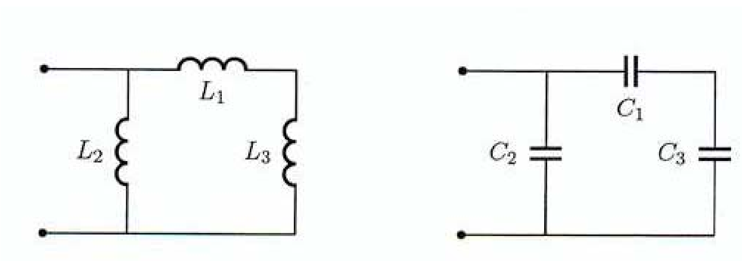
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2007
Homework #9
Handout - S07-45

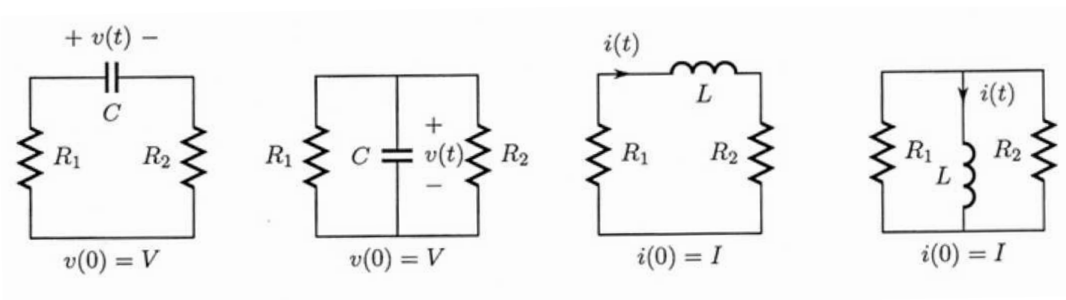
Issued 4/12/2007 – Due 4/20/2007

Helpful Reading for this Homework: Chapter 12.

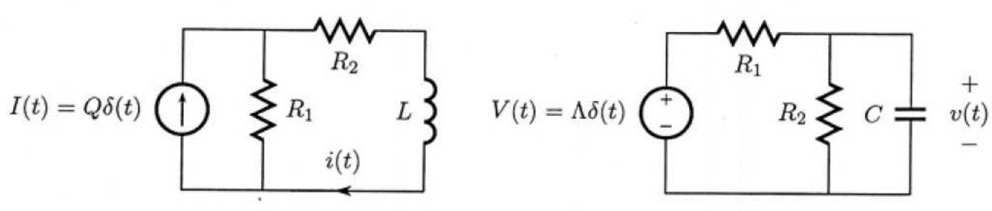
Exercise 9.1. Find the inductance of the all-inductor network, and the capacitance of the all-capacitor network, shown below.



Exercise 9.2. Each network shown below has a non-zero initial state at $t = 0$ s, as indicated. Find the network states for $t \geq 0$ s. Hint: what equivalent resistance is in parallel with each capacitor or inductor, and what decay time results from this combination?



Problem 9.1. At $t = 0^-$ s, the networks shown below have zero initial state; the inductor current $i(t)$ and the capacitor voltage $v(t)$ are both zero at $t = 0^-$ s. At $t = 0$ s, the current source produces an impulse of area Q, and the voltage source produces an impulse of area Λ.



A Derive the differential equation that relates $i(t)$ to $I(t)$ and $v(t)$ to $V(t)$. Hint: consider using Thevenin and/or Norton equivalents to simplify the work.

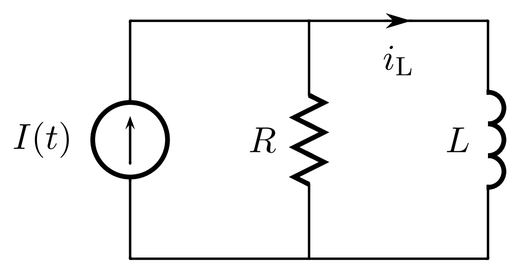
B Find the inductor current $i(t)$ and the capacitor voltage $v(t)$ at both $t = 0^+$ s and $t = \infty$. Feel free to determine the states through either physical or mathematical reasoning. However, explain your reasoning in any case.

C Next, find the time constant by which each state goes from its initial value at $t = 0^+$ s to its final value at $t = \infty$.

D Using the previous results, and without necessarily solving the differential equations directly, construct $i(t)$ and $v(t)$ for $t \geq 0$ s. Alternatively, find $i(t)$ and $v(t)$ by any means you choose, but be sure to explain your reasoning.

E Verify that the solutions to part **D** are correct by substituting them into the differential equations found in part **A**.

Problem 9.2. This problem examines the relation between transient responses of linear systems.



A Find the inductor current $i_L(t)$ for $t \geq 0$ s in response to the current step $I(t) = I_{Step}(t) = I_0 u(t)$. Assume that $i(0) = 0$.

B Find the inductor current $i_L(t)$ for $t \geq 0$ s in response to the current step $I(t) = I_{Ramp}(t) = I_0 \alpha t u(t)$. Again, assume that $i(0) = 0$.

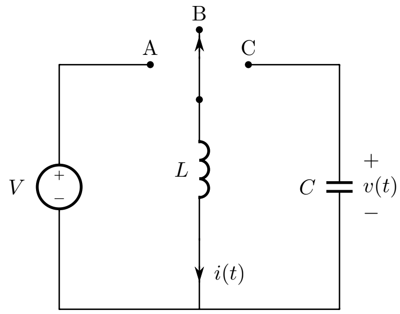
C The step input can be constructed from the ramp input according to $I_{Step}(t) = \frac{1}{\alpha}\frac{d}{dt} I_{Ramp}(t)$. Show that their respective responses are related in a similar manner. (Note: you could have used this relation to solve part **B** given your answer to part **A**.)

D Would the result from part **C** hold if $i(0) \neq 0$? Why or why not?

Problem 9.3. The network shown at the top of next page includes a switch with three positions: A, B and C. Prior to $t = 0$ s, the switch is in Position B, and the inductor current $i(t)$ and the capacitor voltage $v(t)$ are both zero. The voltage source V is constant.

A At $t = 0$ the switch moves to Position A, and it remains there until $t = T_1$. Find $i(t)$ and $v(t)$ for $0 \leq t \leq T_1$.

B At $t = T_1$ the switch moves to Position C without interrupting the current $i(t)$, and it remains there until $i(t)$ goes to zero, at which time the switch moves back to Position B. Define the time at which $i(t)$ goes to zero as $t = T_2$. Determine T_2, and find both $i(t)$ and $v(t)$ for $T_1 \leq t \leq T_2$.



C The switch remains in Position B until $t = T_3$. Find both $i(t)$ and $v(t)$ for $T_2 \le t \le T_3$

D At $t = T_3$ the switch moves again to Position A, and it remains there until $t = T_4$. Find $i(t)$ and $v(t)$ for $T_3 \le t \le T_4$.

E Finally, at $t = T_4$ the switch moves to Position C, and it remains there until $i(t)$ first goes to zero, at which time the switch moves back to Position B. Define the time at which $i(t)$ again goes to zero as T_5. Determine T_5, and find both $i(t)$ and $v(t)$ for $T_4 \le t \le T_5$.

F Sketch and clearly label $i(t)$ and $v(t)$ for $0 \le t \le T_5$

Problem 9.4. This problem is continuation of Problem 9.3. It explores the use of energy conservation to analyze the operation of the network described therein.

A Determine the energy stored in the inductor at $t = T_1$.

B The energy stored in the inductor at $t = T_1$ is fully transferred to the capacitor at $t = T_2$. Use this fact to determine $v(T_2)$. This answer should match your answer in part B of Problem 9.3 when the latter is evaluated at $t = T_2$.

C Determine the energy stored in the inductor at $t = T_4$.

D Use energy conservation to determine the energy stored in the capacitor at $t = T_5$, and then determine $v(T_5)$. This answer should match your answer to part E of Problem 9.3 when the latter is evaluated at $t = T_5$.

E Now let the switch move repetitively through the cycle of Positions B to A to C to B. Assume that in each cycle the switch remains in Position A for the duration T. Further, assume that switch always moves from Position C to Position B when $i(t)$ reaches zero. Assuming that v and i are initial zero, determine v at the end of the n^th switching cycle in terms of n, C, L, T and V.