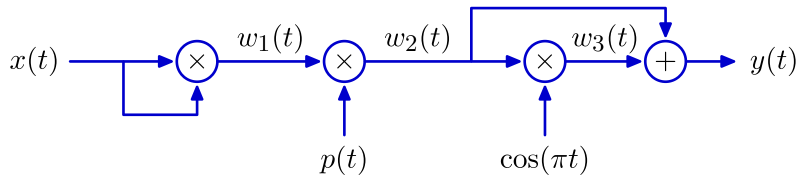
6.003 Homework #13

*Due at the beginning of recitation on **December 7, 2011**.*

Problems

1. Transformation

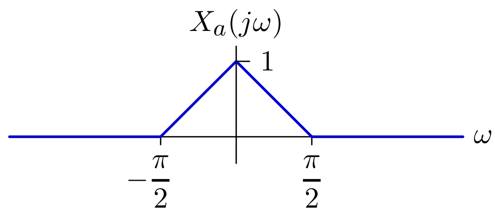
Consider the following transformation from $x(t)$ to $y(t)$:



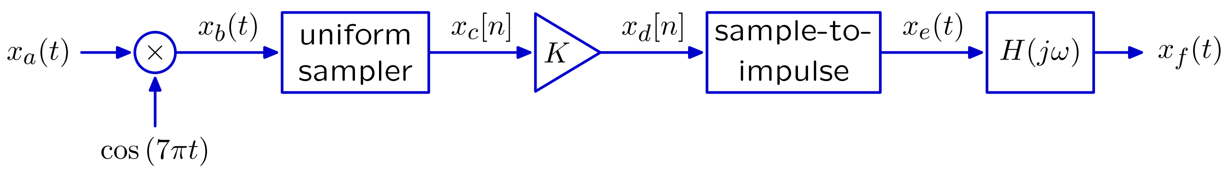
where $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-k)$. Determine an expression for $y(t)$ when $x(t) = \sin(\pi t/2)/(\pi t)$.

2. Multiplied Sampling

The Fourier transform of a signal $x_a(t)$ is given below.



This signal passes through the following system



where $x_c[n] = x_b(nT)$ and

$$x_e(t) = \sum_{n=-\infty}^{\infty} x_d[n]\delta(t - nT)$$

and

$$H(j\omega) = \begin{cases} T & \text{if } |\omega| < \frac{\pi}{T} \\ 0 & \text{otherwise}. \end{cases}$$

a. Sketch the Fourier transform of $x_f(t)$ for the case when $K = 1$ and $T = 1$.

Use your sketch to determine an expression for $X_f(j\omega)$ for the following intervals:

$0 < \omega < \pi/2$:

$\pi/2 < \omega < \pi$:

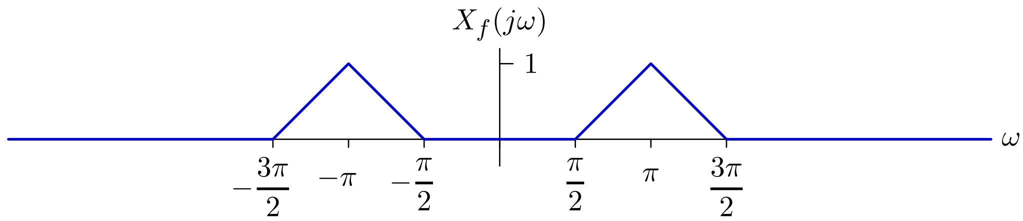
$\pi < \omega < 3\pi/2$:

$3\pi/2 < \omega < 2\pi$:

b. Is it possible to adjust T and K so that $x_f(t) = x_a(t)$?

If yes, specify a value T and the corresponding value of K (there may be multiple solutions, you need only specify one of them). If no, write **none.**

c. Is it possible to adjust T and K so that the Fourier transform of $x_f(t)$ is equal to the following, and is zero outside the indicated range?



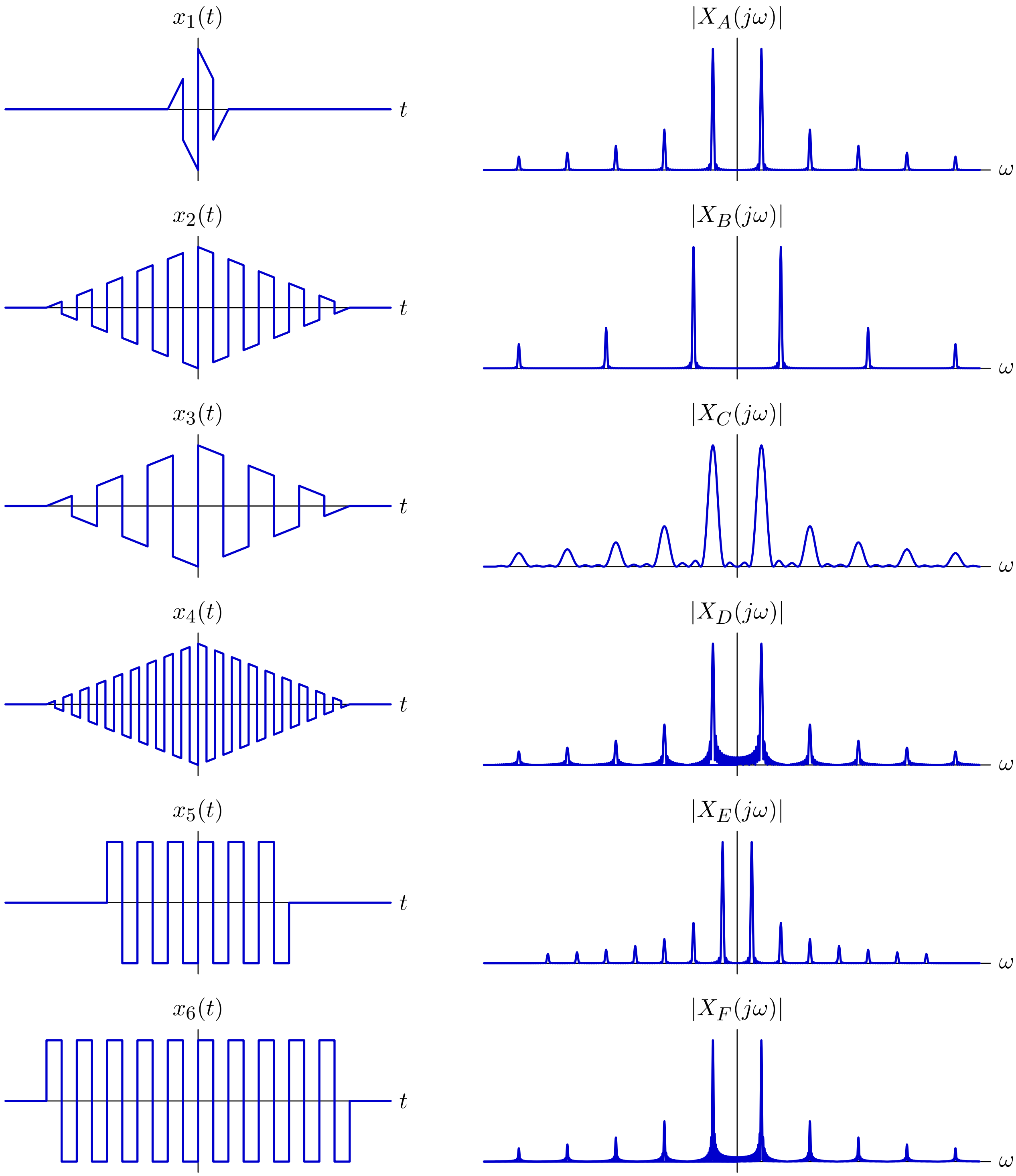
If yes, specify all possible pairs of T and K that work in the table below. If there are more rows in the table than are needed, leave the remaining entries blank. If no, enter **none.**

T	K

3. Patterns

The time waveforms for six signals are shown in the left panels below. The right panels show the magnitudes of the Fourier transforms of $x_1(t)$ to $x_6(t)$, however, the order has been shuffled. For each panel on the left, find the corresponding panel on the right.

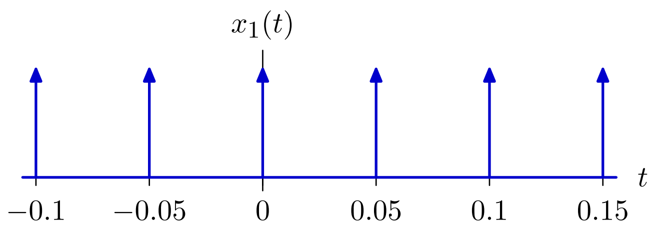
All of the time functions are plotted on the same time scale. Similarly, all of the frequency functions are plotted on the same frequency scale.



4. Inputs and Outputs

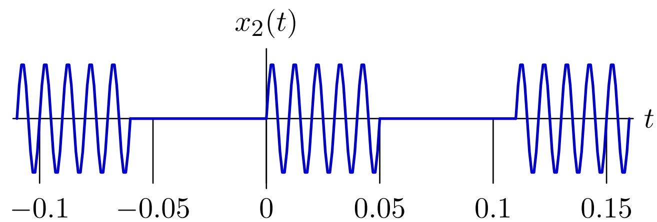
A **causal, stable** LTI system with frequency response $H(j\omega)$ has input $x(t)$ and output $y(t)$. The problem is to determine which of the following inputs can or cannot give rise to the output $y(t) = \sin(2\pi \cdot 100 \cdot t)$. For each part of the problem, determine if the statement is True (T) or False (F) and give an explanation.

Part a. $x_1(t)$ is a periodic impulse train of period 0.05 s.



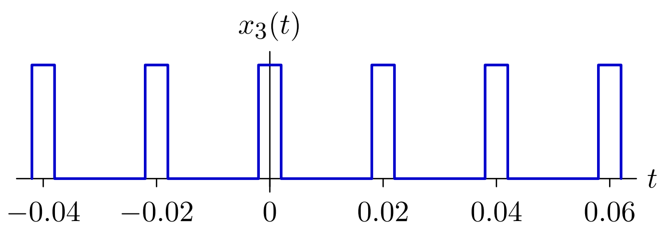
(T or F) $x_1(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

Part b. $x_2(t)$ is a periodic function of period 0.11 s. Each period consists of five cycles of a sinewave of the form $\sin(2\pi \cdot 100 \cdot t)$.



(T or F) $x_2(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

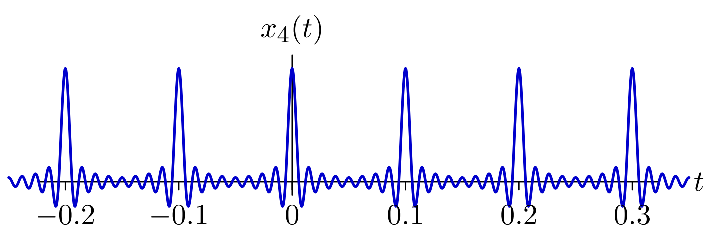
Part c. $x_3(t)$ is a periodic pulse train of period 0.02 s. Each pulse has duration 0.004 s.



(T or F) $x_3(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

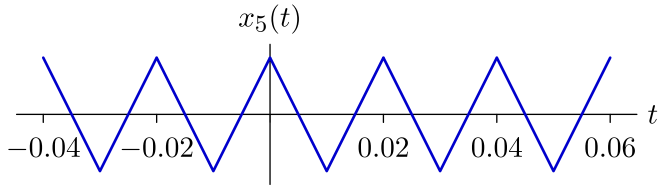
Part d. $x_4(t)$ is a periodic sinc pulse train of period 0.1 s. Each sinc pulse has the formula

$$\frac{\sin(\pi \cdot \frac{t}{0.006})}{\pi \cdot \frac{t}{0.006}}$$



(T or F) $x_4(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

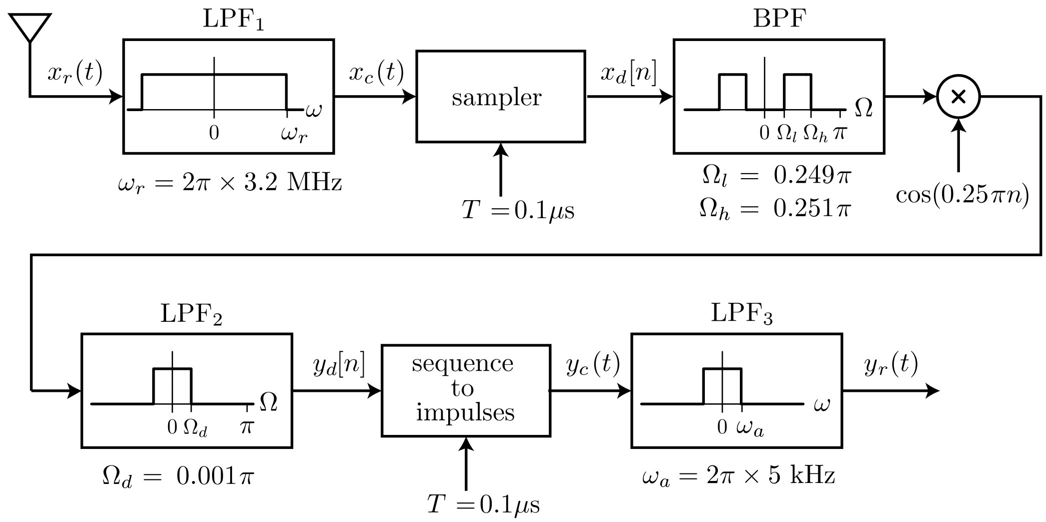
Part e. $x_5(t)$ is a periodic triangular wave of period 0.02 s.



(T or F) $x_5(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

5. DT Radio Demodulation

Commercial AM radio stations broadcast radio frequencies within a limited range: $2\pi(f_c - 5\text{ kHz}) < \omega < 2\pi(f_c + 5\text{ kHz})$, where $f_c = \omega_c/(2\pi) = n \times 10$ kHz and n is an integer between 54 and 160. The system shown below is intended to decode one of the AM radio signals using DT signal processing methods. Assume that all of the filters are ideal.



sampler: $x_d[n] = x_c(nT)$

sequence to impulses: $y_c(t) = \sum_{n=-\infty}^{\infty} y_d[n]\delta(t - nT)$

Part a. Determine the center frequency f_c for the AM station that this receiver will detect.

Part b. Which of the following statement(s) is/are correct?

b1. Increasing the cutoff frequency ω_r of LPF_1 by a factor of 1.5 will cause aliasing.

b2. Decreasing the cutoff frequency ω_r of LPF_1 by a factor of 2 will have no effect on the output $y_r(t)$.

b3. Halving the sampling interval T would have no effect on the output $y_r(t)$.

b4. Doubling the sampling interval T would have no effect on the output $y_r(t)$.

Part c. Which of the following statement(s) is/are correct?

c1. Increasing the cutoff frequency Ω_d of LPF_2 will change $y_r(t)$ by adding signals from unwanted radio stations.

c2. Increasing the cutoff frequency Ω_d of LPF_2 will change $y_r(t)$ because aliasing will occur.

c3. Doubling the cutoff frequency Ω_d of LPF_2 will have no effect on $y_r(t)$.

c4. Halving the cutoff frequency Ω_d of LPF_2 will have no effect on $y_r(t)$.