14.75: The Median Voter Thereom

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Overview

- Example: an experiment
- Theory:
  - Spatial models of voting and single-peaked preferences
  - The Median Voter Theorem
- The Median Voter Theorem in Practice:
  - Expanding the electorate (it works as advertised)
  - Reservations for politicians (maybe it isn't exactly right!)

How to model voting

- In order to introduce models of voting and politicians, we need to start by making some assumptions about people's preferences
- Suppose there are three choices that people are deciding on: $\{A, B, C\}$
- In principle, you could imagine 6 different ways preferences could be ordered:
  - $A > B > C$
  - $A > C > B$
  - $B > A > C$
  - $B > C > A$
  - $C > A > B$
  - $C > B > A$

Single-peaked preferences

- For the moment, we need to make a simplifying assumption on these preferences – we need to assume that they are "single-peaked." This is defined as

Definition (Single-Peaked Preferences)

Preferences are said to be single-peaked if the alternatives can be represented as points on a line, and each utility function has a maximum at some point on the line and slopes away from the maximum on either side.

- I'll come back to what happens if we don't have single-peaked preferences in about 4 lectures

Example of single-peaked preferences

- How do we write down single-peaked preferences?
- Suppose we are making a decision about where to put a public good g on the interval $[0, 1]$
- An example of single-peaked preferences would be something like

$$u_i = -(g - b_i)^2$$

- In this example, b_i is individual i's *bliss point*.
- Some examples:
  - General "liberal vs. conservative" preferences
  - Tax rate and level of spending on public education
  - Where to locate a public good (e.g., I prefer it near my house, and my utility declines in distance from my house – albeit this is the one-dimensional version)

What does single-peakedness rule out?

- Suppose that we had ordered on a line A, B, C. Suppose I told you that we had people that
  - $B > A > C$
  - $B > C > A$

  so if preferences are single-peaked, then clearly B is in the middle.
- If everyone's preferences are single-peaked, could someone have the preferences:
  - $A > C > B$? No. Why? Because B is in the middle.
- In practice, many of the economic things we care about – e.g. tax rates, size of government, how much money to spend on defense, etc – are continuous variables that can sensibly be modeled with single-peaked preferences
- The kinds of things where single-peakedness becomes more of a problem are unordered choices. Examples?
  - Which band should play at a special campus-wide concert?
  - What color should we paint the bridge?

What does single-peakednes buy us?

- Single-peakedness is very useful analytically.
- Suppose I am interested in the question of voting between two levels of funding for education, $e = 1$ and $e = 2$.
- With single-peakedness, I know that everyone whose bliss point $b_i < 1$ will vote for $e = 1$, and everyone whose bliss point $b_i > 2$ will vote for $e = 2$.
- What about someone whose bliss point $b_i = 1.75$?

The Median Voter Theorem

- Suppose preferences are single-peaked over a single-dimensional policy space.
- Suppose there are two candidates, 1 and 2.
- The two candidates simultaneously announce (and can commit) to implement policies p_1 and p_2.
- Voting is by majority rule.
- Then we have the following result:

Theorem (Median Voter Theorem)

If preferences are single-peaked, and there are two candidates who can commit in advance to policies and care only about winning, then in equilibrium, $p_1 = p_2 = b_{median}$.

The Median Voter Theorem

Proof.

- Suppose not.
- Without loss of generality, suppose that p_1 has more votes than p_2 and that $p_1 < b_{median}$.
- Then p_2 will deviate and instead choose $p_2 = p_1 + \varepsilon$, with ε small, so that $p_2 < b_{median}$.
- From single-peakedness, all the voters with ideal points in the interval $[p_2, \infty)$ prefer p_2 to p_1.
- Since $p_2 < b_{median}$, this is more than half of the voters.
- So p_2 would win, and thus would prefer to deviate.So it is not an equilibrium for p_1 to win with $p_1 < b_{median}$.
- Thus the only equilibrium where there is no profitable deviation is $p_1 = p_2 = b_{median}$.

□

What this does and doesn't mean

- This is a key result in voting theory.
  - Why?
  - Because it suggests there is a huge force driving candidates towards the preferences of the median voter – if they need to get more than 50% of the votes, the best way to do that is to have the median preferences – and if you don't, the other guy will
  - There are lots of reasons it may not hold exactly – e.g. Mitt Romney's positions may not exactly equal Barack Obama's – but it is a force driving them to the center
- Note that it assumes that politicians care only about winning – they don't also care about policy
  - E.g. Barack Obama would be happy to move to the right of Mitt Romney if he thought there were more than 50% of the votes there
  - This is probably not strictly true, but it is a useful benchmark

What this does and doesn't mean

- Note that intensity of preferences doesn't matter in this result
  - I may *really, really* dislike Candidate A, but my vote counts just as much as someone who is close to indifferent
  - This is a consequence of one person, one vote, and the inability of voting to collect preference information
  - It is, however, one of the fundamental ways in which elections are different than economic decisions
  - We can also think about ways of relaxing this (e.g. if you pay a cost of voting, people with stronger preferences may be more likely to vote; campaign contributions; etc). But once again, it's a useful benchmark

Example of the median voter

- Suppose we have three restaurant choices:
  - A costs \$5
  - B costs \$10
  - C costs \$20
- There are three people, 1 prefers A, 2 prefers B, 3 prefers C
  - What do single-peaked preferences mean in this case?
  - Who will win? Why?

An example of how the MVT is a powerful tool

Adapted from Meltzer and Richards (1981): "A Rational Theory of the Size of Government"

- Here is a simple example of how the median voter theorem is a powerful tool for analyzing policy
- Revenue:
  - Suppose we have only one variable: the income tax rate, denoted τ
  - So every individual pays fraction τ of their income as taxes.This means that if my income is y, my after tax income is $y(1-\tau)$.
  - Suppose that the income distribution is $f(y)$.
  - What is average tax revenue per person?
  - Revenue per person is

$$R_{avg} = \int \tau y f(y) = \tau \int y f(y) = \tau y_{avg}$$

  So total revenue is just equal to τ times the average income level in the population.

An example of how the MVT is a powerful tool

- Costs of taxation:
  - Almost all taxes are distortionary – e.g., you work less to avoid high taxes, and the higher the taxes, the greater the distortions
  - For simplicity, let's assume that the costs per person of taxes are equal to δt^2
- Expenditures:
  - The government uses taxes for one purpose: to provide some shared good that everyone consumes equally.
  - There is some loss to society from taxation (this is called the "deadweight loss of taxation). This is equal δR^2.
  - The total amount of this good per person is equal to

$$R = \tau y_{avg} - \delta \tau^2$$

An example of how the MVT is a powerful tool

- So each person's final consumption is

$$C = y(1-\tau) + \tau y_{avg} - \delta\tau^2$$

- Here's the policy question: how high will τ be? i.e., how high will the tax rate and amount of government expenditure be? How do we figure this out?
- This is where the median voter theorem comes in.

An example of how the MVT is a powerful tool

- What would the median voter want?
- The median voter would want to solve

$$\max_{\tau} y_{median} (1 - \tau) + \tau y_{avg} - \delta \tau^2$$

- How do we solve this? Take the derivative with respect to τ to find

$$\begin{aligned} y_{avg} - y_{median} &= 2\delta\tau \\ \tau &= \frac{y_{avg} - y_{median}}{2\delta} \end{aligned}$$

- So the tax rate – and hence the size of government – is increasing in the difference between average income and median income.
- Why is this? Because politicians make decisions based on the median voter, but average tax rates are based on the average income.

Examples

$$\tau = \frac{y_{avg} - y_{median}}{2\delta}$$

- Suppose there are 5 people. Let's assume $\delta = \frac{1}{2}$ to make life easy.
- Case 1: Incomes $= \{0, 1, 2, 3, 4\}$
  - What is the median? 2
  - What is the mean? 2
  - What is the tax rate? 0.

Examples

$$\tau = \frac{y_{avg} - y_{median}}{2\delta}$$

- Suppose there are 5 people. Let's assume $\delta = \frac{1}{2}$ to make life easy.
- Case 2: Incomes $= \{0, 1, 2, 3, 9\}$
  - What is the median? 2
  - What is the mean? 3
  - What is the tax rate? 1.

Examples

$$\tau = \frac{y_{avg} - y_{median}}{2\delta}$$

- Suppose there are 5 people. Let's assume $\delta = \frac{1}{2}$ to make life easy.
- Case 2: Incomes $= \{0, 1, 2, 3, 59\}$
  - What is the median? 2
  - What is the mean? 13
  - What is the tax rate? 11.

What's going on?

- What's going on?
  - In this model what's happening is that the policy is driven by the difference between the median and the mean.
  - So when you get a lot of inequality (particularly, if you have some very rich people), the median voter can gain a lot from setting a higher tax rate and taxing the rich.
- Why was there no tax rate in case 1 when median and mean were the same?
  - Median voter gets no benefit. Taxation is costly, so optimum is 0.

More than 2 candidates

- Note that the median voter theorem does not generalize to cases with more than 2 candidates.
- Technical assumption: suppose you have the same policy position as another candidate. Then the two candidates split the votes at that level.
- Suppose that the policy space is $[0, 1]$, and people are uniformly distributed. With 2 candidates the equilibrium is: $\frac{1}{2}$.
- Now suppose that there are three candidates. Suppose candidates 1 and 2 were at $p_j = \frac{1}{2}$. What could candidate 3 do?
- Candidate 3 could announce $\frac{1}{2} + \varepsilon$ and win! So everyone at $\frac{1}{2}$ is no longer the equilibrium.
- Instead, the equilibrium has the candidates spaced out a bit.

The Median Voter Theorem in Practice

- What are the predictions of the Median Voter Theorem? We'll look empirically at two predictions:

1. What happens if I change the electorate?
   - Suppose the electorate had ideal points uniformly distributed on $[0, 1]$. What is the policy outcome?
   - Now suppose we enfranchise new voters, so the electorate shifts to be distributed on $\left[0, \frac{3}{2}\right]$. What is the policy outcome?
   - How might we examine this in the data?
2. What happens if I prevent some people from becoming candidates?
   - E.g., suppose I have a policy that says that the candidates must only be women, or must be poor people, etc
   - What would happen?
   - (Aside: why might I want to do that?)

Changing the electorate and the MVT

Miller 2008: "Women's Suffrage, Political Responsiveness, and Child Survival in American History"

- What does it mean to change the electorate? How could you do that?
- We'll study one dramatic example from the US: the enfranchisement of women

Changing the electorate and the MVT

- What would the predictions of the median voter theorem tell us?
- Suppose we can choose whether to spend public money on roads or clean water. Denote by α the share of municipal expenses on clean water, so $\alpha \in [0, 1]$.
- Suppose that preferences are as follows:

$$u_i = -\left|\alpha - b_i\right|$$

  - For men, $b_i \sim Uniform\left[0, \frac{3}{4}\right]$
  - For women, $b_i \sim Uniform\left[\frac{1}{4}, 1\right]$
- What do these preferences look like? Are they single-peaked?
- What is the policy outcome if only men can vote?
- What is the policy outcome if everyone can vote?

Women's suffrage in the United States

- Women's suffrage in the United States
  - Universal women's suffrage was achieved in 1920 with the ratification of the 19th amendment to the U.S. constitution
  - However, before that, 29 of the 48 states had already extended suffrage to women
  - This happened over a roughly 30 year period

Women's suffrage in the United States

Women's suffrage in the United States

- This is a differences-in-differences approach:

$$\ln\left(d_{sy}\right) = \alpha + \beta v_{sy} + \delta_y + \delta_s + \delta_s \times t + \varepsilon_{sy}$$

  - where d is the outcome, s is a state, y is a year
  - δ_y are year fixed effects and δ_s are state fixed effects.What do they do?
  - $\delta_s \times t$ is a state-specific time trend. What is this?
  - What does β mean if (d) is in logs?
- What would have happened if we had just compared states in cross-section?
- Are the states that are early adopters of women's suffrage a random sample?

Reservations for Politicians

Chattopadhyay and Duflo (2004): Women as Policy Makers: Evidence from a Randomized Policy Experiment in India

- Indian village councils in India which have authority over local public goods decisions
- In 1993, an Indian Constitutional amendment mandated representation for women and minorities.
  - For minorities: in each district, representation in each local council, and among the heads of all the council, must be equal to the share of SC/ST in the district.
  - For women, in each list (reserved for SC, ST, and general), every third list to be reserved for women.
- Since the reservations for women were essentially randomly assigned, they focus on women

Reservations for Politicians

- Question: Does the identity of the leader affect the type of investment decisions made by the Panchayat?
- What would the Median Voter Theorem predict?
    - If democracy is perfect, since the leader must still be elected by everyone, one could expect that his (or her) platform represent the preferences of the median voter.
    - So mandating a woman as candidate wouldn't necessarily matter.

The Impact of Reservations for Women

- The goal is to answer the question: when women have power, do the political decisions better reflect the needs of women?
- Since women live in the same place as men, there is no straightforward way to measure preferences.
- The authors used revealed preferences: what have men and women complained about in the last year?
  - In West Bengal and Rajasthan: women strongly prefer drinking water.
  - West Bengal: men prefer education and irrigation.
  - Rajasthan: men prefer roads.
- The idea is that in areas reserved for women, we should see more investment in water everywhere, less investment in school and irrigation in West Bengal, more investment in roads in West Bengal, and less investment in roads in Rajasthan

Are Reservations Welfare Improving?

- Results suggest that rules which favor election of women ensure that public goods better represent the preferences of women.
- These results are not reverted in the second cycle: women elected for the second time invest in a very similar way to women elected in the first cycle; there is no "backlash" in places where men come back in power after the end of reservation.
- While this is clearly a redistribution towards women, we cannot conclude that the allocation is welfare improving: it depends on the preferences for roads, schools, wells.
- But what does it imply for the median voter theorem?

Reservations for Minorities

Pande (2003): "Can Mandated Political Representation Increase Policy Influence for Disadvantaged Minorities?"

- Setting:
  - State-level legislatures in India
  - Reservations of seats for low-caste legislators
- Empirical strategy:
  - Law requires percent of seats reserved for SC/ST legislators be equal to their percentage in the state's population
  - Census updates the population every 10 years
  - This takes effect at the next state election after the census.
  - Pande exploits the different lag structure caused by the interaction of state election cycles with the census (plus two other nationally-mandated rule changes) to gain identification.
  - What does this mean?

Reservations for Minorities

- She estimates

$$Y_{st} = \alpha_s + \beta_t + \gamma R_{st} + \phi P_{st\_census} + \delta P_{st} + \eta X_{st} + \varepsilon_{st}$$

where R_{st} is share reserved seats, P_{st} is SC/ST population share and P_{st_census} is the latest census estimate of the population share
- This is an example of a difference-in-difference
  - How is that?
  - The key is that it includes state fixed effects (α_s) and year fixed effects (β_t)
  - So we are learning one's going on controlling for the fact that states are different, and that years are different, and just looking at what happens when they change the share of seats that are reserved
- Outcomes:
  - Total spending
  - Education
  - Land reform
  - SC/ST job quotas and targeted welfare spending

Why might there be deviations from the median voter?

- There are several reasons why there might be deviations.
- This is one example, from what's called a citizen-candidate model.
- The idea is that we need to think about who would bother running for office.
- If people have to pay some cost to run for office:
  - We may not necessarily always have 2 candidates
  - And even if we have two candidates, they won't have the same position in equilibrium

Setup

- Setup:
  - Village elects individual who implements policy $\in [0, 1]$
- Each person has preferred preference ω_i
  - median voter's preference if m
- If outcome is x_j, utility is:
  - $-\left|x_j - \omega_i\right|$ if i was not a candidate and
  - $-\left|x_j - \omega_i\right| - \delta_i$ if i was a candidate
- Timing:
  - Each person decides whether to run or not. If no candidates μ' is policy.
  - Citizens vote strategically for candidates.
  - Assuming somebody runs for office, the winner's preference x_j is policy.
- Note that in this model, you can't commit to any policy other than your most preferred policy.

How does this change things?

- Suppose we had the result before that both candidates had the median voter's policy
  - So $p_1 = p_2 = m$
- Is this an equilibrium?
  - Suppose candidate 2 decides not to run.
  - Policy will be unchanged (it will still be m), and candidate 2 will no longer have to pay δ_i
  - So $p_1 = p_2 = m$ is not an equilibrium because candidate 2 will deviate and not run!
- What is the equilibrium with 2 candidates?
  - Symmetric around the median: Positions $m + \varepsilon$ and $m - \varepsilon$. Why? Otherwise, one candidate would always win and the loser wouldn't run.
  - ε cannot be too small (otherwise not worth it). In this example ε must be at least δ.
  - ε cannot be too large (otherwise a third candidate could enter in the middle)

2 candidates

- Note that there are many possible equilibria.
- Why? Suppose we're at an equilibrium with $\varepsilon > \delta$, so $p_1 = m - \varepsilon$ and $p_2 = m + \varepsilon$
- If ε is close to δ, someone just to the left of p_2 won't bother running because the gain is too small.

How does this help us think about reservations?

- Suppose that
  - For women, $\omega_i \in [0, W]$
  - For men, $\omega_i \in [M, 1]$
  - Women also face higher barriers to being candidates $(\delta_m < \delta_w)$
- Then the conditions under which women never run for office without reservations are:

$$1.\ \delta_w - \frac{\delta_m}{2} > \mu' - m$$
$$2.\ \delta_w > m$$

- Proof:
  - Condition 1: No woman runs unopposed $(\delta_w - \frac{\delta_m}{2} > \mu' - m)$. A woman would run unopposed if $\mu' - x_j \geq \delta_w$, so most "man-friendly" woman candidate is $x_j^w = \mu' - \delta_w$. A man would run against this candidate if $x_j^m \geq \delta_m + x_j^w = \delta_m + \mu' - \delta_w$. This man would win if $x_j^m - m < m - x_j^w$.

Analysis

- Substituting we get that

$$\begin{aligned} x_j^m - m &< m - x_j^w \\ \delta_m + \mu' - \delta_w - m &< m - \mu' + \delta_w \\ \mu' - m &< \delta_w - \frac{\delta_m}{2} \end{aligned}$$

  Intuition: If cost of running is high, only women with strong pro-women views are willing to run. But then men can defeat these women.

- Condition 2: No woman runs against a man ($\delta_w > m$). Two candidates must be symmetric around median voter for it to be an equilibrium, and will win with probability $\frac{1}{2}$. So the most you can possibly gain is $2m$ with probability $\frac{1}{2}$, or m. But if $\delta_w > m$, even the most extreme woman's cost of running is higher than her expected gain.

Analysis

- Point of Proposition 1: biases $\delta_w > \delta_m$ and μ' mean that without reservations, it is possible that women may never be candidates, biasing the equilibrium outcome away from women.
- What will happen in this case?
  - We'll either get the default u'
  - Or some man will run unopposed.
- Reservations for women allow women to run and can improve women's welfare under these circumstances.
- Propositions 2 and 3 in the paper:
  - Reservations can increase or decrease women's welfare and that of median voter.
  - Increase intuitive (move implemented policy towards median)
  - How could it decrease? No candidate may run, so get default rather than mix of lobbying and citizen-candidate.

How is this different from the median voter model?

- In this model, candidates
  - Can't commit to policies other than their ideal point. What does it mean? Is this a reasonable assumption?
  - Have to pay a cost to run for office. Is this a reasonable assumption?
- The basic setup gets us away from the starkest version of the median voter results
  - However, in the limit as δ gets small, we get back to the median voter result
  - So how far we are from the median voter depends on how large δ is
- With heterogeneity in δ, we can get groups with high δ systematically excluded from policy – even though they vote.

Concluding thoughts

- The Median Voter Theorem provides a strong benchmark for voting models:
  - With 2 candidates, they tend to be towards the median
  - So thinking about who the median voter can be a useful first approximation for policy
  - And changes in who the median voter is produce predictable changes in policy
- But we should think about this as a guideline, not a solid rule

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