14.452. Topic 7. Nominal rigidities

Olivier Blanchard

April 2007

1. Motivation, and organization

Why introduce nominal rigidities, and what do they imply?

- In monetary models, the price level (the price of goods in terms of money) behaved like an asset price.

$$M/P = CL(i) = CL(r + \pi^e)$$

  Changes in i, either from changes in r or π^e led to a change in the price level today.

- Example: Cagan specification, in discrete time:

$$m_t - p_t = -\alpha(E[p_{t+1}|\Omega_t] - p_t)$$

  Solving forward:

$$p_t = \frac{1}{1+\alpha}(\sum (\frac{\alpha}{1+\alpha})^i E[m_{t+i}|\Omega_t])$$

- The price level is not an asset price.

  It is an aggregate of millions of individual prices, each of them set by a price setter, at discrete intervals in time. So, it is unlikely to adjust in the manner above.

- If P adjusts more slowly, then what will happen? If the equation above still holds, then the nominal interest rate will not move in the same way.

  An increase in M will lead to a *decrease* in the nominal interest rate, and likely a *decrease* in the real interest rate.

- If the demand for goods is given by the same equations as before, it will move differently from before (go back to the FOC for consumers, or the q theory characterization for investment. Both depend on the sequence of current and anticipated real rates.)

- What will happen to output? This depends on how the price (wage) setters decide to respond to shifts in demand.

- The older fix price equilibrium line of research–Barro, Grossman, Malinvaud: Output will be given by the minimum of demand and supply at the given price. Died, rightly so.

- If firms have monopoly power, they may want to accomodate these shifts so long as $P > MC$. So movements in demand, both positive or negative will have an effect on output, at least within some range (so long as $MC < P$).

Proceed in three steps.

- Build a static model (simplified BK). Monopolistic competition, price setting. Effects of demand, money, on output, welfare.

- Look at price staggering more closely. Different rules, different structures. (Fischer, Taylor, Calvo, SS rules)

- Integrate monopolistic competition, and price staggering in a model with C/S, L/N and $C, M/P$ choices. **New Keynesian model.** Modern version of IS-LM-AS.

A one-period model of yeomen farmers

Simplified version of Blanchard-Kiyotaki. Instead of having price and wage setting, just price setting.

1. A description of the economy

- Large number of households, each producing a differentiated good, and each consuming all goods. More specifically, a continuum of households and goods on $[0\ 1]$.
- Each household produces its good using its own labor (this way we integrate producers and suppliers of labor, and have to keep track only of prices, not wages and prices).

The utility function of a household i is given by:

$$U(C_i, \frac{M_i}{P}, N_i)$$

where:

$$C_i \equiv [\int_0^1 C_{ij}{}^{(\sigma-1)/\sigma} dj]^{\sigma/(\sigma-1)} \qquad \sigma > 1$$

$$P = [\int_0^1 P_j{}^{1-\sigma} dj]^{1/(1-\sigma)}$$

The budget constraint is given by:

$$\int_0^1 P_j C_{ij} + M_i = P_i Y_i + \bar{M}_i$$

and the production function for producing good i is given by:

$$Y_i = Z N_i$$

Things to note about the model:

- A one-period problem. No uncertainty. Both relaxed later.
- Each household consumes a consumption basket, composed of all goods.
- Each household needs money for transactions; this is formalized by putting money in the utility function rather a CIA constraint.
- Each household produces a differentiated good using labor and a constant returns technology. Z is the level of technology. We shall think of movements in Z as technological shocks.
- Each household faces a demand curve for its product, which we shall have to derive (the demand for the good by all other consumers.)
- The budget constraint is a short cut to a dynamic budget constraint.

Easy to characterize the equilibrium of the model with a general utility function. Even easier to do it with the following utility function:

$$U(C_i, \frac{M_i}{P}, N_i) = (\frac{C_i}{\alpha})^{\alpha} (\frac{M_i/P}{1-\alpha})^{1-\alpha} - \frac{1}{\beta} N_i{}^{\beta}$$

Yields a very simple relation between consumption and real money balances, and constant marginal utility of income.
To characterize the general equilibrium, proceed in 4 steps:

- Given spending on consumption, derivation of consumption demands for each good by each household.
- Derivation of the relation between aggregate consumption and aggregate real money balances.
- Derivation of the demand curve facing each household, and derivation of its pricing decision, with and without nominal rigidities.
- General equilibrium. with and without nominal rigidities.

2. The demand for individual goods

Suppose household i spends a nominal amount X_i on consumption. How does it allocate this spending across goods?

$$\max C_i \equiv [\int_0^1 {C_{ij}}^{(\sigma-1)/\sigma} dj]^{\sigma/(\sigma-1)} \quad \text{subject to} \quad \int_0^1 P_j C_{ij} dj = X_i$$

Then, with a bit of algebra:

$$C_{ij} = \frac{X_i}{P}(\frac{P_j}{P})^{-\sigma} \quad \text{and} \quad C_i P = X_i$$

where P is the price index we wrote earlier, so we can rewrite the consumption demand for good j as;

$$C_{ij} = C_i(\frac{P_j}{P})^{-\sigma}$$

Given overall consumption C_i, demand for each good function of the relative price P_j/P.

3. The choice of money and consumption

Using what we just learned, we can rewrite the choice of consumption versus money balances as:

$$\max \left(\frac{C_i}{\alpha}\right)^{\alpha} \left(\frac{M_i/P}{1-\alpha}\right)^{1-\alpha}$$

subject to:

$$PC_i + M_i = P_i Y_i + \overline{M_i}$$

Given income and initial money balances, can solve for optimal consumption and money balances:

$$C_i = \alpha \frac{P_i Y_i + \bar{M}_i}{P}, \qquad \frac{M_i}{P} = (1-\alpha)\frac{P_i Y_i + \bar{M}_i}{P}$$

People allocate their initial wealth in proportion α and $1-\alpha$ to consumption and real money balances.

- For future use, the following FOC between the two will be useful: Relation between real money balances and consumption (both endogenous):

$$C_i = \frac{\alpha}{1-\alpha}\frac{M_i}{P}$$

- This implies that the demand for good j by household i can be written as:

$$C_{ij} = C_i(\frac{P_j}{P})^{-\sigma} = \frac{\alpha}{1-\alpha}\frac{M_i}{P}(\frac{P_j}{P})^{-\sigma}$$

- Replacing C_i and M_i/P in the utility function gives an indirect utility function of the form:

$$\frac{P_i}{P}Y_i - (1/\beta){N_i}^{\beta} + \frac{\bar{M}_i}{P}$$

3. Pricing and output decisions

Household i then chooses the price and the level of output of good i:

$$\max \frac{P_i}{P} Y_i - (1/\beta) {Y_i}^{\beta} Z^{-\beta}$$

where $N_i = Z^{-1} Y_i$, and we can ignore $(\bar{M}_i/P)$ in the objective function. Integrating over households j, the demand for good i is given by:

$$Y_i = \int_0^1 C_{ji} \, dj = \frac{\alpha}{1-\alpha} \frac{M}{P} (\frac{P_i}{P})^{-\sigma}$$

where $M = \int_0^1 M_j dj$. Using the fact that, in equilibrium, the money balances households want to hold must be equal to the nominal money stock, so $M = \bar{M}$, then:

$$Y_i = \frac{\alpha}{1-\alpha} \frac{\bar{M}}{P} (\frac{P_i}{P})^{-\sigma}$$

Solving the maximization problem gives:

$$\frac{P_i}{P} = \frac{\sigma}{\sigma - 1} Y_i^{(\beta-1)} Z^{-\beta}$$

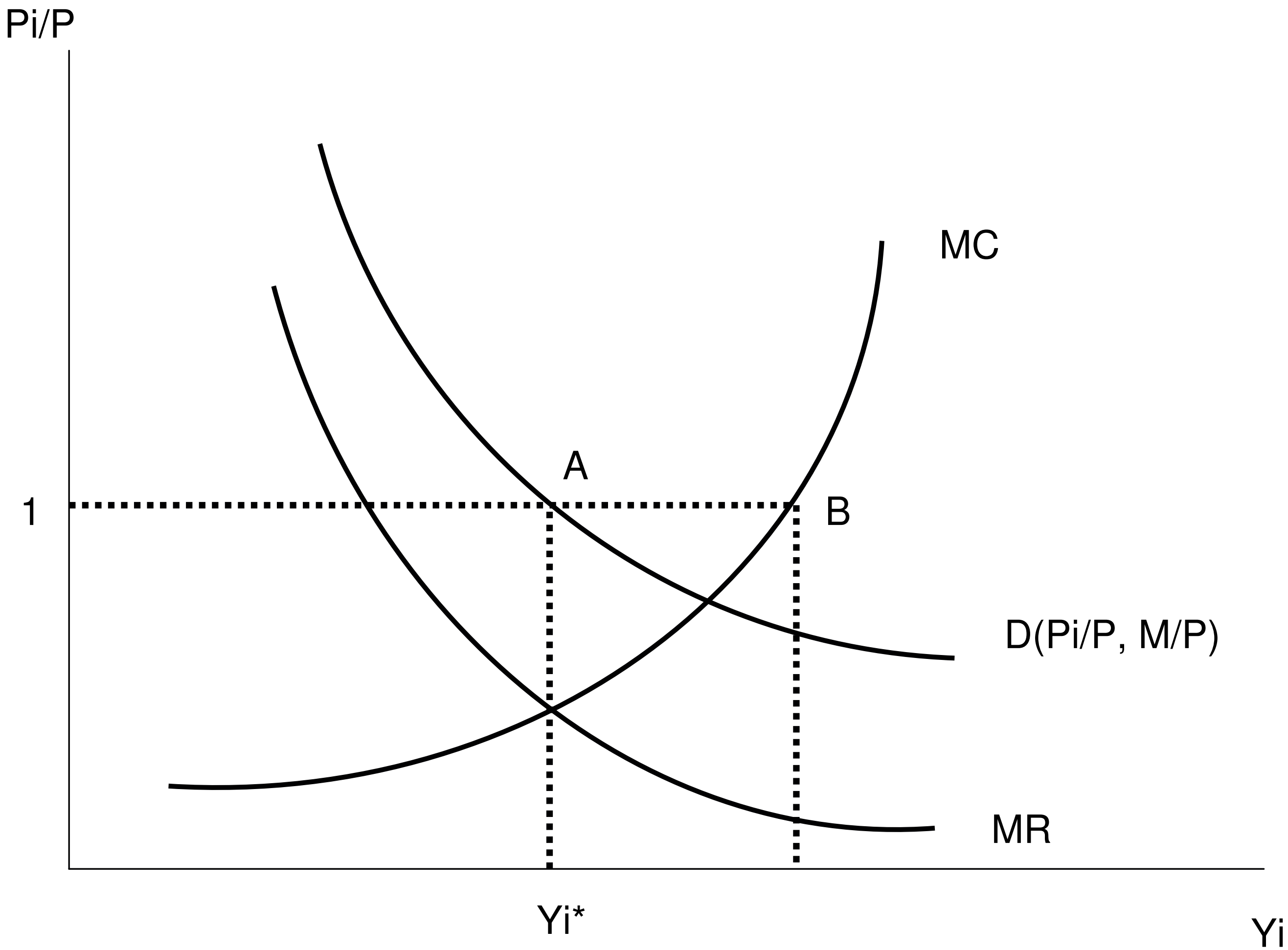
Price equals marginal cost times a markup. Solving for Y_i gives:

$$\frac{P_i}{P} = [\frac{\sigma}{\sigma - 1} X^{(\beta-1)} Z^{-\beta}]^{1/(1+\sigma(\beta-1))} \quad \text{where} \quad X \equiv \frac{\alpha}{1-\alpha}\frac{\bar{M}}{P}$$

An increase in $\bar{M}/P$ leads to an increase in P_i/P. The effect depends on β and σ. The closer β is to unity, the smaller the effect on P_i/P.
Can characterize the equilibrium graphically:

- Demand is a function of relative price, and real money balances. Marginal revenue as well. Marginal cost increasing in output.
- Equilibrium at point A. (In GE, it must be that $P_i/P = 1$. Next step.)

Equilibrium under monopolistic competition.



4. General equilibrium

In general equilibrium, the relative price must be equal to 1. So, output for each household must be such that this holds:

$$1 = \frac{\sigma}{\sigma - 1} Y^{(\beta-1)} Z^{-\beta}$$

so:

$$Y = [\frac{\sigma - 1}{\sigma} Z^{\beta}]^{\frac{1}{\beta-1}}$$

and:

$$N = [\frac{\sigma - 1}{\sigma} Z]^{\frac{1}{\beta-1}}$$

So lower equilibrium output than under perfect competition. But only a small modification, for the presence of a markup. Output is lower than first best. Technological shocks increase both employment and output. The more so, the closer β is to one.

The price level must be such that the real money stock generates the right level of demand:

$$Y = \frac{\alpha}{1-\alpha}\frac{\bar{M}}{P} \;\Rightarrow\; P = \frac{\alpha}{1-\alpha}\frac{\bar{M}}{Y}$$

Little progress?

- Output determined by: marginal cost plus markup equals price.
- Nominal money neutral.

But in fact, closer:

- First, a model with aggregate demand. An effect of real money balances. Clearly simplistic, but we know how to extend it.
- Second, a model with price setters. So we can look at how they set prices, and what determines the price level.

Some intuition for price level determination.

- Consider an increase in nominal money, from M to M'.
- Requires a proportional increase in P, **no change** in relative prices.
- Nobody is in charge of the price level. Each price setter tries to adjust its relative price. If β not too far above 1, then relative prices increase only a little.
- And then a bit more, and so on, until the price level has adjusted. Then: same relative prices, higher price level.

Suggests that the adjustment may be slow, and that the speed depends on how much price setters want to adjust their relative price. Now ready to introduce nominal rigidities.

5. Yeomen farmers and nominal rigidities

Think of the households having to set nominal prices. Two arguments for why they may want to do this at discrete intervals.

- Menu costs. (Akerlof Mankiw) Small changes in prices (equivalently, small deviations of prices from optimum) have only a second order effect on profit.

  But a small change in the price level has a first order effect on output and welfare. Why? Because of the initial wedge created by monopoly power. Back to diagram.

- Desired change in relative price may be small. Go back to the equation for P_i/P earlier. If MC is relatively flat ($\beta - 1$ close to zero), then want to change the relative price by little.

So modify the model as follows:

- Each household chooses the price of its product before knowing the realization of nominal money and productivity this period.
- Consumption decisions, and thus demand, are taken after observing the realization.

Return to the choice of the relative price by households.

$$\max E[\ \frac{P_i}{P} Y_i - \frac{1}{\beta} Y_i{}^{\beta} Z^{-\beta}]$$

subject to:

$$Y_i = \frac{\alpha}{1-\alpha} \frac{\bar{M}}{P} (\frac{P_i}{P})^{-\sigma} \equiv X\ (\frac{P_i}{P})^{-\sigma}$$

The difference is that $\bar{M}$ and Z are now random variables. The FOC is given by:

$$E[X(1-\sigma)(\frac{P_i}{P})^{-\sigma} + \sigma X^{\beta} Z^{-\beta} (\frac{P_i}{P})^{-\beta\sigma-1}] = 0$$

Or, rearranging:

$$\frac{P_i}{P} = \left(\frac{\sigma}{\sigma-1} \frac{E[X^{\beta} Z^{-\beta}]}{E[X]} \right)^{1/(1+\sigma(\beta-1))}$$

The higher expected nominal money, the higher the relative price.

6. General equilibrium with nominal rigidities

In equilibrium, all price setters must set prices so that the relative price is equal to 1. So, the price level is implicitly determined by:

$$1 = \frac{\sigma}{\sigma - 1} \frac{E[X^{\beta} \; Z^{-\beta}]}{E[X]}$$

where $X \equiv (\alpha/(1-\alpha))\bar{M}/P$.

Demand and output (as long as $MC < P$) are given by:

$$Y = (\alpha/(1-\alpha))\bar{M}/P$$

and employment is given by:

$$N = Z^{-1} Y$$

Implications

- Unanticipated movements in $\bar{M}$ affect $\bar{M}/P$ one for one and so affect C and Y one for one.
- Demand affects output, so long as $MC < P$—so suppliers willing to supply. Up to B in earlier diagram.
- No systematic movement in relative prices (in real wages in a model with a labor market). Fits the data well.
- Welfare goes up and down with Y. Indeed, higher than expected money is good. Temptation to increase welfare by unexpectedly increasing money?

- Unanticipated technological shocks have no effect on demand and thus on output this period.

  (Extreme result. No longer true in more realistic model where demand depends on more than real money balances).

- Unanticipated technological shocks decrease employment initially (i.e during the period during which prices are predetermined).

7. A useful log linear version

Can either do a log linearization around the equilibrium, or under the assumption that M and Z are jointly log-normal, can derive exact log linear relation (in which second moments appear). Shall follow the first route.

$$(p_i - p) == \frac{\beta - 1}{1 + \sigma(\beta - 1)}(Em - p) - \frac{\beta}{1 + \sigma(\beta - 1)}Ez$$

Role of β in effect of $Em - p$ on $p_i - p$. So, if $p_i = p$:

$$p = Em - \frac{\beta}{\beta - 1}Ez$$

$$y = m - p = (m - Em) + \frac{\beta}{\beta - 1}Ez$$

$$n = y - z$$

where lower case letters indicate log deviations from steady state.

- Absent nominal rigidities (we shall call this level of output the second best level and denote it by a hat):

$$\hat{y} = \frac{\beta}{\beta - 1} z, \qquad \hat{n} = \frac{1}{\beta - 1} z$$

- Second-best output does not respond to money, but responds to technological shocks.

- Actual output responds to unexpected money, but not to unexpected technological shocks.

Optimal monetary policy? A very first pass.

Say, minimize distance from second best, $y - \hat{y}$. If central bank can adjust money after having observed z, then easy:

$$m - Em = \frac{\beta}{\beta - 1}(z - Ez) \Rightarrow y - \hat{y} = 0$$

Increase money in the face of positive productivity shocks, so as to increase demand in line with supply, and get employment to increase rather than decrease.

Why not do even better and try to further increase welfare and achieve first best $y_{FB} =$ constant $+ \hat{y}$?

Can clearly do it ex-post by increasing m further. What is the problem? What will agents expect ex-ante? (This is the problem known as time inconsistency. More on this later.)

8. Summary

- Price setting. Relative prices function of aggregate demand.
- Money still neutral.
- With nominal rigidities, money affects output.
- Increases in money increase output and welfare.

Next steps:

- More realistic price setting. Dynamics.
- Richer specification of aggregate demand.