Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.011: Introduction to Communication, Control and Signal Processing

QUIZ 1, March 16, 2010

ANSWER BOOKLET

Your Full Name:	
Recitation Time :	o'clock

- This quiz is **closed book**, but **two** sheet of notes are allowed. Calculators will not be necessary and are not allowed.
- Check that this **ANSWER BOOKLET** has pages numbered up to 14. The booklet contains spaces for **all** relevant reasoning and answers.
- **Neat work and clear explanations count; show all relevant work and reasoning!** You may want to first work things through on scratch paper and then neatly transfer to this booklet the work you would like us to look at. Let us know if you need additional scratch paper. **Only** this booklet will be considered in the grading; **no additional answer or solution written elsewhere will be considered.** Absolutely no exceptions!
- There are **4 problems, weighted as shown**. (The points indicated on the following pages for the various subparts of the problems are our best guess for now, but may be modified slightly when we get to grading.)

Problem	**Your Score**
1 (5 points)	
2 (15 points)	
3 (10 points)	
4 (20 points)	
Total (50 points)	

Problem 1 (5 points)

$H(e^{j\Omega}) = \exp\{-j(60\Omega + 25\Omega^3)\}$ for $|\Omega| < \pi$. In this frequency range,

1(a) (1 point) the magnitude $|H(e^{j\Omega})|$ is:

1(b) (1 point) the phase $\angle H(e^{j\Omega})$ is:

1(c) (1 point) the group delay $\tau_g(\Omega)$ is:

1(d) (2 points) The impulse response $h[n]$ of this filter is given by the plot in (choose A or B or C or D or E):

Two different features of this response that support choosing it as $h[n]$ are:

Problem 2 (15 points)

The deterministic autocorrelation function of $x(t)$ is

$$\overline{R}_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t-\tau)\,dt = 9\frac{\sin(2\tau)}{\pi\tau}\ .$$

2(a) (3 points) The energy spectral density $\overline{S}_{xx}(j\omega)$ of this signal is given by the following — careful and fully labeled! — sketch:

The energy of the signal, $\mathcal{E}_x = \int_{-\infty}^{\infty} x^2(t)\,dt$, is:

$\mathcal{E}_x =$

(Show in both the time domain and frequency domain how you computed this.)

2(b) (2 points) The magnitude of the Fourier transform of $x(t)$, i.e., $|X(j\omega)|$, is given by the following — careful and fully labeled! — sketch (be sure to explain your answer):

2(c) (2 points) One possible signal $x(t)$ that has the specified deterministic autocorrelation function $\overline{R}_{xx}(\tau)$ is:

$x(t) =$

(Be sure to explain your reasoning!)

2(d) (2 points) The relation (in either the time domain or the frequency domain) between any other answer to 2(c) and the specific one you gave in 2(c):

2(e) (3 points) The $x(t)$ above is the input to an ideal lowpass filter that has gain 1 for frequencies ω satisfying $|\omega| < 1$, and gain 0 elsewhere. The corresponding output of the filter is $y(t)$, with energy spectral density $\overline{S}_{yy}(j\omega)$ given by the following — careful and fully labeled! — sketch (be sure to explain your answer):

The energy of $y(t)$, namely $\mathcal{E}_y$, is:

$\mathcal{E}_y =$

2(f) (3 points) Suppose another signal $f(t)$ has deterministic autocorrelation function

$$\overline{R}_{ff}(\tau) = \int_{-\infty}^{\infty} f(t)f(t-\tau)\,dt = \cos(10\tau)\,\frac{\sin(2\tau)}{\tau}\,.$$

The magnitude of the Fourier transform of the signal, i.e., $|F(j\omega)|$, is derived below, and given by the following — careful and fully labeled! — sketch:

The value of $\int_{-\infty}^{\infty} x(t)f(t-\tau)\,dt$ is:

(Explain your reasoning carefully.)

Problem 3 (10 points)

Note $x_c(t) = \sin(2\pi f_1 t)$, with $f_1 = 1300$ Hz.

3(a) (1 point) The minimum value that the sampling frequency $1/T$ has to exceed, in order to avoid aliasing at the C/D converter, is:

For each of the following parts, fully specify what the output $y_c(t)$ is for the indicated choice of the sampling/reconstruction frequency $1/T$. Show how you obtain your answers!

3(b) (3 points) When $1/T = 8000$ Hz, the output $y_c(t)$ is:

$y_c(t) =$

3(c) (3 points) When $1/T = 4000$ Hz, the output $y_c(t)$ is:

$y_c(t) =$

3(d) (3 points) When $1/T = 1600$ Hz, the output $y_c(t)$ is:

$y_c(t) =$

Problem 4 (20 points)

$$\dot{\mathbf{q}}(t) = \begin{bmatrix} 0 & 1 \\ 6 & 1 \end{bmatrix} \mathbf{q}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(t)$$

$$y(t) = [\ 1 \quad 1\ ]\, \mathbf{q}(t)\ .$$

4(a) (3 points) The eigenvalues and associated eigenvectors are:

$\lambda_1 =$ $\qquad\qquad$ $\mathbf{v}_1 =$

$\lambda_2 =$ $\qquad\qquad$ $\mathbf{v}_2 =$

Check your answers carefully!

4(b) (1 point) Is the system asymptotically stable? Explain your answer.

4(c) (3 points) The transfer function $H(s)$ of the system is:

$H(s) =$

What feature of this $H(s)$ tells you the system is reachable and observable?

4(d) (4 points) When the initial state of the system is at zero, i.e., when $\mathbf{q}(0) = \mathbf{0}$, and when the input for $t \geq 0$ is $x(t) = 5e^{-t}$, the output $y(t)$ of the system for $t \geq 0$ is

$$y(t) = e^{3t} - e^{-2t} \; .$$

(You can use this fact as a check on some of your preceding answers!) What initial condition $\mathbf{q}(0)$ would we need in order to have $y(t) \equiv 0$ for $t \geq 0$, with this same input $x(t) = 5e^{-t}$ for $t \geq 0$. Be sure to explain your reasoning!

$\mathbf{q}(0) =$

4(e) (3 points) Suppose we implement a state feedback control of the form

$$x(t) = \mathbf{g}^T \mathbf{q}(t) + p(t)$$

on the given system, where the feedback gain vector is $\mathbf{g}^T = [g_1 \quad g_2]$, and $p(t)$ is some new external input. Write down the resulting state-space model, with input $p(t)$, state vector $\mathbf{q}(t)$, and output $y(t)$. You need to show the model in detail, making explicit the entries of all matrices and vectors involved in the state evolution equation and the output equation.

$$\dot{\mathbf{q}}(t) = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \mathbf{q}(t) + \begin{bmatrix} \quad \\ \quad \end{bmatrix} p(t)$$

$$y(t) = [\qquad\qquad] \, \mathbf{q}(t) + [\qquad] \, p(t) \,.$$

4(f) (4 points) For your closed-loop system in (e), determine what choice of g_1 and g_2 will result in a closed-loop characteristic polynomial of

$$\nu(s) = s^2 + 3s + 2 \ .$$

Is the resulting system observable? Be sure to show your reasoning.

$g_1 =$ $\qquad\qquad\qquad\qquad$ $g_2 =$

Is the resulting system observable?

4(g) (2 points) With the state feedback gains picked as in (f), suppose $p(t)$ in the closed-loop system is kept constant at the value $\overline{p} = 6$ for all time. What is the corresponding equilibrium value $\overline{\mathbf{q}}$ of the state vector $\mathbf{q}(t)$?

$\overline{\mathbf{q}} =$

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