14.662 Recitation 3

The Task Model

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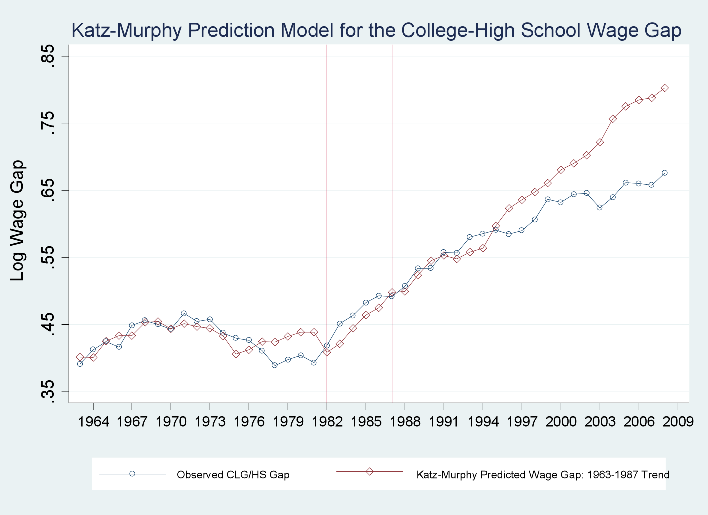
How Does the Canonical Model Fall Short?

1. Wage inequality has risen less than predicted



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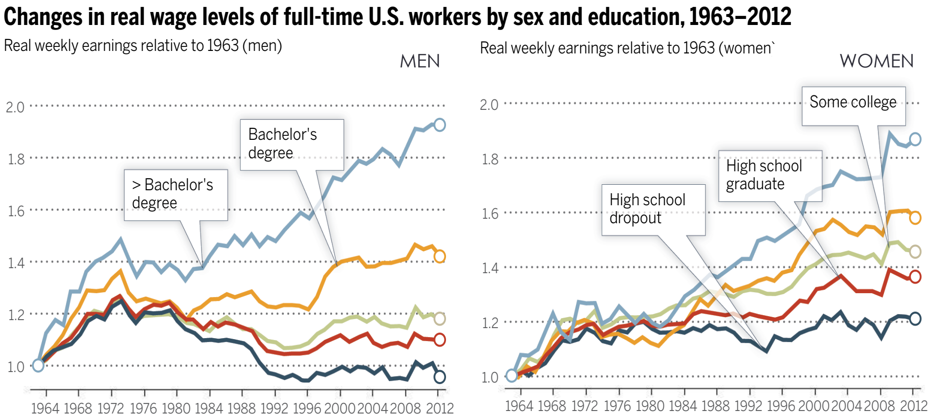
How Does the Canonical Model Fall Short?

2. Real wages have fallen for some education groups



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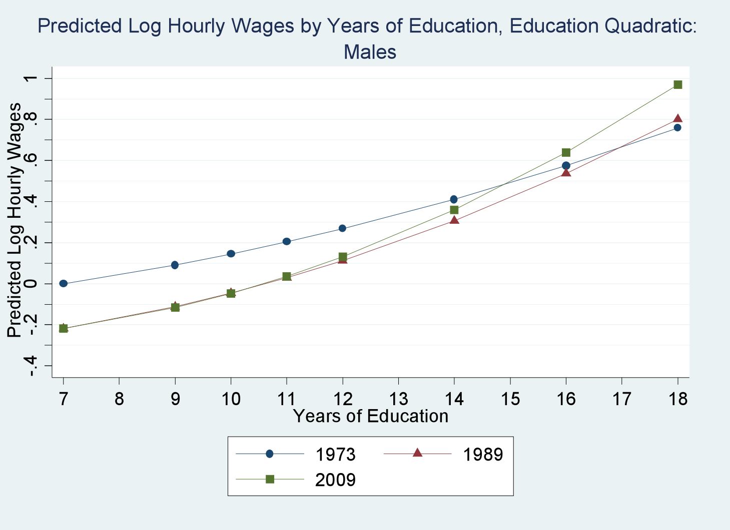
How Does the Canonical Model Fall Short?

3. The returns to education have "convexified"



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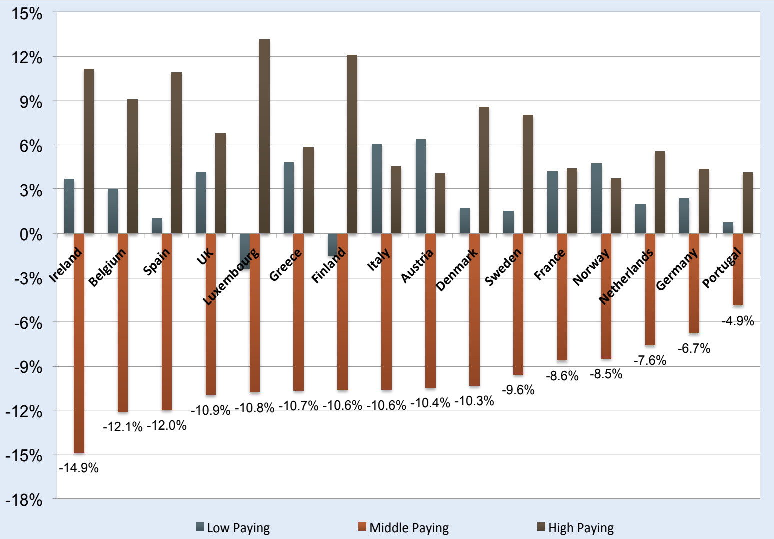
How Does the Canonical Model Fall Short?

4. Occupations and wages have "polarized"



Simplified Model (Acemoglu and Zilibotti, 2001)

- Unique final good produced by a continuum of tasks $i \in [0,1]$:

$$Y = \exp \int_0^1 \ln y(i) di$$

and a fixed, inelastic supply of H and L workers

- Suppose

$$y(i) = (1-i)A_L l(i) + iA_H h(i)$$

$l(i)$, $h(i)$: amount of high-, low-skilled labor set to task i

- Key questions:
  1. How are tasks assigned?
  2. What are equilibrium wages?
  3. What are assignment/wage comparative statics for H, L, A_H, A_L?

Equilibrium Task Assignment

- In eq'm there exists I s.t. $l(i) = 0$ for all $i > I$ and $h(i) = 0$ for all $i < I$
  - I an (endogenous) equilibrium object
- All $i < I$ tasks must pay the same wage:

$$(1 - i)A_L p(i) = (1 - i')A_L p(i')$$

- Equal-share Cobb-Douglas technology implies:

$$\begin{aligned} p(i)y(i) &= p(i')y(i') \\ \Longrightarrow (1 - i)A_L p(i)l(i) &= (1 - i')A_L p(i')l(i') \\ \Longrightarrow l(i) &= l(i') = L/I \end{aligned}$$

- By the same logic, $h(i) = H/(1 - I)$

Characterizing the Equilibrium

- At I, firm must be indifferent between hiring L and H workers

$$(1-I)A_L\frac{L}{I} = IA_H\frac{H}{1-I}$$
$$(1-I)\sqrt{A_L L} = I\sqrt{A_H H}$$
$$\implies I = \frac{\sqrt{A_L L}}{\sqrt{A_H H}+\sqrt{A_L L}}$$

- Wages equal marginal products:

$$w_L = (1-i)p(i)A_L, w_H = ip(i)A_H$$
$$\implies w_L\frac{L}{I} = w_H\frac{H}{1-I}$$
$$\frac{w_H}{w_L} = \frac{1-I}{I}\frac{L}{H} = \sqrt{\frac{A_H L}{A_L H}}$$

What's New with Tasks?

$$\frac{w_H}{w_L} = \frac{1-I}{I}\frac{L}{H}$$

- Suppose instead Y is Cobb-Douglas in L and K:

$$\begin{aligned} Y &= L^{\alpha}H^{1-\alpha} \\ \Longrightarrow \quad w_H &= (1-\alpha)(Y/H) \\ w_L &= \alpha(Y/L) \\ \frac{w_H}{w_L} &= \frac{1-\alpha}{\alpha}\frac{L}{H} \end{aligned}$$

- With tasks "α" (low-skill labor share) is "endogenous" – reacts to supply and technology
- As we'll see this can lead to different (perhaps more realistic) comparative statics

General Setup (Acemoglu and Autor, 2011)

- Three labor types: L, M, and H

$$y(i) = A_L \alpha_L(i) l(i) + A_M \alpha_M(i) M(i) + A_H \alpha_H(i) H(i)$$

- $\alpha_L(i)/\alpha_M(i)$, $\alpha_M(i)/\alpha_H(i)$ continuously differentiable and strictly decreasing in i

- As before,

$$\frac{w_H}{w_M} = \left(\frac{1 - I_H}{I_H - I_L}\right)\left(\frac{H}{M}\right)^{-1}$$

$$\frac{w_M}{w_L} = \left(\frac{I_H - I_L}{I_L}\right)\left(\frac{M}{L}\right)^{-1}$$

Task Model Comparative Statics "Bingo"

	∂I_H	∂I_L	$\partial \frac{I_H}{I_L}$	$\partial \frac{w_H}{w_M}$	$\partial \frac{w_H}{w_L}$	$\partial \frac{w_M}{w_L}$
∂H						
∂M						
∂L						
∂A_H						
∂A_M						
∂A_L						

- $+$, $-$, or ?

Task Model Bingo: Supply and Tasks

	∂I_H	∂I_L	$\partial \frac{I_H}{I_L}$	$\partial \frac{w_H}{w_M}$	$\partial \frac{w_H}{w_L}$	$\partial \frac{w_M}{w_L}$
∂H	$-$	$-$	$-$			
∂M	$+$	$-$	$+$			
∂L	$+$	$+$	$-$			
∂A_H						
∂A_M						
∂A_L						

- Increased supply expands set of tasks performed

Task Model Bingo: Technology and Tasks

	∂I_H	∂I_L	$\partial \frac{I_H}{I_L}$	$\partial \frac{w_H}{w_M}$	$\partial \frac{w_H}{w_L}$	$\partial \frac{w_M}{w_L}$
∂H	$-$	$-$	$-$			
∂M	$+$	$-$	$+$			
∂L	$+$	$+$	$-$			
∂A_H	$-$	$-$	$-$			
∂A_M	$+$	$-$	$+$			
∂A_L	$+$	$+$	$-$			

- Technology increases effective supply; same effects on assignment

Task Model Bingo: Supply and Wages

	∂I_H	∂I_L	$\partial \frac{I_H}{I_L}$	$\partial \frac{w_H}{w_M}$	$\partial \frac{w_H}{w_L}$	$\partial \frac{w_M}{w_L}$
∂H	$-$	$-$	$-$	$-$	$-$	$-$
∂M	$+$	$-$	$+$	$+$	?	$-$
∂L	$+$	$+$	$-$	$+$	$+$	$+$
∂A_H	$-$	$-$	$-$			
∂A_M	$+$	$-$	$+$			
∂A_L	$+$	$+$	$-$			

- Demand curves are downward-sloping

Task Model Bingo: Technology and Wages

	∂I_H	∂I_L	$\partial \frac{I_H}{I_L}$	$\partial \frac{w_H}{w_M}$	$\partial \frac{w_H}{w_L}$	$\partial \frac{w_M}{w_L}$
∂H	$-$	$-$	$-$	$-$	$-$	$-$
∂M	$+$	$-$	$+$	$+$	?	$-$
∂L	$+$	$+$	$-$	$+$	$+$	$+$
∂A_H	$-$	$-$	$-$	$+$	$+$	$-$
∂A_M	$+$	$-$	$+$	$-$	?	$+$
∂A_L	$+$	$+$	$-$	$+$	$-$	$-$

- SBTC shrinks med/low workers task set; increases skill premium

Middle-Skill Technology and Wages

- For $\beta_H(I) \equiv \ln \alpha_M(I) - \ln \alpha_H(I)$ and $\beta_L(I) \equiv \ln \alpha_L(I) - \ln \alpha_M(I)$:

$$\frac{\partial \ln(w_H/w_L)}{\partial \ln A_M} \lesseqgtr 0 \quad \text{and}$$
$$\frac{\partial \ln(w_H/w_L)}{\partial \ln M} \lesseqgtr 0$$
$$\Longleftrightarrow \mid \beta_H'(I_H)(1-I_H) \mid \lesseqgtr \mid \beta_L'(I_L)I_L \mid$$

- When $\beta_L'(I_L)$ is relatively high, low skill workers have a strong comparative advantage for tasks below I_L.
- Effective medium-skill workers will not be displacing low-skill workers as much as they displace high-skill workers
- w_H/w_L must decline.

What Does the Task Model Buy Us?

- Acemoglu and Autor (2011) show it's possible to have

$$\frac{\partial w_M}{\partial A_H} < 0$$

That is, factor-augmenting increase in productivity can reduce the *level* of wages for other groups (by shrinking the set of assigned tasks)

- Machines replacing subset of medium-skill tasks can lead to (Prop 4)

$$\frac{\partial w_H}{\partial w_M}, \frac{\partial w_H}{\partial w_L} > 0$$
$$\frac{\partial w_M}{\partial w_L} < 0$$

which could explain job/wage polarization

Problem Set #1

- Questions?

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