

6.002

CIRCUITS AND ELECTRONICS

Introduction and Lumped Circuit Abstraction

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ADMINISTRIVIA

- ☠ Lecturer: Prof. Anant Agarwal
- Textbook: Agarwal and Lang (A&L)
- Readings are important!
Handout no. 3
- Web site —
<http://web.mit.edu/6.002/www/fall00>
- Assignments —
Homework exercises
Labs
Quizzes
Final exam

- Two homework assignments can be missed (except HW11).

- Collaboration policy

Homework

You may collaborate with others, but do your own write-up.

Lab

You may work in a team of two, but do you own write-up.

- Info handout

- Reading for today —
Chapter 1 of the book



What is engineering?

Purposeful use of science

What is 6.002 about?

Gainful employment of
Maxwell's equations

From electrons to digital gates
and op-amps

Nature as observed in experiments

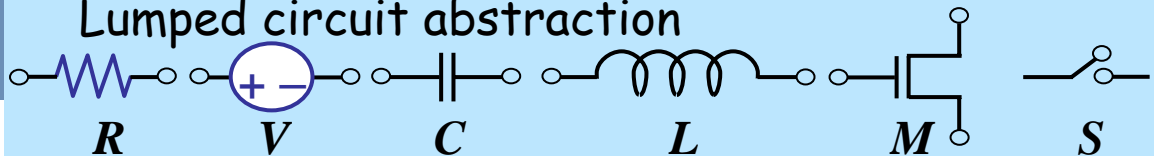
| | | | | | |
|-----|-----|-----|-----|-----|-----|
| V | 3 | 6 | 9 | 12 | ... |
| I | 0.1 | 0.2 | 0.3 | 0.4 | ... |

Physics laws or "abstractions"

- Maxwell's
 - Ohm's
- abstraction for tables of data

$$V = RI$$

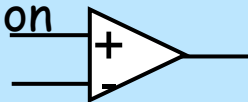
Lumped circuit abstraction



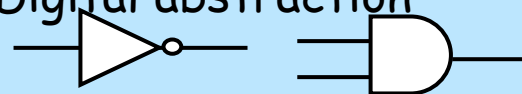
Simple amplifier abstraction



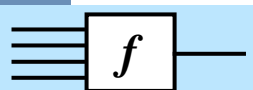
Operational amplifier abstraction



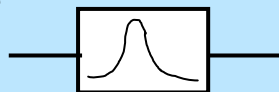
Digital abstraction



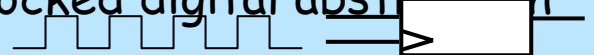
Combinational logic



Filters



Clocked digital abstraction



Analog system components:

Modulators,
oscillators,
RF amps,
power supplies 6.061

Instruction set abstraction

Pentium, MIPS 6.004

Programming languages

Java, C++, Matlab 6.001

Software systems 6.033

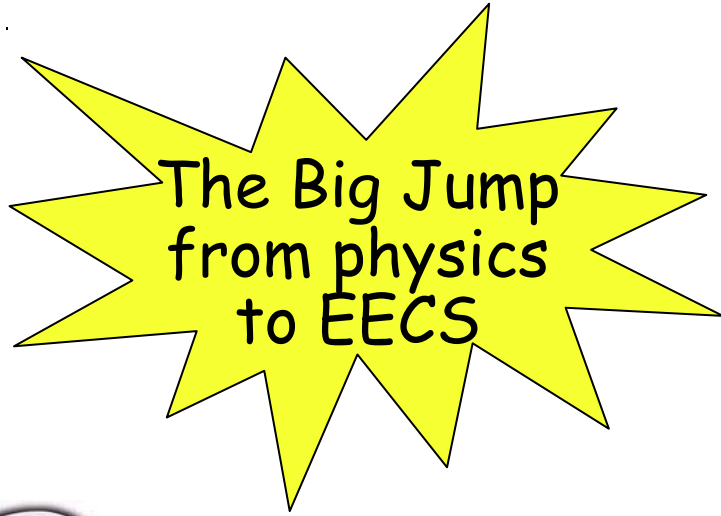
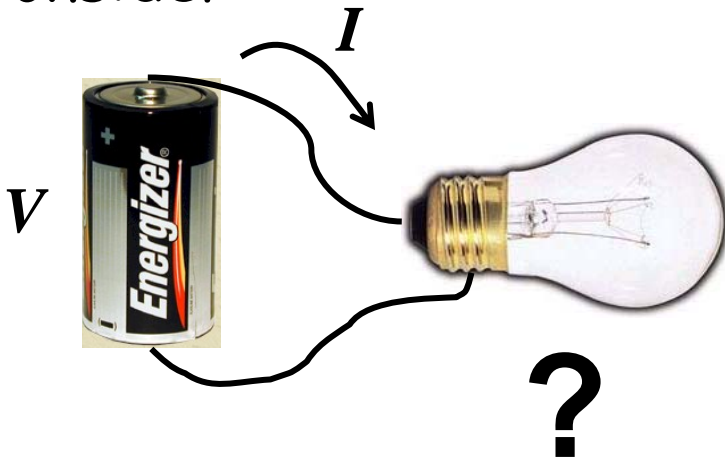
Operating systems, Browsers

Mice, toasters, sonar, stereos, doom, space shuttle
6.455 6.170

6.002

Lumped Circuit Abstraction

Consider



Suppose we wish to answer this question:
What is the current through the bulb?

We could do it the Hard Way...

Apply Maxwell's

Differential form Integral form

| | | |
|-----------|--|--|
| Faraday's | $\nabla \times E = -\frac{\partial B}{\partial t}$ | $\oint E \cdot dl = -\frac{\partial \phi_B}{\partial t}$ |
|-----------|--|--|

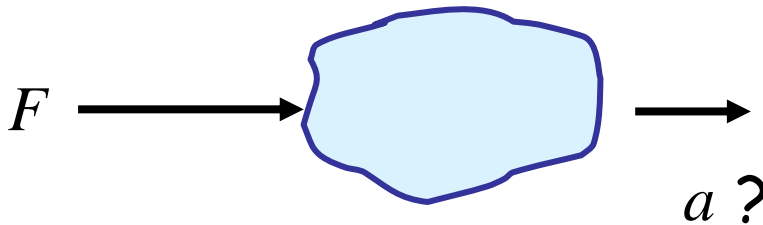
| | | |
|------------|--|---|
| Continuity | $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$ | $\oint J \cdot dS = -\frac{\partial q}{\partial t}$ |
|------------|--|---|

| | | |
|--------|--|---|
| Others | $\nabla \cdot E = \frac{\rho}{\epsilon_0}$ | $\oint E \cdot dS = \frac{q}{\epsilon_0}$ |
| | • | • |
| | • | • |
| | • | • |

Instead, there is an Easy Way...

First, let us build some insight:

Analogy



I ask you: What is the acceleration?

You quickly ask me: What is the mass?

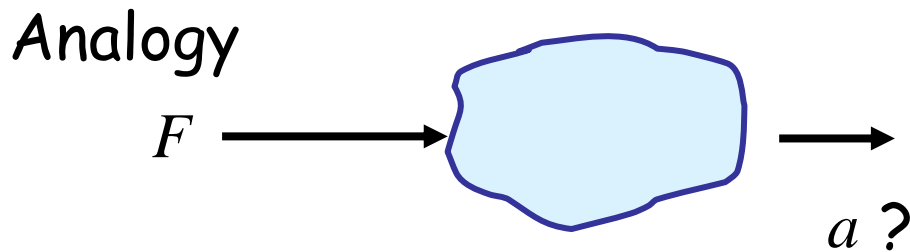
I tell you: m

You respond: $a = \frac{F}{m}$

Done!!!

Instead, there is an Easy Way...

First, let us build some insight:



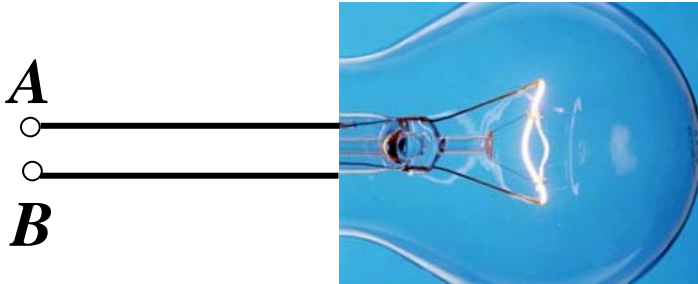
In doing so, you ignored

- the object's shape
- its temperature
- its color
- point of force application

→ Point-mass discretization

The Easy Way...

Consider the filament of the light bulb.



We do not care about

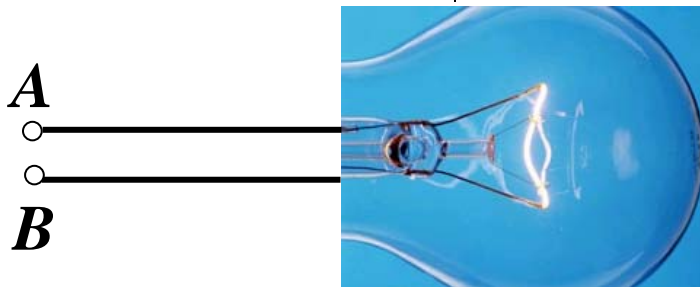
- how current flows inside the filament
- its temperature, shape, orientation, etc.

Then, we can replace the bulb with a

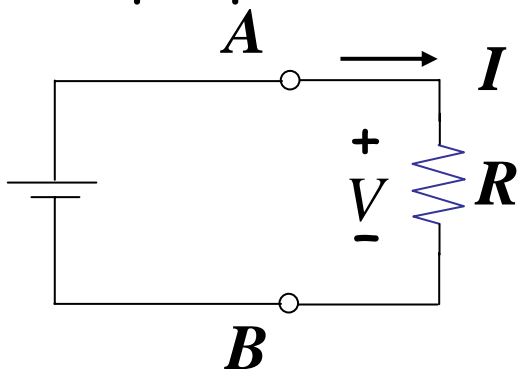
discrete resistor

for the purpose of calculating the current.

The Easy Way...



Replace the bulb with a
discrete resistor
for the purpose of calculating the current.

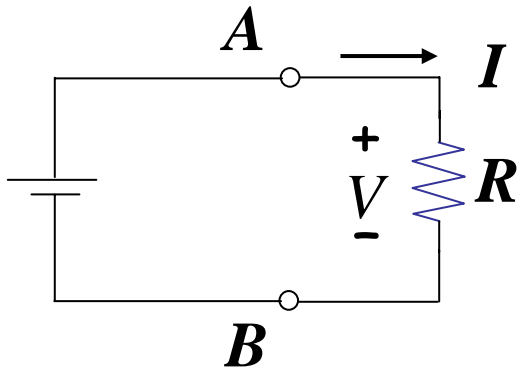


and $I = \frac{V}{R}$

In EE, we do things
the easy way...

R represents the only property of interest!
Like with point-mass: replace objects
with their mass m to find $a = \frac{F}{m}$

The Easy Way...



and $I = \frac{V}{R}$

In EE, we do things the easy way...

R represents the only property of interest!

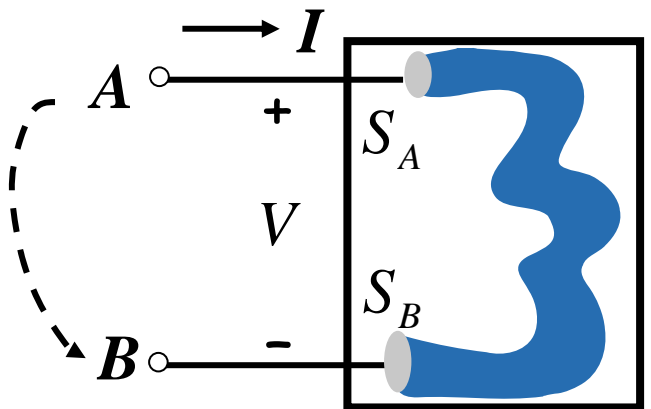
R relates element v and i

$I = \frac{V}{R}$ called element v - i relationship

R is a lumped element abstraction
for the bulb.

R is a lumped element abstraction for the bulb.

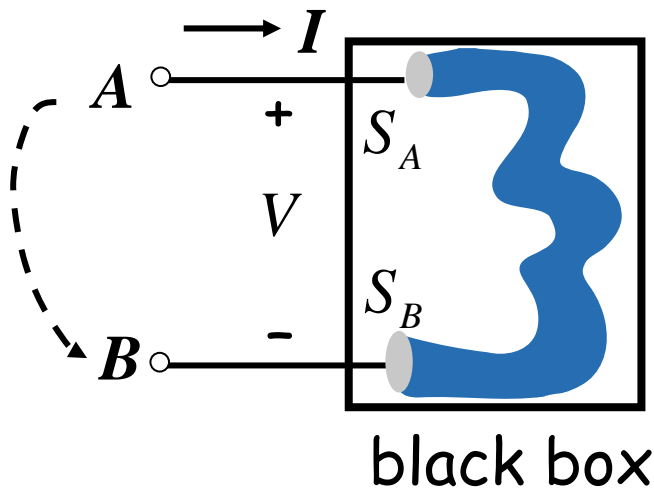
Not so fast, though ...



black box

Although we will take the easy way using lumped abstractions for the rest of this course, we must make sure (at least the first time) that our abstraction is reasonable. In this case, ensuring that \boxed{V} \boxed{I}

are defined
for the element



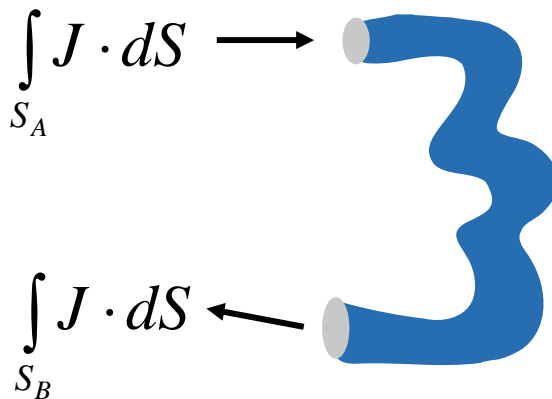
V I

must be defined
for the element

I must be defined. True when

$$I \text{ into } S_A = I \text{ out of } S_B$$

True only when $\frac{\partial q}{\partial t} = 0$ in the filament!



$$\int_{S_A} J \cdot dS - \int_{S_B} J \cdot dS = \frac{\partial q}{\partial t}$$

I_A I_B

from Maxwell

$$I_A = I_B \text{ only if } \frac{\partial q}{\partial t} = 0$$

So let's assume this

V Must also be defined.

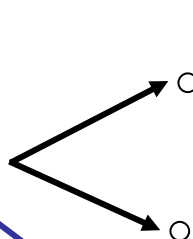
see
A & L

So let's assume this too

V_{AB} defined when $\frac{\partial \phi_B}{\partial t} = 0$

So $V_{AB} = \int_{AB} E \cdot dl$ outside elements

Lumped Matter Discipline (LMD) Or self imposed constraints:


$$\begin{aligned} \frac{\partial \phi_B}{\partial t} &= 0 && \text{outside} \\ \frac{\partial q}{\partial t} &= 0 && \text{inside elements} \\ &&& \text{bulb, wire, battery} \end{aligned}$$

More in
Chapter 1
of A & L

Lumped circuit abstraction applies when elements adhere to the lumped matter discipline.

Demo → Lumped element examples whose behavior is completely captured by their $V-I$ relationship.

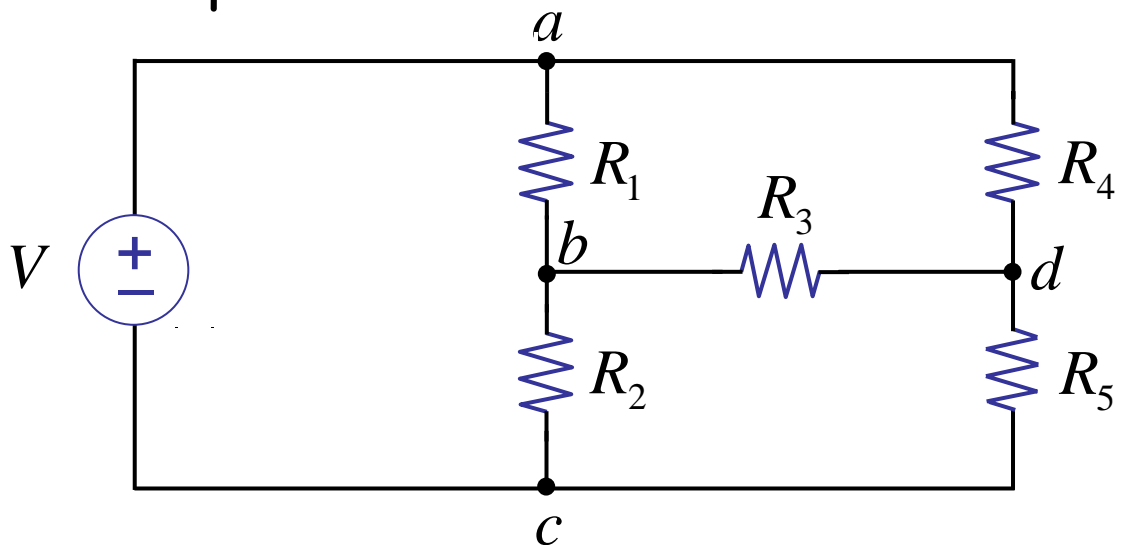
only for the sorts of questions we as EEs would like to ask!

Demo → Exploding resistor demo
→ can't predict that!
Pickle demo
→ can't predict light, smell

So, what does this buy us?

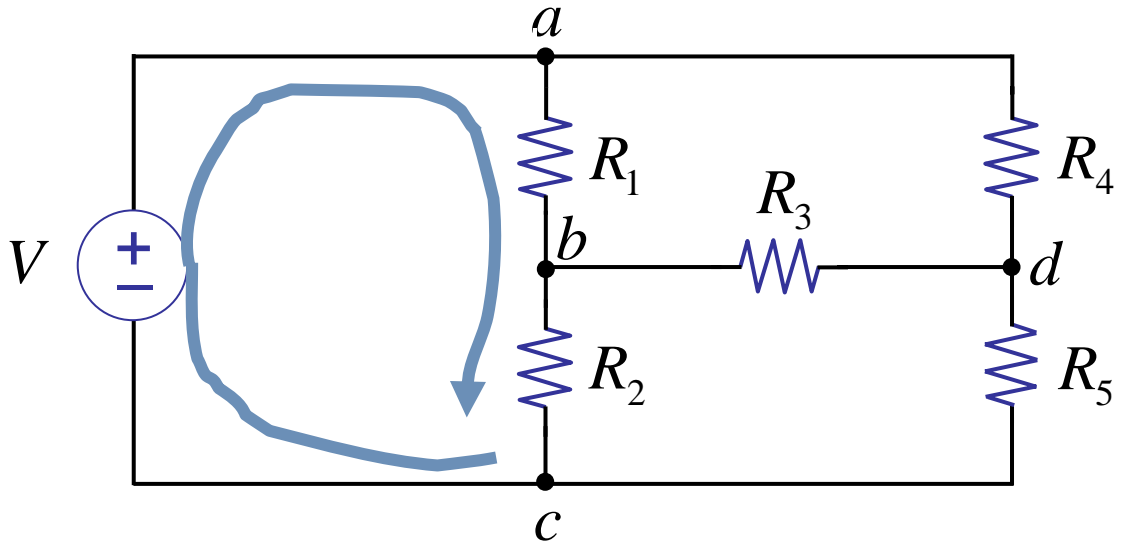
Replace the differential equations with simple algebra using lumped circuit abstraction (LCA).

For example —



What can we say about voltages in a loop under the lumped matter discipline?

What can we say about voltages in a loop under LMD?



$$\oint E \cdot dl = -\frac{\partial \phi_B}{\partial t} \rightarrow 0 \text{ under DMD}$$

$$\Rightarrow \int_{ca} E \cdot dl + \int_{ab} E \cdot dl + \int_{bc} E \cdot dl = 0$$

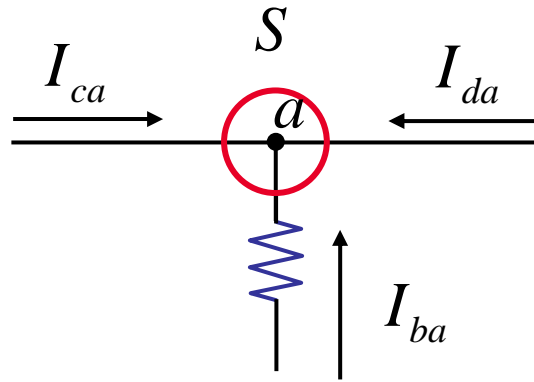
$$\Rightarrow + V_{ca} + V_{ab} + V_{bc} = 0$$

Kirchhoff's Voltage Law (KVL):

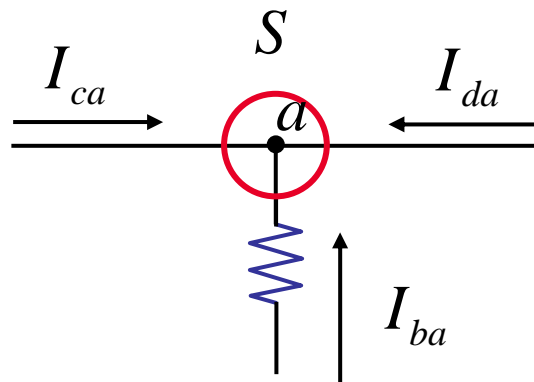
The sum of the voltages in a loop is 0.

What can we say about currents?

Consider



What can we say about currents?



$$\oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial q}{\partial t} \xrightarrow{\text{under LMD}} 0$$

$$\Rightarrow I_{ca} + I_{da} + I_{ba} = 0$$

Kirchhoff's Current Law (KCL):

The sum of the currents into a node is 0.

simply conservation of charge

KVL and KCL Summary

KVL:

$$\sum_j v_j = 0$$

loop

KCL:

$$\sum_j i_j = 0$$

node