MITOCW | L09b-6002

Good morning, all.

Let's get going. So I guess you had your quiz review yesterday. I hope you guys didn't beat up on (name of TA) and who else was it?

(Name of TA) too much. As you know, the quiz is tomorrow. And unfortunately MIT couldn't give us one big room so we are broken up into three rooms.

And you will go to your room based on the first letter of your last name. OK, so today we shall cover a topic called "Large Signal Analysis".

So in the last couple lectures we looked at one dependent sources abstractly, and then we looked at an amplifier built using a practical dependent source called the MOSFET. Now, the MOSFET had to be operated in a given region of its operation in order to behave like a current source. And while it behaved like a current source you would get amplification or a FET amplifier. So that was in the past two lectures. What you are going to do today is called large signal analysis, and this is a loaded term.

So large signal analysis means something very specific in our business, and I will describe to what that is.

This analysis involves looking at a circuit containing, for example, a MOSFET and figuring out how to get that device to operate in a way that the MOSFET was always in saturation. So you had to figure out, based on parameters that you could control, how to establish those parameters so that the circuit operated in a way that the MOSFET was always in saturation.

So large signal analysis involves that.

And although the examples we use, use the MOSFET, the same kind of analysis can apply to any other device.

Remember, your MOSFET is a primitive element that we use as an example in this course. There are other primitive elements that you can use. The course notes, for example, discusses a couple other devices. One is the "bipolar junction transistor", the BJT, and works through a complete example from start to finish involving a circuit containing a bipolar junction transistor. And you can do a large signal analysis of that device as well. It turns out that you need to operate that device in an interesting region of its operating space, and so you can conduct a large signal analysis of a circuit containing that device and figure out how best to operate that circuit.

So that is large signal analysis, and we will do an example and explain how this is done using an example today.

So to quickly review where we have been so far, we looked at this little structure here, our MOSFET amplifier.

Notice that when I write a voltage at a node, that's a short form for saying I am looking at the voltage between the ground node and the node at which the voltage is written down. So VO here and VI applied here.

This is a very, very common circuit that we use. To emphasize one more point, in general, in the kind of circuits we look at both in this course and in real life, there are a few patterns that we use very commonly that keep repeating all the time.

Very often you don't have to look at every possible permutation and combination of how things could be connected.

This sort of connecting thing is very, very, very common. And you will see a lot of this pattern. And we do the equivalent circuit for this. In the equivalent circuit we replace the MOSFET with a dependent source provided this operated in the saturation region.

So I will just say while operating under saturation the equivalent circuit would look like this, VO, VI.

And IDS for the dependent source was given by K/2 (VI-VT)^2. So this was an amplifier.

Here was the equivalent circuit while this device was in saturation. And to operate in saturation, I said that certain properties need to be true for the MOSFET.

And there are two properties that need to be true for this to be operating in saturation. One is that its gate to source voltage needs to be greater than VT, so VGS for the MOSFET should be greater than VT. And the second one was that the output voltage needed to be greater than the input voltage minus one threshold drop. And this was the same as VDS for the MOSFET, this was the same as VGS for the MOSFET. So what are we really saying here? What we are saying is that look, we built this circuit using a MOSFET, and it is up to us as engineers to choose its operating points in a way that these two properties hold.

For example, to make the first condition true, I can discipline myself to operate such that VI is always greater than VT. Similarly, I can choose VS, RL and VI in a way that this condition is true, which says that the drain to source voltage across my MOSFET drain and source should be greater than VI minus VT.

As an example, if VI was 2 volts and VT was, say, 1 volt, then what I am saying is that VO should be greater than or equal to 2 minus 1 or 1 volt.

So I need to keep this high, 2, 3, 4, 5, whatever, a high voltage so that this guy stays in saturation.

The relevant readings for the material that we are going to cover in the course notes are in 7.5.1 and 7.6.

So that is pretty much a review of where we were.

We said we could build an amplifier.

Its equivalent circuit was shown on the right.

And, provided that, I discipline myself to operate in the saturation region or to have the MOSFET operating in the saturation region, then this would work like an amplifier and all would be good with the world.

So today -- -- we look at large signal analysis of a circuit.

And an example would be this circuit up here containing a MOSFET. And, again, as I mentioned earlier, a large signal analysis is a loaded term in 6.002, or for that matter in circuits. And large signal analysis involves two steps.

The first step involves writing down the transfer function of your little circuit. In our case, VO is the output, VI is the input, so involves writing down VO versus VI.

Simply write down the transfer function.

In other words, the relationship between the output and the input for that circuit.

And, in our case, again, we've disciplined ourselves to adhere to the "saturation discipline".

And the second part of large signal analysis is to find out the valid input operating range.

Find out for the given circuit parameters, let's say I apply a VS and I use some value of RL and I use a given MOSFET, which has a given value of VT. The question then is that what is a valid set of input voltages that would operate the circuit in a way that I would be in saturation.

And so find out the valid input range, and this would give me a corresponding output range -- -- for saturation operation of the MOSFET.

That is what we will dwell on in the lecture today.

So what we are saying here is that if I am careful with how I apply VI for a given value of RL and VS and for a given choice of my MOS transistor then I can stay within saturation provided I select my input voltages carefully.

And the analysis that we will go through today will figure out what that range of input voltages is.

And, again, I will use this as a motivating example, the MOSFET amplifier. But in general large signal analysis would apply to any other circuit as well.

For example, in recitation you may learn about other circuits containing a MOSFET.

And you can do a large signal analysis of other circuits containing a MOSFET or you might learn about some other devices like the bipolar junction transistor, and you could do the same kind of analysis for that device.

So remember that the MOSFET amplifier here is an example.

I will be using that as a driving example to explain large signal analysis. So the first step, as I mentioned earlier, is to get the VO versus VI.

And in general for some circuit that you build the output will not even be a voltage. There are certain circuits where the output might be some kind of a current.

Let's say I am building some kind of a circuit where I would like the output current or the current through some edge of the circuit to depend on some input. In that case the transfer function would be the output current versus VI.

And if I had an input current here it would be output current versus input current, you know, whatever the given problem tells you. So this is under the saturation discipline. And I will not rederive this for you. You can apply a good old technique like the analytical method.

Or you can use the graphical method to get the appropriate answer here. I wanted to point out in a quick aside that why do we care about graphical analysis?

Once you have the analytical method, why do you care about the graphical method? And a student asked me a question after lecture last Thursday, and it occurred to me that it's not obvious why you need the graphical method.

So it turns out that often times you do not have an equation describing the device. So let's say, for example, I am a manufacturer.

Let's say I am AMD. As AMD I sit down and my semiconductor division builds a MOSFET.

And when you build a MOSFET your experiments and your fabrication division often times doesn't give you an equation with the MOSFET. They build something and then you look at it and you experiment with it.

You apply various input voltages and you measure currents and output voltages and so on.

And so what you end up getting is a graph that describes the behavior of the MOSFET. And you have seen this in your lab as well, your 2N7000 or was it 2000?

the MOSFET you use in the lab also gives you a data sheet.

And in that data sheet you see a bunch of curves.

So very often devices come with data sheets.

And when you have a data sheet but no equation then you can apply the graphical method and solve your circuits.

In this example, assuming I can apply the analytical method, here was the expression that I had derived for you in the last lecture.

So VO was related to VI using the square law relationship.

And we can plot and do other fun stuff with this equation.

So here is the input voltage VI.

That is my VT. So notice that VO is VS.

This is true when VI greater than or equal to VT and VO greater than or equal to VI minus VT.

So these are the constraints of the saturation discipline.

And in our particular situation when VI was less than VT output would simply be VS. If VI is less than VT the MOSFET would turn off, switch off, and I would have no current flowing through RL, and VS would appear at the output. So until VT, I have VS, and then following that I get the square law behavior articulated by this equation.

And this was simply VS-K/2 (VI-VT)(RL^2).

So that's the first part. You have seen this before.

The transfer function shows that I have a square law dependence between VI and VO. So now I can embark on the second step of my large signal analysis, and my goal is to find the valid input operating range. So what does that mean?

What I am looking to do here is, for this little circuit, is drain, source, gate, VI, VO, RL and VS. What I am looking to do is that given the value of the supply VS, RL and a MOSFET, in our case given a MOSFET implies that it is a given value of K and a given value of VT for that MOSFET.

So what I am going to do is find out, let's assume VI is my free variable here. So my goal will be to find out the range of VI for which this device stays in saturation.

And I will use a couple of methods to do that, and I will use both a combination of a graphical method to give you intuition and then apply analytical analysis to get down to specific answers. So let's start with the intuitive part. So here is VI, VO. I will use the transfer curve VO versus VI to help build intuition here.

So that is what it looks like. So the first step, looking at this graph, we know that this point here, that VI needs to be greater than VT to satisfy the first equation. Let me just write down both equations here. So VI greater than or equal to VT is one of them, and VO is greater than VI minus VT is a second equation. And just remember that this is the same as VDS and this is the same as VGS.

So VI must be greater than VT for the MOSFET to turn on.

And so therefore the valid operating range starts at this point and is somewhere up here. So the first part is pretty easy. Somewhere here -- Somewhere at that point, my output voltage VO.

I'm not quite sure what that point is.

My output voltage VO, as this keeps falling down, my output voltage VO goes lower than one threshold below VI.

And at that point it goes into its triode region, and I need to find out what that point is.

So somewhere here I go into my triode region and begin to show a different behavior than the amplifying square law relationship there and go into my triode behavior.

So I need to find out what this point is.

Once I find out what that point is then this will be my valid operating range. So let's figure out what that point is. At that point the following is true. Certainly VI is greater than VT. And at that point the output goes below one threshold, the input minus one threshold.

So at this point the following is true, VO is equal to VI minus VT. At that point the output voltage is equal to the input minus VT.

And if the output goes lower then it will violate this equation here. It is no longer greater than that number. So how do we find out what this point is? The principle intuition.

Let's draw some lines here. Let's assume that VI and VT use the same scale, say, volts.

So if I draw a straight line at 45 degrees then that is a curve representing VI equals VO. We all know that.

No big shakes. So the line at 45 degrees here is the line at which VI equal VO.

And if I take that line now, the VI equals VO line, and I begin translating it to the right.

So let's take a line here. Let's take a line there.

That line will be simply equal to VO equals VI minus VT.

I have translated that to the right.

And so this line is simply VO equals VI minus VT.

So this line is a locus of points at which VO is equal to this value. This minus VT shows up as a translation to the right. So I take my VO equals VI line, translate that to the right and it becomes VO equals VI minus VT. Elementary geometry 101 or whatever. So what do we have here?

Above this we have the condition VO greater than or equal to VI minus VT, and below that we have VO less than VI minus VT. If we look at this graph here, this is the valid input operating range.

Starting at this point greater than VT, and at this point my output equals VO equals VI minus VT.

This must be the valid operating range for the input here to here. And correspondingly the outputs are from here to this point to here like so.

So this is my valid input operating range and this is my valid output operating range or the corresponding valid output operating range. So what does this say?

What this is saying is that if I, as the designer of the circuit, am disciplined enough to apply inputs that are in this range, VT to some value here, graphically shown here.

Then my MOSFET will remain in saturation.

And correspondingly my outputs will go between VS and some value here. So hopefully that gives you some of the intuition behind how we get it.

And let's continue. Let me label this point X.

So continuing with two to get the valid operating range.

I have shown you intuitively where that point is, but what I will do next is actually compute that for you.

It is a pretty simple computation.

Note that point X is the intersection of two curves VO equals VI minus VT. And the second curve is VO equals VS minus K divide by 2, VI minus V2 all squared RL.

So the point X iss at the intersection of these curves, and I can very easily get that as follows.

What I will do is I will simply substitute for VI minus VT from this equation here and then solve for it, so I get VO equals VS minus K divide by 2 VO squared RL.

And so this gives me a quadratic in VO.

And I can solve for it pretty easily.

And I get for a quadratic AX squared plus BX plus C equals 0.

The solution is given by VO is minus B plus/minus square root of B squared minus 4AC divided by 2A.

And so I am just going to get those numbers here.

So the coefficient of VO, that is B, is minus 1.

Take the positive root because we are up in the positive voltages here. And square root of B squared, that is 1, minus 4AC. So I get a plus 4 times K divide by 2 RL. And 2A is simply 2 times K divided by 2 times RL. So that is what I get.

That gives me VO. So it tells me that VO, at the point where the output just equals one threshold drop below VI is given by the other circuit perimeter such as VS, RL and so on. Oh, I am missing a VS here.

I just forgot the VS up here. That is my VO.

So what is VI equal to? Remember that at this point VO equals VI minus VT, so VI is simply VT plus -- I have not taught you anything earth shattering here.

I have just done some grubby math here to solve these two equations. So this is a straight line at 45 degrees from VT and this is the transfer function.

And I need to find the intersection.

And the intersection is given by this point.

So that point, VI being VT plus something, is simply the second dot on the X axis.

So therefore I am pretty much done.

My valid input range for VI goes from VT.

So it starts at VT. That is where the transistor just turns on. And then goes all the way to this point, VT plus minus 1 plus square root of 1 plus 2 K RL VS K RL. So this is my valid operating range. And again remember I won't dwell on this equation because, in some sense, you will get a different set of limits for other devices, for other circuits containing a MOSFET.

Or, for that matter, for other outputs that one may be focusing on. So what is more important here is not so much the results that you see but the process that I have gone through. So what is more important here is how did I get here? And the way I got here was looked at the graph and said look, the MOSFET is in saturation in that regime. And I am finding the bounding points of the regime of saturation operation.

So now, as an engineer, I can say that hey, look, if you build a MOSFET circuit like so, with a given value of RL, a given MOSFET and a given VS, then if you limit yourself you are operating with input voltages in this range thou shalt be happy.

If you go beyond that range then you will be violating the saturation discipline. So the corresponding output range -- I can write the corresponding output range, and that goes from VS, when the input is at VT the output is at VS and goes from VS down to the input minus VT.

Which is simply minus 1 plus -- Let me go back and quickly show you a little MOSFET circuit, amplified circuit so you can stare at a real transfer curve yourselves. And indeed convince yourselves that roughly at the point where proportionately shown in the curve up there the MOSFET indeed goes into its triode region and begins heading out of its saturation region.

Notice that here that is the same curve, the transfer function. And the amplified output is at VS until input reaches a threshold voltage VT.

And once input goes beyond VT the output begins to drop precipitously. And at some point here this begins to go into its triode region.

And what I am going to do is simply increase the input voltage VI, and you will see that the output them begins to go into its triode region. It keeps dropping.

And, as you can see, the output begins to go into a space where the gain is no longer more than 1.

And this is a triode region of MOSFET operation.

So the MOSFET is in saturation, things are going great.

As I increase my VI notice at some point I begin to go out of my saturation region of the MOSFET.

And somewhere here I go from the saturation region and transition into the triode region.

And this value shown here gives you the corresponding input voltage and the output voltage. Other practical devices like bipolar junction transistors or MOSFETs and other circuits and so on can be subjected to a similar analysis.

And you can find out the valid operating regions for that device as well, or for that circuit.

So as a next step what I would like to do -- Out here I began by looking at the transfer function, the VO versus VI curve, and used that to drive the intuition behind how we calculated the bounding regions.

You can do the same kind of analysis intuitively looking at yet another curve, another set of graphs that you are familiar with, and that is a load line characteristic. And it is interesting to get two interpretations. And you can use whichever one you feel comfortable with. So I will do two alternatively and show you another set of curves that you can use to get that.

Here I am going to plot IDS versus VDS, which is the same as VO. This was the load line graph that we had seen earlier. And, just for our reference, remember that VI must be greater than VT for saturation operation. Similarly VO should be greater than or equal to VI minus VT for saturation operation.

Those are my limits. The way we got the load line graph was we superimposed the load line equation over the device characteristics. And so let me plot the device characteristics in the saturation region.

Remember that this constraint could be related to the current as I derived for you in the last lecture as follows.

IDS being less than or equal to K divided by 2 VO squared.

So in terms of my IDS versus VDS relation, this lateral constraint is equivalent to IDS being less than K by 2 VO squared. So this is that equation.

So this line is IDS equals K by 2 VO squared.

And in this region I have the valid operating region where IDS is less than that quality. So here are all my other curves for various values VGS. So here are my devices curves, IDS versus VDS. Remember that these curves come down like this, for the MOSFET, right? Just that we focus on the right-hand side because that is where the MOSFET is in saturation. And on this side the MOSFET is in its triode region, and we discipline ourselves not to operate the MOSFET such that it is in its triode region.

So those were the device characteristics.

And then I could plot my load line equation.

My load line equation, if you recall, was IDS = VS/RL - VO/RL. This was simply obtained by writing KVL at the loop containing the output node and the supply VS. Notice there that VO is equal to VS minus IDS times RL. And that is simply by dividing by RL on both sides and moving IDS to the left-hand side we get this equation. And this equation gives rise to a curve that looks like this. And what is this point here?

This point is where VO is 0. So when VO is 0 my IDS is simply VS divided by RL. And this point is obtained when IDS is 0. And under those conditions VS and VO are equal so this is VS. This is my saturation region and this is the triode region. This was another interesting graph. We often times fondly call it the load line graph. So here is a load line superimposed on the MOSFET device IDS versus VDS curves for a variety of values of VGS. So by looking at this curve, we can also intuitively determine the valid operating range. So what are the two points here? I will let you stare at it for a couple of seconds yourselves to figure out what two points here bound the valid operating range of the MOSFET, the valid operating range of the circuit.

I will start. One is this point, because at this point the output is VS and VGS has just begun to equal VT. So think about where the second point is for valid operation. It is here, and, somewhere along that load line. Remember the load line is a constraint that must be met by the output VO.

It is the constraint imposed by KVL on the output.

So the output is constrained to operate in this regime for various values of VGS. So as the output keeps going from here all the way here, at some point I exit my saturation region. And that other point is given by this one. So notice that this is the curve that bounds. On the left-hand side of this the MOSFET is no longer in saturation.

It is on the right-hand side, and so therefore this is the valid operating region.

Here to here. This is good.

This is VS. That is good to know.

And for this point I know that VI, which is VGS, equals VT. I know VO is equal to VS.

And IDS, at this point, is 0.

So VO and IDS being VS and 0 correspondingly are the output operating perimeters when VI equals VD.

So that is one point. And let's find out what this point is. At that point I get my output just entering the range of the MOSFET triode region operation.

Notice that this point is the intersection of two curves, this line and this curve. So this curve here is given by IDS equals K divided by 2 VO squared.

And this is my load line equation.

So that is VS divided by RL minus VO divided by RL.

That's it. So I won't go ahead and solve that for you. You can go and check it out and convince yourselves that if you solve these two equations and find out the VO for this, it should be the same VO that you obtained using the other graph.

What I have done here, obtaining the valid regions of operation is no different from what I did here.

The two are alternative approaches to getting to the same place. Just that over the years what I have discovered is that there are one class of people that are output transfer function people, this graph, and another set of people that are load line people that like to think that way.

I have always been a transfer function person myself, but some of you may be load line people and so you can use that to drive your intuition. It is pretty amazing.

As we get into this business and keep going down the path, it is amazing how some people really kind of get the load line thing and others feel much more comfortable with the transfer function. So pick whatever you want.

So what we have so far is we have conducted a large signal analysis of a MOSFET amplifier. It is an analysis of a circuit, and we found two things. One is the transfer function under saturation operation, and we found the valid input operating ranges and the corresponding valid output operating ranges for the circuit.

In the last five or six minutes let me talk about a couple of other issues. And the first issue is what we have done so far is intuitively and mathematically shown you what the valid regions are. Now you are thinking that's fine, but how do I get there? This region is good, VT through that other point, that's good, but how do I get there? How do I make my amplifier operate in that region? The answer is pretty simple, and let me drive the intuition again using a graph.

So this is a graph. And I showed you that -- That was my valid region here. Take a 45 degree line, find out where it intersects the transfer function, then this is the valid region here, VT through that coordinating function that we developed out there.

If I have an input that looks like so, some input whose gyrations fall within this range, will constantly keep the MOSFET in saturation. And the corresponding output will look like this. If my input is in this range, my output will be within this range.

And how do I get my input to be here?

Let's say I have a sinusoid that is 1 volt peak to peak or whatever. How do I get my sinusoid up there? Well, you have learned the trick on how to boost things. Remember boost?

All you have to do is boost up your signal by some value capital VI. And the way you do that is as follows. VS, RL, VO.

What you do is you apply a DC offset to your input.

You take your sinusoid and boost it up so that all the gyrations of the input are in the valid range.

This is my input, some VA.

Then I apply some DC offset capital VI given by this value here. And boost up the interesting input? My interesting input is the VA.

And I boost it up by capital VI so that this guy is always in saturation. I would like to show you a little demo now. I am going to show you an input that is a triangular wave. And what we will do is I'll play with a wide variety of offset voltages.

This guy is a triangular wave. And what I am going to do is apply a triangular wave and we'll look at the output and convince ourselves that I get amplification when VI is big enough that the input goes into a valid operating range.

And we will look at a variety of ranges here.

You can put it a little larger.

OK. So the triangular wave is my input. And this is my output.

This looks nothing like a triangular wave.

And the reason is that I do not have the right offset.

So what I will do is gradually increase the offset on the MOSFET. So at this point the offset is very low, a very small near zero offset.

And so therefore my output is a disaster.

My MOSFET is not in saturation all the time.

So what I will do here is apply some sort of offset.

Is this the one? We want to switch.

This is the input. You can see I am applying an offset by bumping and boosting up the input.

I don't have clipping happening at both ends, but I get something. And I get amplification.

Now let me apply way too much of an offset.

With this offset I am kind of operating here.

What I will do now is apply an even higher offset so that this triangular wave begins to move here.

If I apply a very high offset what I am doing is overdriving the amplifier, boosting it so high that the MOSFET is going to go into its triode region and you are going to see that I won't have any gain.

My output is going to shrink noticeably if I overdrive the input. You will notice the input going higher and higher. Pull the trigger point down.

There you go. Notice that as I boost up my input even higher notice that the output is a really small image of what the right input should be.

The right answer here, of course, is that I apply some right amount of offset to boost up the input into the right regime so that the output is seen to be some amplified version of this input. So I showed you three things.

One is very little offset. That was like so, as the thing comes down. A very high offset, it gets killed again. And the right amount of offset.

But notice that we still have a problem, even with the right offset. The output is not linearly related to the input. It is nonlinear.

And the answer to get a linear response is good old small signal stuff. And we will be looking at the small signal part in the next lecture. So your 2N7000,