14.471: Fall 2012: Recitation 11: Dynamic insurance

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1 Highlights/Questions on pages 1-3 of Lecture Notes on Dynamic Insurance?

- Idioyncratic, privately observed taste shocks affecting the MU of consumption
- The FB is history independent but not incentive compatible
- Create incentives by making consumption history dependent: incentivize agents with low current marginal consumption to report this by promising high consumption in the future at the expense of lower consumption today
- Solve the dual version of the SB planning problem
  - minimize discounted expected ressource cost
  - s.t. planner can keep promise of delivering utility v_0
  - IC to report type
- Recursive planning problem

2 Four Discussion points from pages 4-5 of Lecture Notes on Dynamic Insurance

Agent's value function/promised utility v and consumption c are geometric random walks with drifts

- A geometric random walk $\{x_t; t \geq 0\}$is a time series such that the relative increments are i.i.d. $R_t = \frac{x_t}{x_{t-1}}$

There exists a shadow interest rate q that makes the drift zero

- If there is such a interest rate, then we solved the original problem

Growing inequality is optimal and immiseration in the limit

The allocation is history contingent

- unlike the first best
- this is not surprising since for example an Aiygari incomplete market allocations is also history dependent
- just like there, however, there is a state variable that summarizes the past; instead of assets, it is utility

3 Generalization: Dynamic programming for a dynamic mechanism design problem

3.1 Most General set-up

- Define v as the utility promised by the planner to the agent
- Write the Bellman equation where we:
  - minimize the net present value of the ressource cost to promise utility v
  - the promised utility v is the expected net present value of the current flow utility and the discounted continuation utility
  - the agent with taste shock θ has no incentive to misreport his type

$$K(v) = \min \mathbf{E}\left[C(x(\theta)) + qK(w(\theta))\right]$$

$$v = \mathbf{E}\left[u(x(\theta), \theta) + \beta w(\theta)\right]$$

$$u(x(\theta), \theta) + \beta w(\theta) \geq u(x(\theta'), \theta) + \beta w(\theta')$$

- x can be a vector

3.2 Leading case 1: Atkeson-Lucas (Restud,1992)

- x is just consumption level/a scalar
- Then we have:

$$u(x(\theta), \theta) = \theta u(x)$$

- The ressource cost of consumption $C(x) = x$

3.3 Leading case 2: Dyamic mirrlees: Albanesi-Sleet

- x is a vector with consumption and output

$$x = (c, y)$$

- Utility depends on output, consumption and type

$$u(x, \theta) = U(c, y; \theta)$$

- The net ressource cost to deliver a vector x is given by$C(x) = c - y$
- Let us use the usal trick for characterizing the IC

$$K(v) = \min \mathbf{E}\left[c(x(\theta)) + qK(w(\theta))\right]$$

$$v = \mathbf{E}\left[v(\theta)\right]$$

$$v'(\theta) = u_\theta\left(x(\theta), \theta\right)$$

$$v(\theta) = u(x(\theta), \theta) + \beta w(\theta)$$

- In Mirrlees we write

$$K(v) = \min_{v(\theta), y(\theta), w(\theta)} \mathbf{E}\left[e(v(\theta), y(\theta), \theta) - y(\theta) + qK(w(\theta))\right]$$

$$v'(\theta) = u_\theta\left(x(\theta), \theta\right)$$

$$v(\theta) = u(e(v(\theta), y(\theta), \theta) + \beta w(\theta)$$

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