DENNIS FREEMAN: Hello, and welcome. So like last time, the important announcement is of course that we have an exam coming up. So exam 2, Wednesday, everything's very similar, analogous to last time, with the exception that we are in Walker. Don't go to Building 26. Go to Walker.

Other than that things look very similar to before. The idea will be that the exam will focus on things since exam 1. However, this subject builds on itself. So the official coverage is 1 the 12, lectures 1 to 12, recitations 1 to 12, and homeworks 1 to 7. Questions? Comments? All the logistical issues are clear?

AUDIENCE: I had one question. [INAUDIBLE] how much do we have to know about [INAUDIBLE].

DENNIS FREEMAN: So there was an introduction to root locus. Sorry, so everybody knows what root locus is? You can find out after class. You know, it will just making me feel a lot better if you nod your head yes. Root locus is the way you figure out the locus of possible points that a pole could move to.

There is an enormous number of rules for thinking about how root locuses evolve. And if you're really good signals and systems person, you know 50 rules. That's not the point of this class.

You should know that poles move around when there is feedback. We'll have examples of that in class today. You should be able to calculate how they move around for simple problems.

You're not expected to know the rules. OK, so you shouldn't be surprised by the rules. If we ask you to derive one of the rules, you probably could. But we don't expect that you've memorized the rules. Does that sounds roughly correct?

OK, other questions or comments? Content, administration, anything like that?

OK, so about a week ago, Russ introduced the idea of using feedback for the purpose of controlling. And the example in lecture was controlling a robot. You've seen that before. Right? That's 6.01.

And then in recitation yesterday, he ran over some more ways of thinking about more sophisticated ways to do control with feedback. And the point of those more sophisticated ways to do control is really performance. The idea is to enhance performance. And that's the theme that I want to build on today, today and on Tuesday. How do you enhance performance by using feedback?

And we'll go through a number of examples. You can use feedback to increase speed and bandwidth. You can use feedback to change the control variable from-- I'll do an example-- speed to position. You can reduce distortion, reduce sensitivity to parameter variation. You can stabilize unstable systems.

We'll do a bunch of examples. The point is that you can use feedback to accomplish a lot of different tasks that generally speaking improve performance. So with that, let me think about an op-amp. You all know about op-amps, right? That's also a 6.01 sort of thing.

So an ideal op-amp would have a bunch of properties that we would like. We'd like it to have-- it we'd like it to be very high speed. We'd like it to have a very broad bandwidth. We'd like it to be fast. We'd like it to have a very low output impedance so it works like a perfect voltage source. We like it have a very high input impedance so that when we hook it up to something it doesn't change the something that we hooked it up to.

There's lots of things we'd like it to have. Unfortunately, it's difficult to build a circuit that has all of those things. And so here is an example of that. Here's one of the world's most popular op-amps, the LM741, invented eons ago and still used today.

The idea that's plotted here is the frequency response. You remember frequency response. What's the frequency response of an LM741? Well, if you plot as a function of frequency, how does the gain change?

Well, the gain is enormous. High gain, that's what we'd like it to have. The gain is enormous at low frequencies, at 1 radian per second. But the gain becomes very small out at high speeds at the speeds we think we think about when we're thinking about communication systems or when we're thinking about computation or things. When we think about speeds that are electronic speeds, the gain is actually pretty small.

Similarly, the phase changes. The phase of very low frequencies is near 0. That's kind of ideal. 0 would mean in phase. The output reliably tells you what's the input doing. So that's good.

But as you go to higher frequencies, you start to pick up a phase delay, which means the output is telling you what the input did a little while ago. OK, that's less good. So the performance in terms of providing a lot of gain is quite good for some frequencies, but not for others.

Unfortunately, the range where it's not very good is most of the range. So if you thought about an op-amp doing even a very low frequency problem, like audio signal processing, most of the frequencies that are of interest to audio are in the region where the op-amp is not working too well.

We hear sounds from about 20 Hertz to about 20 kilohertz. And that range is indicated by the red arrow. And you can see that's not where the op-amp is working ideally.

Similarly, we would like the op-amp to be fast. Fast is indicated here by thinking about the step response. So if you thought about having an ideal step at the input, what would the output look like?

Well, we'd like it to be an ideal step, but it generally isn't. Generally, there's some lag. That's associated with that phase delay that I talked about. And in fact, what I'd like you to do is figure out exactly how it's associated with that phase delay.

Think about-- so figure out-- it will turn out that the step response can be characterized by a time constant tau. The time constant tau is the time required for the signal to get within 1 over e of this final value determined from this. I've already told you the performance parameters in terms of frequency. You should be able to use 6.003 to figure out what that implies about time.

So look at your neighbor-- OK, look, look, look. And now, figure out which answer-- what's tau for the 741 op-amp?

[SIDE CONVERSATION]

OK, it's dead quiet. So I assume that means convergence. So what's the answer? What's the time constant tau of the 741? Raise your hand with a number between 1 and 5. Don't look at other people, just raise your hand.

OK, I can see a couple of answers I don't like. How do I think about converting this representation to that representation? What should I do first? Shout. I start with the thing on the right. What should I do next?

AUDIENCE: [INAUDIBLE] of like doing a 2 over tau or something like that?

DENNIS FREEMAN: So we're looking for something like e to the minus t over tau. That's right. We're looking for something like that. How do I get that starting with the thing on the right?

AUDIENCE: It's like using poles.

DENNIS FREEMAN: Poles, what a good idea. How do I figure out poles from the thing on the right? 6.003, poles, that should be like hard wire. Right? So when you hear the word 003, you should think the word pole. How do you associate a pole? Can somebody tell me what the poles of the right-hand system should look like?

AUDIENCE: Where the line is.

DENNIS FREEMAN: Where the line is-- there's a couple of lines there.

AUDIENCE: The vertical line.

DENNIS FREEMAN: The vertical line-- how many poles are on the right? How many poles are in the system represented by the right-hand figure?

AUDIENCE: One.

DENNIS FREEMAN: One. How many zeros? Zero, wonderful. So we have to think about a pole, just one pole. So we think about the s-plane. Right? Where's the pole? You already told me.

AUDIENCE: 80 halves, so it was 40.

DENNIS FREEMAN: 40-- OK, where's 40 over here? Somewhere in there. Got it. Where's the pole over here? It's at location 40. That's actually correct, well, sort of. Negative 40, that's exactly correct. So negative 40, so the pole is right here.

OK, so the pole is at minus 40 radians per second. How do you know that? Well, that's that Bode stuff we talked about last time. Right?

So if the pole were here, you would think about a vector. So this is the j omega axis. Right? You would think about a vector connecting the pole to some location on the j omega axis. If the location is omega small, it's going to have some-- that vector is going to have some length like that. It's going to have an angle of 0.

And as frequency goes up, the vector gets longer, which means that the magnitude gets smaller because poles and bottom. Right? And the phase goes increasingly toward pi over 2. But it's in the bottom. So it's minus pi over 2.

Right, so the frequency at which you get to pi over 4-- OK, 0pi over 2-- the frequency at which you get to pi over 4 is-- so pi over 4 is here. Right? So the distance here, the critical frequency, the frequency at which the phase is pi over 4, minus pi over 4, is going to be the same as the frequency of the pole. So that's how we know it's at minus 40. Right?

So we know that this then is a characterization of that system. It's not quite done. So what's the h of s for this system? If I've got a pole at minus 40, what's h of s look like? Shout, shout. These answers coming up, this is perfect practice.

AUDIENCE: I've already talked too much.

DENNIS FREEMAN: What's h of s look like?

AUDIENCE: Is this something over s plus 40.

DENNIS FREEMAN: Exactly, something over s plus 40. What would you like the something to be? We'd like it to be some k. We'd actually like it to be related, for example, to the open-loop gain. How big is the gain when frequencies go towards 0? What's the gain at 0?

Let's say the gain of 0-- let's define the gain of 0, k0, let's define the gain at 0 to be-- so what is the gain at 0? 10 to the 5, 2 times 10 to the 5. Right? It's a little bit above the 10 to the 5 line.

OK, so now if I define k0 to be that gain at 0, what's the numerator over here? If the numerator over here were 1, what would be the low-frequency gain? 1 over 40, right?

To think about frequency, we think about 1 over j omega plus 40. Right? You substitute s equals j omega to think about frequency. And if omega is very small, you get 1 over 40. Right? So if you wanted that to be 1, you'd have to make it 40.

But we don't want it to be 1. We want it to k0. So you'd want it to be 40 k0. Or to be a little bit more general, we'd like this to be of the form alpha k0 divided by s plus alpha. Right?

So now, we've got the Laplace transform. We can figure out the time response. What's the time response associated with that Laplace transform?

You remember the impulse response of a system is related to the system function via the Laplace transform. Right? So all we need to do is take the inverse Laplace transform of that thing. And we get h of t. So what's h of t?

OK, it's such a strain, all those weeks ago. e to the minus t, e to the minus something t. So we need something like e to the minus what? Alpha. e to the M alpha t u of t. And there's going to be that-- that thing is going to come out here too. And it's going to be like that.

And then if you wanted to find the step response what would you do? h of t is the impulse response. How would you find the step response? A step is related to the impulse by integrating. The step response is related to the impulse response by, surprisingly, integrating.

So you would integrate that thing, and you would end up with something. Right? So you would get something like k0 1 minus e to the minus alpha t u of t, something like that. I think that's right.

OK, the time constant is given by alpha. Right? So what is the time constant now? 1 over 40, right? So it's alpha t. But the time constant is t over tau.

So the time constant is the inverse of frequency. Right? Frequency is per second. Time constant is seconds. They're reciprocals of each other.

OK, so the idea then is that we can associate a time response with that. And that time response is pretty slow. A 40th of a second, 40 things per second kind of. That's my speed. Right, that's not computation speed. Right? So that's pretty slow.

So we'd like to improve that. And one way that we can improve it is by thinking about feedback. So the idea would be we don't use the op-amp just out of the package the way it comes. We put it in a feedback loop.

So for example, here you could put it into a non-inverting amplifier where you take the output divided by a voltage divider and feed that back in. In 6.003, the way we'll think about that is that the effectiveness plus and minus input is to take the difference between two signals. Right, we're going to take a circuit diagram and turn it into a integrator, gain-adder diagram, a block diagram. We're going to take a circuit and turn it into a block diagram.

So the way we think about the plus and minus input of the op-amp is as a subtractor. And then this k of s that characterized-- so this thing, right, this thing here, we plug-in there just as a box. And the effect of the resistors is to divide the output signal by some constant beta.

So we can reduce the circuit to a 6.003 equivalent. And then we can use Black's equation and all those other things that we do in 6.003. So we can write an expression for the closed-loop system the looks like k of s over 1 plus beta k of s. Make sense?

So then, as we've already seen, the op-amp by itself looks like a pole at minus 40. So we can substitute this expression into that expression for the closed loop. And we get a new closed loop. So h of s, which had been alpha k0 over s plus alpha becomes-- the new value is alpha k0 over s plus alpha plus alpha beta k0. OK, so the point is that we can think about the way the feedback changes the system function by just formulating what's the circuit look like in terms of a 6.003 block diagram.

So now, the effect of that, it looks as though what happens is the pole went from here to here. How big of a change is that? What's the biggest change in the position of the pole that you can achieve using feedback?

AUDIENCE: [INAUDIBLE] distance.

DENNIS FREEMAN: I'm sorry.

AUDIENCE: [? Is ?] [? beta ?] the [INAUDIBLE] distance. Or where are they located?

DENNIS FREEMAN: I want a factor by which you multiply alpha to find the biggest place that you could have put the new pole. I didn't say that right. I didn't say that right. I want to know where is the new pole. Sorry, I confused myself.

Where is the new pole? How left can you put the new-- how left can you get the new pole to be? What's the furtherest-- left is good, right? Left is good. Everybody knows that, right? So right is bad.

So small distances in the s-plane correspond to slow responses. Big distances correspond to fast responses. That's what we saw in part one.

So we're trying to push the pole that way as far as we can. How far can we get it to go? OK, everybody raise your hands. The answer is-- and again, I see lots of answers that I don't quite like. With a minor perturbation, all those people who I don't like can become right by saying-- so a lot of people have said this, which I don't quite like.

What's the biggest distance that I can move it? OK, new vote-- I didn't like tw0. And so therefore, I will change my answer to four. Wrong, no. How big can beta be? How big can beta be?

What's the biggest possible value of beta? One. So how far can I push it to the left? So I can push it this far. Right? So if I make beta 1, that's as far as it goes. Beta 0 is kind of the rightmost position. Beta 1 is the leftmost position. I can get it that high.

But that's very good. Right? 1 plus k0-- k0 is a number like 10 to the 5th. So I can get at like 10 to the 5th bigger than it was. That's enormous.

That means that I can stretch the frequency response. If the pole is the thing that determines the frequency response by that construction up there, I can stretch the frequency response by 10 to the 5th. That's huge. Pure win, right?

Well, that doesn't sound right. So I've stretched the region where the gain is constant. I've stretched the region where the phase is close to 0. It's a 100% win except that I've cheated when I drew the plot because I normalized it by the DC gain.

DC, I probably didn't define that yet. DC is direct current. It has nothing to do with current or direct. DC means zero frequency. I have no idea why we insist on using that word, but we do. DC means zero frequency. OK, so you just do that automatic mapping in your head. Every time I say DC, I mean zero frequency.

Everybody else talks that way too. So the DC gain, I've divided the magnitude by the DC gain, h of j0. So this is the relative frequency magnitude, h of j omega, divided by the magnitude at omega equals 0. Had I not done that, the picture would have looked different.

If you think about what happens-- so the top curve here shows what happens when the-- shows the response curve for a 741 op-amp with no feedback. It has a pole at 40. It has a pole at minus 40 to be a little more correct. So there's a single pole at minus 40. That corresponds to beta 0. That's open loop, no feedback.

OK, and what you can see is the DC gain is 2 times 10 to the 5th. And there's an AC gain defined by the cutoff frequency. If you compare this expression and that expression, the interesting thing that happens is that the DC response depends critically on beta.

But the high-frequency asymptote doesn't depend on beta at all. The high-frequency asymptote of this is alpha k0 over s. The high-frequency asymptote of this is alpha k0 over s. They're the same high-frequency asymptote.

What happened was that as I increased the feedback, I lopped off the low-frequency gain, which had the effect of making the frequency look like it got very big. So pure win? No. Almost pure win? Yes.

OK, I've traded-- the trick is I've traded gain for bandwidth. That's important because the folks who know how to microfabricate things know how to make things with very high gain. Making things that are fast is harder. This is a way that we can take advantage of the relative simplicity of making things with high gain to make them behave as though they're fast.

OK, you can see the same thing if you think about the response in time. If you think about the response in time, we're going to go from e to the minus alpha t to-- I guess I didn't write it down. It's going to be e to the minus alpha 1 plus alpha beta k0 t, same factor. It's going to get 10 to the 5th times faster.

Again, it looks like pure win. But that's because I've cheated because I'm plotting via this normalized final value that depends on beta. If beta were 0, the final value is 2 times 10 the 5th. If beta is anything else, the final value is smaller.

If I plot it not normalized, you get a kind of different picture. So if you were to use the op-amp open loop, the response has a time constant of 1 over 40, as we said before. If you include feedback, it decreases the final value, but has the property that it does not change this slope.

So if you think about this, this is an analytic expression for the closed-loop step response. I get that by integrating one of these expressions. I didn't write the beta one down. I need the beta equivalent one of these. If you right the beta equivalent one of these, which is on a previous slide, and integrate it, you you get this expression.

And the if you differentiate this expression with regard to time to find the slope, the slope at 0 is unchanged by beta because the exponent here, 1 plus beta k0, has the same form as the denominator here. So when you differentiate it, this comes down. The two cancel. And you end up with a slope that is independent of beta. The effect of that is that if you change the final value, it appears to go much faster.

So pure win? No. Apparent win? Yes. We've traded gain for speed. OK, so that's ways that we can use feedback to improve the performance of an op-amp.

OK, the big performance parameters that I talked about today were bandwidth and speed. You could do the same kind of analysis with many different kinds of metrics for op-amps like output impedance, input impedance, distortion reduction. We'll do distortion next time.

| there's lots of other things that feedback improves. I've illustrated it with two. In both of those, as will be the case in all of the others, the effective feedback is to trade gain for something else of interest.

OK, the next example I want to think about is thinking about a motor controller where what I want to think about is changing the way we think about the input-output relationship. So I take a motor. That's my motor. And what I want to do is have it control position.

So it's not too easy to see probably because it's such a tiny little motor. Actually, it's not a tiny little motor. It's an enormous motor.

So the red bar, that's the shaft. Right? And so the first question I want to think about is what's the relationship for the motor? What's the relationship for the motor for the transformation from voltage to angle of the shaft? So what's the relation between v of t and theta of t for DC motor?

This is very much the same problem you worked in in the head-turning problem in 6.01. So you should be able to remember this answer from 6.01. What's the relationship between the voltage into a DC motor and the angle out? Is it one, two, three, four, or none of the above? Talk to your neighbor. Figure out the right answer.

[SIDE CONVERSATION]

So what's the answer-- one, two, three, four, or none of the above? Everybody votes. You're on such a streak. More than half the answers, I really don't like. OK, so we'll do a demo, and we'll figure out.

So the question is, what is the relationship between voltage and speed? So I've got a motor, and I've got a knob that controls the voltage that goes to the motor. OK, so now, I turn the motor on, and nothing happens because I preset the voltage to be 0.

Now, I turn the voltage to the right to make the voltage a bit positive, more positive, back towards 0, at 0, negative, more negative. OK, so what's the relationship between voltage and angle?

AUDIENCE: Is that velocity or voltage?

DENNIS FREEMAN: v is voltage, sorry. v is voltage. Theta is angle. So here's 0. Here's 90. Here's 0, pi over 2, pi. Right? The question is, what's the relationship between the voltage input of the motor and the angle of the shaft the output?

And now, the answer is-- OK, the number correct has not changed. So tell me in words-- forget the one, two, three, four-- what's the relationship between voltage and angle? What is [INAUDIBLE] of the voltage?

AUDIENCE: [INAUDIBLE]

DENNIS FREEMAN: Angular velocity. So how do I get angular velocity?

AUDIENCE: [? Measure ?] of the angle.

DENNIS FREEMAN: So I'm giving you the angle, theta of t. How do I get angular velocity?

AUDIENCE: Derivative of the angle-- the derivative.

DENNIS FREEMAN: Take the derivative, yes. So what I want is that theta dot proportional to v. OK? Which of those relationships say that? None of them. OK, it was a trick question of course.

So what do I want to have as my model for the DC motor? I want my model to take a voltage in and give me theta out. What should be in the model? k, it's always good to throw a k in. That always works. Excellent answer. k always works, sure. In any physical system, you never get one, right? Yeah.

AUDIENCE: [INAUDIBLE]

DENNIS FREEMAN: You're going to need an integrator, exactly right. So it's going to need something like I think I call it a gamma, a being the accumulator guy. Right? A different way of saying that would be--

or I write it in terms of Laplace transforms, I would get gamma over s. So if I put v in, and I want to get theta out, right, I need to put it through an integrator.

So the idea is that if I put a constant voltage in-- say the voltage is 0-- theta was flat. But if I put a voltage at the blue line, theta increased at some rate. And if I changed it to red, it increased faster just like an integrator would do. So what I want to do is think about the motor being an integrator.

OK, that lets me then cast the motor, which is a physical thing, into 6.003 terms again. So now, I can think about the problem. I'd really like it not to be a speed controller. I'd really like it to be a position controller.

So feedback to the rescue-- what I'll do is I'll put it in a loop. So the idea is going to be that I take my motor, which looks like an integrator, gamma a, feedback a signal that's proportional to the angle, which I derive by having a potentiometer strapped to the back of the shaft-- so there's a pot. I've taken off the little stop so that it spins around doesn't hit the stop. Right? Otherwise, it would be kind of bad for the first part of the demo.

So it's a surgically altered potentiometer that spins around forever. But over most of the range, it's reporting angle. So that's the way I get this feedback loop in.

And then I put it into this thing, which is really just a 741 wired up like I showed in the first example, so that it would take the difference between two things, the desired input, which is a potentiometer that I'm turning here, and the actual position, which is the potentiometer up here, the same kind of potentiometer, just one that I turn, the other that the motor turns. And the op-amp figures out the difference, multiplies it by a gain, alpha, and presents it to the motor. That's the idea.

And if you just think about 003, it's pretty easy to see that you can represent this relationship by a single pole. Right, there's some annoying constants. Right, you have to worry about the gain of the amplifier, the gain of the motor, and how much feedback. That's alpha, beta, gamma. But pretty much, it's just a pole.

So what happens then as you change the gain? Well, if the gain is 0, the pole-- so if the gain, beta, is 0, the pole is at 0. OK, it works just like an integrator. That's what we said before. That's a sanity check where you check something you already know the answer to. Right?

So if you set beta to 0, you get that the transformation is an integrator, good. If you set beta and anything else, what happens? Well, the pole goes screaming out that way. That's good, right?

Remember good is that way. Right? That's not always true. But when you're trying to make something work fast, that's the rule. Right? Fast is over there.

So the idea then would be that if I hook up this circuit-- right, I take the feedback, run it into an op-amp and multiply it, use that to control the motor instead of my turning the knob, then this thing should have the behavior that the pole goes from the origin to the right. So the question is, if that happened, what would happen?

OK, just doing the same kind of analysis we did for the op-amp, the step response goes from being the response of a pole, which is an integrator, to being this kind of a response, which has a final value. That converts it from being voltage to velocity into voltage to angle. Does that make sense?

It used to when the feedback was 0, the pole was at 0. So the output continuously rose. There was no steady-state position.

But now with feedback, I get a step response that has a final value. The effect of moving the pole is the same as the effect of moving the pole. In the op-amp example, I get a step response that becomes increasingly like a step. It goes from being a sluggish step to a fast step.

OK, so that's the way I use 003 to use feedback to fix the motor. And I've done that. So I have to move the input to the amplifier from my pot to the op-amp, which is there hopefully. So now when I turn it on, it becomes a position.

OK, and I can test that by turning the pot. And in fact, when I turn the pot, the motor moves. Wonderful, voila, I won. Right? Great. Yes, applause please. Come on.

[APPLAUSE]

Exactly, exactly. OK, well, it's pretty wimpy actually because I've got this motor that weighs about, I don't know, three pounds. And it's exerting about a quarter of an ounce of torque. It's not really very impressive. So how do I fix it? What do I do? How do I fix it?

I have a wimpy controller. Yes.

AUDIENCE: More gain.

DENNIS FREEMAN: More gain, of course. So remember that response. So now, I turn off the motor and increase the gain by a factor of three. And the answer is much better, well, maybe.

So what do I do? More gain, of course. So another factor of three-- much better, well, maybe. So another factor of three. OK, another factor of three. Hmm, another factor of three.

Some of you close may hear that there's a little buzzing sound now. I can hear it. That's the op-amp killing itself.

OK, it's not working. Why is it not? Is it working? I just gave it away. It's not working. How do I know it's not working?

Here's my theory. Here's my answer. How do they match or mismatch? Is it an angle controller? Yes or no? Yes.

AUDIENCE: Didn't we just do this?

DENNIS FREEMAN: Yeah, it's an angle controller. What's different about this behavior and that behavior?

AUDIENCE: There's oscillation.

DENNIS FREEMAN: There's oscillation. Right, this thing, turn it back on. Let the thing kill itself. There's an oscillation. Turn the gain down so it doesn't kill itself quite as quickly. It's oscillating.

Watch the oscillation. Think about characterizing the oscillation. Make the gain smaller. That's bad. Ignore that. Think about the oscillation. It's still oscillating. Factor of three smaller, it's still oscillating.

That doesn't oscillate. What's wrong? OK, so the $64,000 question, what did I do wrong? Right, the goal is to make a model of the motor, analyze the model, figure out how to put feedback around it, make it perfect. Yes.

AUDIENCE: The model on the board was created without any inertia.

DENNIS FREEMAN: I made too simple of a model for the motor. I ignored inertia. I ignored all manner of things. I ignored friction. So how can you tell that I ignored those things?

So one way you can tell-- let's see. Is it off or on? It's on. Or turn it off. Go back to the original configuration where it's a speed controller.

Turn the speed on a little bit. Turn it off. Grab it with my fingers. Turn it on. Now, release it. What did you see? Grab it with my fingers. Turn it on. Let go.

We're trying to poke a hole in this. There is my model. My model is wrong. I claim I just told you the key experiment for figuring out why my model is wrong. Now you have to figure out how to interpret that experiment.

So what does my theory say would happen if I turn on the power, hold the bar, and release it?

AUDIENCE: [INAUDIBLE].

DENNIS FREEMAN: It should instantly reach the terminal velocity. What's it doing?

AUDIENCE: I has just like a rise time [INAUDIBLE].

DENNIS FREEMAN: It's ramping up. That's what's wrong with my model. OK, so the model is too simple. It's not just an integrator. It's a slow integrator.

OK, I can model that too. If I model the slow integration, I can think about the motor doesn't really behave like an integrator. It works like a sluggish integrator. That might be a lot easier to see if I write that in s. So that's the same as gamma p over s plus gamma. No, s times-- no, close, wrong.

So it used to be gamma over s. Now, it's gamma over s s plus p. OK, so I'm replacing what had been an integrator here with a sluggish integrator.

Sluggish is another pole. So instead of representing it by a single pole at the origin, I represent it by two poles. If you calculate the step response for the single pole at the origin, you get the straight line. That was model one.

If you calculate the step response for the modified model, that's this one. Ultimately, they become parallel. So the speed becomes the same. But there's sluggishness because of inertia and so forth so that it takes a while for the real motor to achieve that.

So you can see that in the model two response, because the angle does not immediately change when I let go of it, it takes a while for it to start to change. That's inertia. So if I put that model, model two, into the feedback loop, then I make a different prediction about the way things should work.

That prediction is shown here. Right? That's just thinking about system functions, saying that I've got two poles, trying to figure out where the poles are, figuring out where the time domain response is, integrating it to turn it into a step response. Do all those things, and you get a response.

The point is that the new model looks like two poles. And when you analyze the root locus, those poles move when you change the feedback. So as I change the feedback number, beta, as beta goes from 0 to big, the poles go toward each other and then split off.

That's just math. That's the math that was showed over here. All I did was write the second model up there, figure out the system function, find the poles of the system function. They depend on beta. And that dependence is shown here.

And so the response then, that's where the oscillations come from. If the gain is big enough, I get a pair of poles that are off the real axis. The fact that they have an imaginary component, the poles, means that there's an oscillation. And what happens is I change the feedback is not that the oscillation goes away. In fact, the real part of the envelope doesn't change.

That's roughly what I was seeing here. If you watched what was going on in the closed-loop configuration, it always oscillated for a couple of seconds-- like a second or two seconds or three seconds. That second or two seconds didn't change. What did change was the bumps-- how many bumps there were in the two seconds.

And that's what the theory says. Because the real part of the imaginary pair of poles doesn't change, the envelope doesn't change. But because the imaginary part does change, the oscillation frequency does change. So the result then is something whose envelope doesn't really improve.

So that model then to a large extent does predict the behavior that I have measured. And then what that motivates is if I want to make the motor work better, I need some even better controller because my simple proportional controller, that's as good as it's going to do.

OK, so what I tried to illustrate today was ways of thinking about feedback to enhance performance. We looked at an op-amp and saw how you could think about feedback increasing the bandwidth or making it faster. We looked at a motor and found out how you could think about changing the motor from being a voltage-controlled velocity to a voltage-controlled position. And we saw how the feedback could be used for both of those. Next time, we'll look at a couple other examples. Thanks.