14.462 Lecture Notes
Complementarities and Coordination

George-Marios Angeletos

MIT Department of Economics

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1 Motivation

Why care?

A large number of economic interactions are characterized by strategic complementarities, can be modeled as coordination games, and often exhibit multiple equilibria sustained by self-fulfilling expectations. Prominent examples of coordination games in macroeconomics include:

- self-fulfilling bank runs (Diamond and Dybvig, 1983)

- currency crises (Obstfeld, 1986, 1996; Velasco, 1996)

- debt crises (Calvo, 1988; Cole and Kehoe, 1996)

- financial crashes (Freixas and Rochet, 1997; Chari and Kehoe, 2003a,b)

- regime switches (Cooper and John, 1988; Chamley, 1999).

Strategic complementarities also arise in the following contexts:

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- business cycles with production externalities or other complementarities (Bryant, 1983; Baxter and King, 1991; Benhabib and Farmer, 1994; Cooper and Johri, 1997; Shleifer, 1986)

- endogenous growth with spillovers (Romer, 1986; Lucas, 1988)

- network externalities, technology adoption (Katz and Shapiro, 1985, 1986; Farell and Saloner, 1985)

- imperfect market competition (Milgrom and Roberts, 1990; Vives 1999; Kiyotaki, 1988)

- thick-market externalities (Diamond, 1982; Murphy, Shleifer and Vishny, 1989),

- restricted market participation (Azariadis, 1981)

- incomplete financial markets (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Angeletos, 2003).

- lobbying, political reforms, revolutions, riots, and social change (Kuran, 1987; Atkeson, 2000; Battaglini and Benabou, 2003).

And so? What do complementarities deliver?

- amplification

- persistence

- commovement

- synchronization

- multiplicity

- animal spirits

2 Framework

We consider a simple canonical model of investment complementarity. The economy is populated by a continuum of measure one of agents, indexed by i and uniformly distributed over the $[0,1]$ interval. Each agent engages in some economic activity (investment). Like in Bryant (1983), Cooper and John (1988), Acemoglu (1993), etc., the individual return to investment is increasing in the aggregate level of investment, thus introducing strategic complementarity in agents' investment choices.

Payoff. The payoff for agent i is given by

$$u_i = U(k_i, K, \theta) = A(K, \theta)k_i - c(k_i), \tag{1}$$

where $k_i \in \mathcal{K} \subseteq \mathbb{R}$ is the action of agent i, $K = \int k_i di$ is the corresponding aggregate, $\theta \in \mathbb{R}$ is an exogenous random variable, $A : \mathcal{K} \times \mathbb{R} \to \mathbb{R}$ and $c : \mathcal{K} \to \mathbb{R}$. We interpret k_i as investment (or effort, or other economic activity), A as the gross return to investment, c as the cost of investment, and θ as exogenous productivity (the underlying economic fundamentals). It is then natural to assume that A is increasing in θ and c is increasing in k_i. We finally introduce an investment complementarity by assuming that A is increasing in K.[1]

Best Response. We are interested in the case that agents have heterogeneous information about θ. For a moment, however, suppose θ is common knowledge. The best response of agent i is then given by[2]

$$k_i = g(K, \theta) \equiv \arg \max_{k \in \mathcal{K}} U(k_i, K, \theta). \tag{2}$$

It follows that, if A increases with K, so does g. Hence, complementarity means that the optimal investment for agent i increases with the aggregate investment in

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[1] Provided differentiability, this is equivalent to imposing $U_{kK} > 0$.

[2] For expositional simplicity, it is useful to assume that c is strictly convext, so that the best response is a continuous function. But similar arguments can be made when the best response is a correspondance.

the economy, or equivalently that the best response function is positively sloped. Without serious loss of generality, we can let $c(k) = k^2/2$, in which case

$$k_i = g(K, \theta) = A(k, \theta).$$

g thus inherits exactly the properties of A.

Equilibrium. A symmetric equilibrium is an equilibrium where all agents choose the same action. The set of symmetric equilibrium thus coincides with the set of fixed of g. That is, in a symmetric equilibrium, aggregate investment solves

$$K = g(K, \theta).$$

Complementarity and Multiplicity. The steepness of the best response represents the strength of the complementarity. Two possibilities then arise. Either the slope of the best response function is always less than one, in which case the equilibrium is unique; or the slope is higher than one over some range, in which case multiple equilibria are possible. It is in this sense that we draw a distinction between *weak* and *strong* complementarity. The two cases are illustrated in Figure 1. The case of weak complementarity corresponds to the black solid curve, which interests only once with the 45-degree line; point A then gives the unique equilibrium of the economy. The case of strong complementarity corresponds to the grey solid curve, which intersects three times with the 45-degree line; points B and C then give the two stable equilibria of the economy, whereas point A now represents an unstable equilibrium.

Ideally, one would like to allow for a general specification of investment complementarities as described above. However, when agents have heterogeneous information about θ, such level of generality makes the model totally intractable. For that reason, in the lectures to follow, we restrict attention to two special specifications which turn out to be tractable. The one specification leads to a linear best response like the black dashed line in Figure 1 and is meant to capture the case of weak complementarity (unique equilibrium). The other specification leads to a best response like the grey dashed line in Figure 1 and is meant to capture the case of strong complementarity
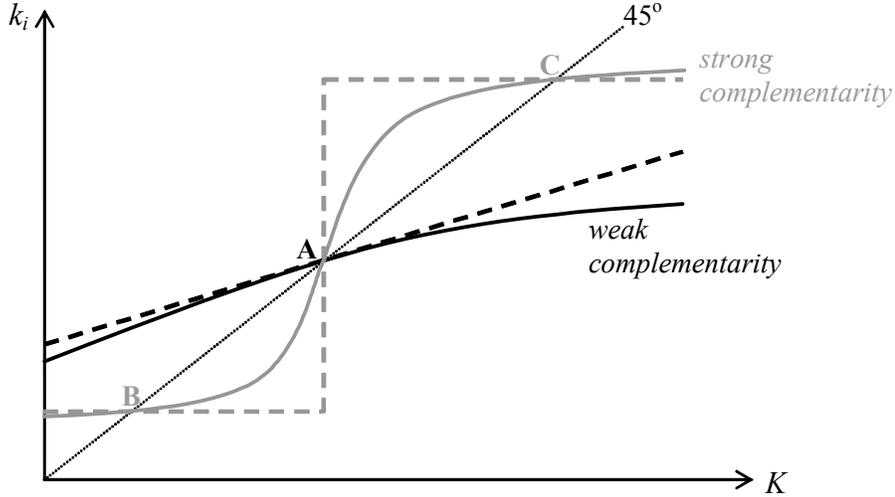
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Figure 1: Best responces with weak and strong complementarity.

(multiple equilibria). For the time being, however, we continue discussing some of the generic properties of models with complementarities.

Pareto Ranking. In this framework, multiple equilibria can be Pareto ranked. In particular, consider two equilibria, the first associated with aggregate investment $K = K_1$ and the second with $K = K_2 > K_1$; the second equilibrium Pareto dominates the first. This ranking is because the complementarity is associated with a positive externality. Indeed, in this framework, $U_{kK} > 0$ if and only if $U_K > 0$. See Cooper and John (1988) for more discussion.

Amplification. Let K solve $K = g(K, \theta)$ and suppose $g_K \neq 0$. Then, by the IFT, K is (locally) a function of θ, that is, $K = K(\theta)$. Moreover,

$$\frac{dK}{d\theta} = \frac{g_\theta}{1 - g_K}.$$

Suppose $g_\theta > 0$, so that we can interpret higher θ with higher productivity. In any stable equilibrium, $g_K < 1$, and because of the complementarity, $g_K > 0$. Hence, $\frac{dK}{d\theta} > g_\theta$, whereas in the absence of complementarities $\frac{dK}{d\theta} = g_\theta$. The quantity $\frac{1}{1-g_K}$

represents a (local) multiplier effect.

Persistence. Suppose there are many periods and $A_t = A(K_t, K_{t-1}, \theta_t)$, with all derivatives of A positive. In equilibrium,

$$K_t = g(K_t, K_{t-1}, \theta_t),$$

so that

$$K_t = K(K_{t-1}, \theta_t),$$

Hence, a dynamic complementarity can give rise to endogenous persistence.

Commovement. Suppose productivity is idiosyncratic, so that

$$k_i = g(K, \theta_i),$$

where θ_i is distributed with density $f(\theta)$. An equilibrium is now a rule $k(\theta)$ such that

$$k(\theta) = g\left(\int k(z)f(z)dz, \theta\right).$$

In the absence of complementarity, $k(\theta_i) = g(\theta_i)$ so that the investment of i depends only his own productivity. If instead there are complementarities, the investment of i depends also on the productivities of other agents. In particular, if θ increases for a (non-zero-measure) subset of the agents, investment will increase for all agents.

Synchronization. If agents take discrete choices (lumpy investment, attack a peg, etc.) and there are complementarities, then there is a incentive to synchronize actions.

3 Empirical Evidence

Discuss Bartelsman, Caballero and Lyons (1994): Demand- versus supply-driven complementarities in the panel of industries.

Discuss Cooper and Haltiwanger (1996): Commovement and seasonality in the panel of industries.

Discuss Baxter and King (1991) and Cooper and Johri (199?): Complementarities in the aggregate production function.