14.451: Introduction to Economic Growth
Problem Set 2

Due date: February 28, 2007.

Question 1: Consider the augmented Solow model as in Mankiw, Romer and Weil (1992):

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}, \quad \text{with } 0 < \alpha, \beta < 1.$$

In what follows, let lower case letters denote quantities scaled by effective labor, e.g. $k \equiv \frac{K}{AL}$. Let *s denote steady state values.

1. State the accumulation equations for physical and human capital.

2. Log-linearize the accumulation equation in the neighborhood of the steady state. Write them in the form:

$$\begin{bmatrix} \frac{\dot{k}}{k} \\ \frac{\dot{h}}{h} \end{bmatrix} = M \begin{bmatrix} \ln\left(\frac{k}{k^{*}}\right) \\ \ln\left(\frac{h}{h^{*}}\right) \end{bmatrix}$$

for some matrix M. Interpret this matrix. Why does the growth rate of one accumulable factor depend upon the deviation of the other factor from its steady state value?

3. Use the production function to write the deviation of y from its steady state value as a function of the deviations of physical and human capital from their steady state values. That is, write:

$$\ln\left(\frac{y}{y^{*}}\right) = N \begin{bmatrix} \ln\left(\frac{k}{k^{*}}\right) \\ \ln\left(\frac{h}{h^{*}}\right) \end{bmatrix}$$

for some matrix N.

4. Derive the growth rate of the economy as a function of the deviations of physical and human capital from their steady state values:

$$\frac{\dot{y}}{y} = P \begin{bmatrix} \ln\left(\frac{k}{k^{*}}\right) \\ \ln\left(\frac{h}{h^{*}}\right) \end{bmatrix}$$

Find matrix P.

5. Can you write the growth rate $\frac{\dot{y}}{y}$ as a function of $\ln\left(\frac{y}{y^*}\right)$ only? Why/why not? Compare the dynamics here to the case in the lecture notes (page 56).

Question 2: Consider the following continuous time discounted infinite horizon problem:

$$\max \int_0^\infty \exp(-\rho t)\, u\,(c\,(t))\, dt$$

subject to

$$\dot{x}\,(t) = g\,(x\,(t)) - c\,(t)$$

with initial condition $x\,(0) > 0$.

Suppose that $u\,(\cdot)$ is strictly increasing and strictly concave, with $\lim_{c \to \infty} u'\,(c) = 0$ and $\lim_{c \to 0} u'\,(c) = \infty$, and $g\,(\cdot)$ is strictly concave with $\lim_{x \to \infty} g'\,(x) = 0$ and $\lim_{x \to 0} g'\,(x) = \infty$.

1. Set up the current value Hamiltonian and derive the Euler equations for an optimal path.

2. Show that the standard transversality condition and the Euler equations are necessary and sufficient for a solution.

Question 3: Consider a variant of the neoclassical economy in which preferences are given by

$$U\,(0) = \int_0^\infty \exp(-\rho t)\, \frac{(c\,(t) - \gamma)^{1 - \frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}}$$

with $\gamma > 0$. There is no population growth. Assume that the production function is given by $Y\,(t) = F\,[K(t), A\,(t)\, L\,(t)]$, which satisfies all the standard assumptions and $A\,(t) = \exp(gt)\, A\,(0)$.

1. Interpret the utility function.

2. Define the competitive equilibrium for this economy.

3. Characterize the equilibrium of this economy. Does a balanced growth path with positive growth in consumption exist? Why or why not?

4. Derive a parameter restriction ensuring that the standard transversality condition is satisfied.

5. Characterize the transitional dynamics of the economy.