

**14.54 Fall 2016**  
**Recitation: IRS**

1. The objective of this exercise is to re-derive some of the expressions in Lecture 17, and then work out a particular numerical example (if time allows). The demand function faced by a producer of differentiated varieties is given by:

$$Q = S \cdot [1/n - b(P - \bar{P})] \quad (1)$$

where  $\bar{P}$  is the average price in the industry. Each firm has a fixed cost of production equal to  $F$  and a constant marginal cost  $c$ .

- (a) Express the average cost of a firm as a function of  $F$ ,  $c$ , and  $Q$
- (b) Assume that you are in a symmetric equilibrium in which  $P = \bar{P}$ . Use (1) to express average costs in a symmetric equilibrium as a function of  $F$ ,  $c$ ,  $S$ , and  $n$ .
- (c) Invert (1), without imposing symmetry, to express  $P$  as a function of  $Q$ . Use this expression to compute the partial derivative of  $P$  with respect to  $Q$ . Is this positive or negative? Why?
- (d) Remember that marginal revenue is  $MR = P(Q) + P'(Q)Q$ . Use your answer in (c). to express  $MR$  as a function of  $P(Q)$ ,  $Q$ ,  $S$ , and  $b$ .
- (e) Use the optimality condition  $MR = MC$ , to express the optimal price as a function  $c$ ,  $Q$ ,  $S$ , and  $b$ .
- (f) Assume now that you are in a symmetric equilibrium in which  $P = \bar{P}$ . As in (b), use (1) to express the optimal price in a symmetric equilibrium as a function of  $c$ ,  $n$ , and  $b$ .
- (g) Use your answers in (b). and (f) to derive the equilibrium values of  $n$  and  $P$  in a long-run equilibrium in which free entry drives each firm's price down to the value of average cost. Express your answers as functions of the parameters  $F$ ,  $S$ , and  $b$ .
- (h) Solve for the implied sales  $Q$  by each firm as a function of  $F$ ,  $S$ , and  $b$ .
- (i) Consider the case in which, for the Home country, we have  $S = 1440$ ,  $b = 2$ ,  $F = 5$ , and  $c = 0.1$ ; and for the Foreign country, we have  $S^* = 250$ ,  $b = 2$ ,  $F = 5$ , and  $c = 0.1$ . Compute the long-run equilibrium values of  $n$ ,  $n^*$ ,  $P$ ,  $P^*$ ,  $Q$  and  $Q^*$  under autarky.
- (j) Compute the long-run equilibrium values of  $n$ ,  $P$ , and  $Q$  in a world integrated equilibrium with a value of  $S$  equal to  $S + S^*$ , i.e.,  $S^W = 1690$ .

- (k) How does trade integration affect the prices charged by firms and the overall number of varieties available in the world economy? Suppose consumers value being able to consume a larger number of varieties. Is trade welfare-enhancing?
- (l) What happens to the number of firms (or varieties produced) in each country? Can they both increase?

2. We consider the same monopolistic competition model as in Exercise 1, but let the demand function faced by a producer of differentiated varieties be given by:

$$Q = \frac{S}{n} \cdot \left(\frac{P}{\bar{P}}\right)^{-2} \quad (2)$$

where  $\bar{P}$  is the average price in the industry. Each firm has a fixed cost of production equal to  $F$  and a constant marginal cost  $c$ .

- (a) What is the market share of each firm in a symmetric equilibrium in which all firms charge the same price?
- (b) Use (2), without imposing symmetry, to express  $P$  as a function of  $Q$ ,  $S$ ,  $n$ , and  $\bar{P}$ .
- (c) Use this expression to compute the partial derivative of  $P$  with respect to  $Q$ , i.e.  $P'(Q)$ . Use the expression you derived in (a) to express  $P'(Q)Q$  as a function of  $P(Q)$  only.
- (d) Use your result in (c) to express  $MR = P(Q) + P'(Q)Q$  as a function of  $P(Q)$  only.
- (e) Use the optimality condition  $MR = MC$ , to express the optimal price as a function of  $c$  only. By how much do prices go up when the marginal cost doubles? Compare your result to that obtained in Exercise 1 (e). What explains the difference?
- (f) Derive the equilibrium values of  $n$  and  $P$  in a long-run symmetric equilibrium in which free entry drives each firm's price down to the value of average cost.
- (g) Solve for the implied sales  $Q$  by each firm as a function of  $F$  and  $c$ .
- (h) Discuss the effects of an trade integration (and increase in  $S$ ) on the equilibrium values of  $n$ ,  $P$ , and  $Q$ . How are these effects different from the ones derived in class and in Exercise 1? What explains the difference?
- (i) Discuss the welfare effects of trade integration assuming that consumers value being able to consume a larger number of varieties.

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