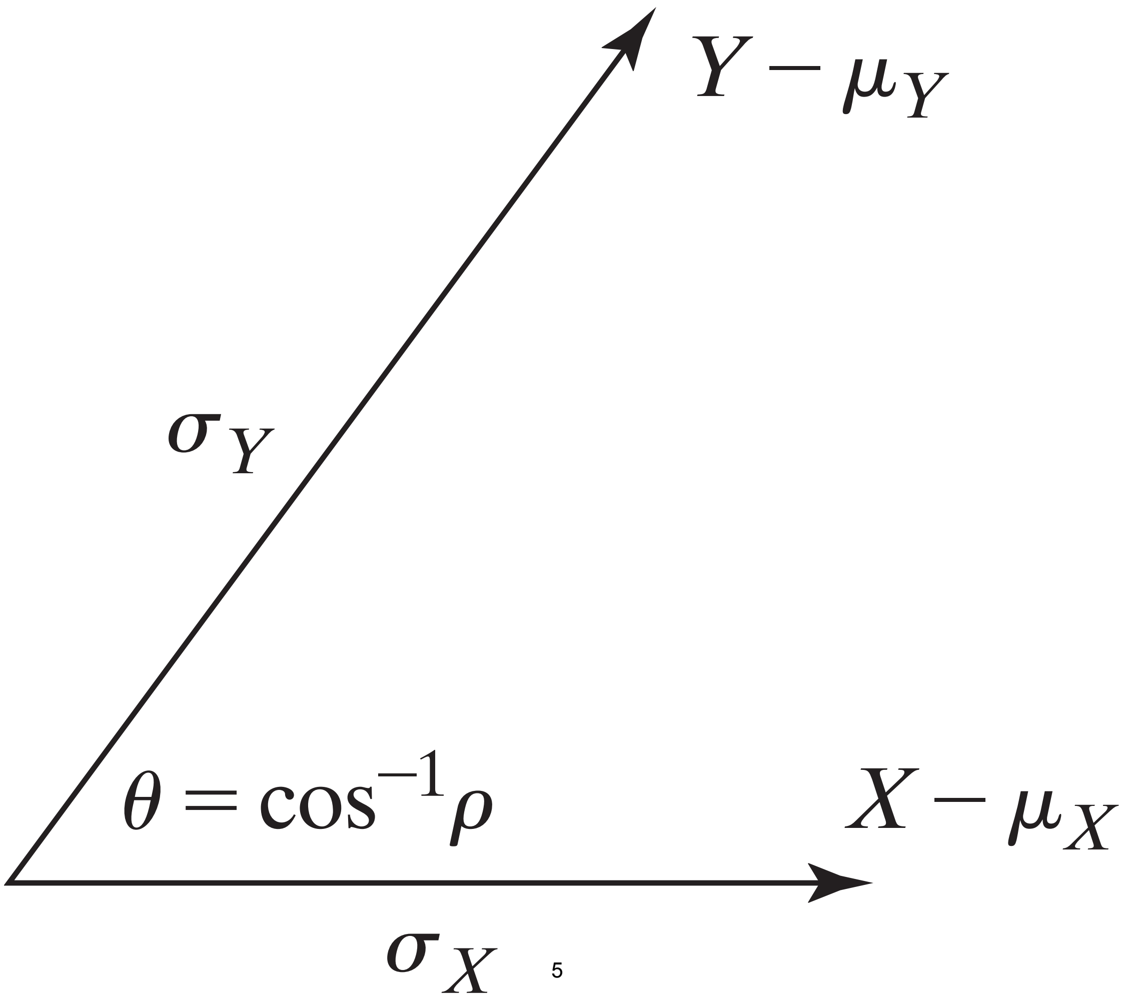
$\mu_X = E[X.1]$: inner product of X and "random variable" 1

$E[X^2]$: squared length of X

$\widetilde{X} = X - \mu_X$: vector difference between X and "random variable" μ_X

σ_X : length of $\widetilde{X}$

Geometric interpretation of correlation coefficient



Orthogonality

$$E[XY] = 0$$

Correlation is 0, but not uncorrelated!

Uncorrelated = zero **covariance**, i.e., $E[XY] = E[X]E[Y]$