1 The consumer's budget constraint and his/her assets

- One can imagine the models in two ways:

1. First, with **positive supply of loans:**
    - There is an innovation sector, which borrows to finance its input costs (either in units of labor or final goods), and it pays interest on its loans
    - Loans are held by agents
2. Second, with **zero supply of loans:**
    - In this case, the research part of the labor force L_R is paid not in terms of current wages, but in terms of future profits (think of this as an "equity" stake in a firm in the innovation sector)

2 Example: Knowledge spillovers model

- Reminder: Equilibrium quantities and prices were

$$\begin{aligned} Y &= \frac{1}{1-\beta} N L_E \\ X &= (1-\beta) N L_E \\ w &= \frac{\beta}{1-\beta} N = \eta N V \\ \pi &= \beta L_E \end{aligned} \tag{1}$$

- Let us first determine C from the resource constraint of the economy,

$$C + X = Y$$

so

$$C = \frac{1-(1-\beta)^2}{1-\beta} N L_E = \frac{2\beta - \beta^2}{1-\beta} N L_E$$

- We now write down the specific budget constraints corresponding to the two interpretations above

2.1 First interpretation: Positive supply of loans

- We can express the consumer's budget constraint as

$$C + \dot{A} \leq rA + wL$$

where $A \equiv NV$ is the total value of outstanding loans. Notice:

$$rA = N\pi + N\dot{V} = N\pi + \dot{A} - \dot{N}V$$

giving

$$C = rA - \dot{A} + wL = N\pi + wL - \dot{N}V = N\pi + wL_E,$$

where in the last equality we used that

$$\dot{N} = \eta N L_R = \frac{wL_R}{V}$$

using the expression for the wage (1).

- This yields the exact same expression for consumption,

$$C = N\pi + wL_E = \beta N L_E + \frac{\beta}{1-\beta} N L_E = \frac{2\beta - \beta^2}{1-\beta} N L_E.$$

2.2 Second interpretation: Zero supply of loans and equity stakes

- In this case, consumption is given by

$$C = \underbrace{wL_E}_{\text{current labor income}} + \underbrace{N\pi}_{\text{dividends from equity holdings}}$$

and this is again the same as before,

$$C = N\pi + wL_E = \beta N L_E + \frac{\beta}{1-\beta} N L_E = \frac{2\beta - \beta^2}{1-\beta} N L_E.$$

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