

14.41 Midterm Solutions

October 6, 2010

1 Question 1

Please write whether the following claims are true, false, or uncertain. No credit will be awarded without a clear, well-reasoned explanation. In the interest of time, be concise!

- (5 pts) **Claim:** Majority voting may violate transitivity.
 - True. Majority voting violates transitivity if agents preferences are left unrestricted. For example, suppose we have three agents, A, B, C , and 3 choices, 1, 2, 3. Suppose

$$\begin{aligned}1 &\succ^A 2 \succ^A 3 \\3 &\succ^B 1 \succ^B 2 \\2 &\succ^C 3 \succ^C 1\end{aligned}$$

Then, a majority prefers 1 to 2, a majority prefers 2 to 3, and a majority prefers 3 to 1.

- (5 pts) 1,000 residents live in the town of Madison, where city planners are deciding whether to build a \$10,000 public park. A survey has asked residents how much they would be willing to pay for the park. **Claim:** As long as residents indicate their willingness to pay to be, on average, at least \$10 per resident, the park should be built.
 - False. Surveys may not elicit an individuals true willingness to pay.

2 Question 2

The city of Newtown is considering building a public subway system to help relieve urban traffic congestion. Newtown politicians in favor of the project argue that the system would benefit the population by drastically reducing commuting time, not to mention the jobs the project would create during the subway construction process. The city planning office has hired you to perform

a cost-benefit analysis to determine whether the project should be undertaken. You have been provided with the following information:

- In 2008, Oldtown (a city neighboring Newtown) built a subway system similar to the one proposed in Newtown. The following chart shows total annual commuting hours for both cities in the years 2007 and 2009:

	2007	2009
Oldtown	10,000,000	9,500,000
Newtown	7,500,000	8,000,000

1. (5 pts) Using the available data, construct the best estimate of the effect of the subway on commuting hours in Oldtown. Describe any concerns about the validity of this estimate.

- A difference-in-differences estimate is given by $(9.5M-10M)-(8M-7.5M)=-1M$, implying a reduction of 1,000,000 commuting hours. The estimate is valid under the assumption that Oldtown and Newtown were on parallel trends and would have remained on parallel trends in the absence of the subway being built.

2. (7 pts) Labor and construction materials are needed to build the subway. Newtown's labor market is competitive, and the wage rate is \$20/hr. Total labor costs would come to \$90,000,000. However, monopolies own the materials factories, and therefore material costs would come to \$150,000,000. A reputable economist has estimated that the cost of materials would fall to \$100,000,000 if they could be purchased in a competitive market (which they can't, politicians lament).

All costs would be paid upfront and the subway would be built instantaneously. After project completion, Newtown would annually experience the time-savings reduction in commuting hours found in part 1. (Note: if you were unable to solve part 1, denote the estimate of time-saving by X and continue for partial credit.) If the interest rate is $r=10\%$, calculate the PDV of this project and use this value to recommend whether or not the subway system should be built.

- Labor costs are \$90M. Material costs are \$150M, but \$50M of this is a transfer to union workers, and therefore not a social cost. Therefore, the total costs are $\$90M+\$100M=\$190M$. The benefits are $\$1Mhr*\$20/hr=\$20M/year$, so that the PDV of the project is $\$20M/0.1-\$190M=\$200M-\$190M=\$10M>0$, so the project should be built.

3 Question 3

Scientists and economists predict that climate change will eventually have detrimental effects on global output. For this question, suppose there are only two countries: US and China. Suppose that

each additional unit of carbon emitted causes a reduction of future incomes. In particular, the best estimates suggest that every additional unit of carbon, c , causes a future reduction in output of both 9 units in the US and 1 unit in China, for a total of 10 units. The benefits of abatement do not depend on where it takes place (this means that one unit of carbon reduced in the US leads to an increase in output of both 9 units in the US and 1 unit in China; similarly, one unit of carbon reduced in China leads to an increase in output of both 9 units in the US and 1 unit in China)

Suppose there are only two firms in the world that emit carbon: one that operates in the US and another that operates in China. Each firm has a capability of reducing its carbon output, but can do so only at a cost. For the US firm, the cost of reducing carbon is given by $C^{US}(x) = x + 2x^2$, where x is the amount of carbon that is abated. In China, the cost of reducing carbon is given by $C^{China}(x) = x + x^2$.

- (7 pts) From a world perspective, calculate the socially efficient level of abatement for the US and China. Which country produces more abatement? Why?

- The social benefit of carbon reduction is 10. Therefore, the optimal levels of abatement are

$$\begin{aligned}\frac{dC^{US}}{dx} &= 10 \\ \frac{dC^{China}}{dx} &= 10\end{aligned}$$

or

$$\begin{aligned}1 + 4x^{US} &= 10 \\ x^{US} &= \frac{9}{4}\end{aligned}$$

and

$$\begin{aligned}1 + 2x^{China} &= 10 \\ x^{China} &= \frac{9}{2}\end{aligned}$$

China produces more abatement.

- (4 pts) Suppose the UN can institute a global carbon tax. What level of tax should it impose to ensure that all firms reach the social optimum?.

- The optimal subsidy would be $\tau = 10$ (or a tax of -10 units).

- (7 pts) Now, suppose only private governments will choose their taxes taking into account the benefit in their own country. What tax will they choose (assuming no cooperation between the US and China)?

- The US faces a benefit of 9 units and would choose to subsidize $\tau^{US} = 9$. China faces a benefit of only 1 unit and would choose to subsidize $\tau^{China} = 1$.

4. (7 pts) Suppose that the taxes in (4) are implemented. Calculate the level of abatement in each country. Which country produces more abatement? Why? Calculate the DWL.

- Facing a subsidy of 9 units, the firm in the US would choose

$$\begin{aligned}\frac{dC^{US}}{dx} &= 9 \\ 1 + 4x^{US} &= 9 \\ x^{US} &= 2\end{aligned}$$

The firm in China will face a subsidy of 1 unit and would choose

$$\begin{aligned}\frac{dC^{China}}{dx} &= 1 \\ 1 + 2x^{China} &= 1 \\ x^{China} &= 0\end{aligned}$$

The social welfare is given by

$$\begin{aligned}SWF &= 2 * 10 - C^{US}(2) - C^{China}(0) \\ &= 20 - 2 - 2 * 4 \\ &= 10\end{aligned}$$

and under the socially optimal level, social welfare is

$$\begin{aligned}& 10 * \left(\frac{9}{4} + \frac{9}{2}\right) - C^{US}\left(\frac{9}{4}\right) - C^{China}\left(\frac{9}{2}\right) \\ &= 90 * \frac{3}{4} - \frac{9}{4} - 2 \left(\frac{9}{4}\right)^2 - \frac{9}{2} - \left(\frac{9}{2}\right)^2 \\ &= \frac{243}{8}\end{aligned}$$

so that the DWL is given by

$$\begin{aligned}DWL &= \frac{243}{8} - 10 \\ &= \frac{163}{8}\end{aligned}$$

Question 4 (38 points)

In the small town of Barksdale live only two families, each with a single child. Barksdale has a single private school with a single teacher providing education to these children, whose names are Abigail and Bernard. At the start of each school year, a teacher with E units of quality is hired, where each unit of quality costs $p=\$10$. Each family contributes toward the teacher quality, so that if Abigail's family pays for E_A quality units and Bernard's family pays for E_B quality units, a teacher of quality $E = E_A + E_B$ is hired.

Each family has an annual income of \$300, and splits their income between paying for education quality and all other consumption spending X . Family utility functions are

$$\begin{aligned} \text{Abigail :} & \quad U_A = \frac{2}{3} \log(X_A) + \frac{1}{3} \log(E_A + E_B) \\ \text{Bernard :} & \quad U_B = \frac{2}{3} \log(X_B) + \frac{1}{3} \log(E_A + E_B) \end{aligned}$$

1. (10 pts) Under private provision of teacher quality, what quality of teacher is hired?

We look for a Nash Equilibrium solution E_A^* and E_B^* . Abigail takes Bernard's purchase E_B as given and chooses E_A to solve

$$\max_{E_A} U_A = \alpha \log(W - pE_A) + (1 - \alpha) \log(E_A + E_B), \quad \text{where } \alpha = \frac{2}{3}.$$

By the first order condition to this maximization problem, Abigail's choice E_A solves

$$\frac{\alpha p}{W - pE_A} = \frac{(1 - \alpha)}{E_A + E_B} \quad (1)$$

By symmetry, Bernard choice E_B solves

$$\frac{\alpha p}{W - pE_B} = \frac{(1 - \alpha)}{E_A + E_B} \quad (2)$$

We obtain the Nash Equilibrium solution by solving Equations 1 and 2 for E_A and E_B . Since the right-hand sides of both equations are equal, it must be that $E_A^* = E_B^*$. Plugging this condition back into Equation 1, we rearrange and get

$$E_A^* = E_B^* = \frac{(1 - \alpha) W}{(1 + \alpha) p} = \frac{1/3 \cdot 300}{5/3 \cdot 10} = 6$$

So a teacher of quality $E^* = E_A^* + E_B^* = 12$ is hired.

2. (7 pts) If instead Barksdale's government mandates how much each family must pay toward education, how much should each family be required to spend on education in order to maximize social surplus? Does quality of teacher will be hired differ from the quality hired in part 1? Why or why not?

The government chooses E_A and E_B to solve

$$\max_{E_A, E_B} U_A + U_B = \alpha \log(W - pE_A) + \alpha \log(W - pE_B) + 2(1 - \alpha) \log(E_A + E_B),$$

where again $\alpha = \frac{2}{3}$. The first order conditions with respect to E_A and E_B are

$$E_A : \frac{\alpha p}{W - pE_A} = \frac{2(1 - \alpha)}{E_A + E_B}$$
$$E_B : \frac{\alpha p}{W - pE_B} = \frac{2(1 - \alpha)}{E_A + E_B}$$

We solve these two equations for E_A and E_B . As in part 1, $E_A^* = E_B^*$. We plug this constraint into either of the first order conditions and solve to get

$$E_A^* = E_B^* = (1 - \alpha) \frac{W}{p} = \frac{1}{3} \frac{300}{10} = 10$$

So the government requires both families to purchase 10 units of teacher quality, resulting in a teacher quality of $E^* = 20$ being hired. Each family pays more toward education than in part 1, because here the government mandated level internalizes the positive externality that purchases of education quality units exert on other families.

Job changes for Abigail's parents force them to move to the very large city of Hugeville, where private schools come in all different levels of quality E . The cost per unit of quality is $p=\$10$. Abigail's family splits their income of $\$300$ between paying for a school of quality E and all other consumption spending X , giving family utility

$$U_A = \frac{2}{3} \log(X) + \frac{1}{3} \log(E)$$

3. (6 pts) What quality of education will Abigail's family purchase?

Abigail solves

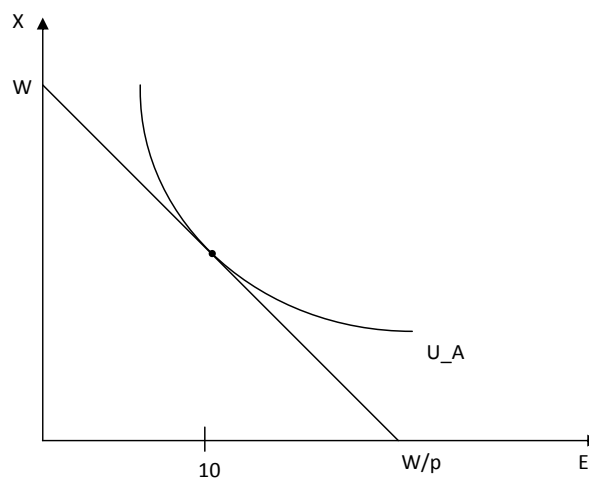
$$\max_E U_A = \alpha \log(W - pE) + (1 - \alpha) \log(E), \quad \text{where } \alpha = \frac{2}{3}.$$

The first order condition to this problem gives

$$\frac{\alpha p}{W - pE} = \frac{(1 - \alpha)}{E}$$

We solve for E to get

$$E^* = (1 - \alpha) \frac{W}{p} = \frac{1}{3} \frac{300}{10} = 10$$



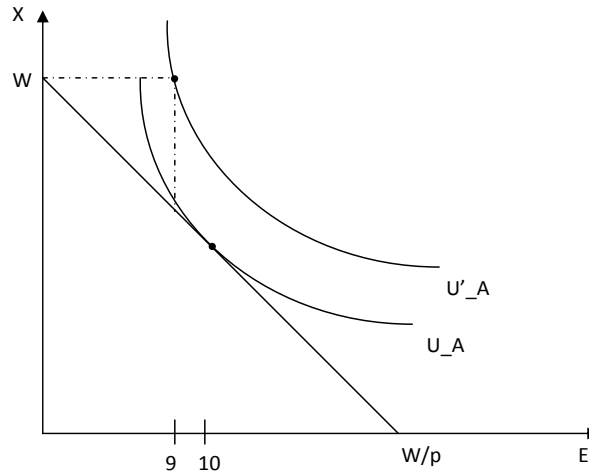
4. (8 pts) Due to credit market failures, many poor students in Hugeville receive an education of quality less than 6. In response, Hugeville has decided to offer a public school option which provides $E_{pub} = 9$ units of quality at no charge. Students may choose to enroll in private school as before, or they may choose to forgo private education and accept the public option.

- How much education quality will Abigail's family purchase? Illustrate their budget set and optimal allocation graphically.
- Briefly discuss: Will this intervention increase every student's education quality? Why or why not?

Abigail either pays to go to private school (and chooses $E = 10$ as shown in part 3) or goes to public school for free. I.e., Abigail will go to public school if and only if $U_A(W, 9) - U_A(W - p * 10, 10) \geq 0$. By properties of the log function, we calculate

$$\begin{aligned}
 U_A(W, 9) - U_A(W - p * 10, 10) &= \frac{2}{3}[\log(W) - \log(W - p * 10)] + \frac{1}{3}[\log(9) - \log(10)] \\
 &= \frac{2}{3} \log\left(\frac{W}{W - p * 10}\right) + \frac{1}{3} \log\left(\frac{9}{10}\right) = \frac{2}{3} \log\left(\frac{3}{2}\right) + \frac{1}{3} \log\left(\frac{9}{10}\right) \\
 &= \frac{1}{3} \log\left(\left(\frac{3}{2}\right)^2 * \frac{9}{10}\right) = \frac{1}{3} \log\left(\frac{81}{40}\right) \\
 &> 0
 \end{aligned}$$

So Abigail goes to public school. Note that for families with preferences and wealth similar to Abigail's family, the public school option actually *decreases* the amount of education quality received. This is because while Abigail receives slightly less education quality, her family gets the public option for free, resulting in a net private gain.



5. (7 pts) Suppose that instead of offering a public school option, the government of Hugeville simply gave each student a voucher worth 9 units of education quality.

- How much education quality will Abigail's family purchase? Illustrate their budget set and optimal allocation graphically. Does this intervention crowd out Abigail's family's private education spending, and if so by how much?
- Briefly discuss: Will this intervention increase every student's education quality?

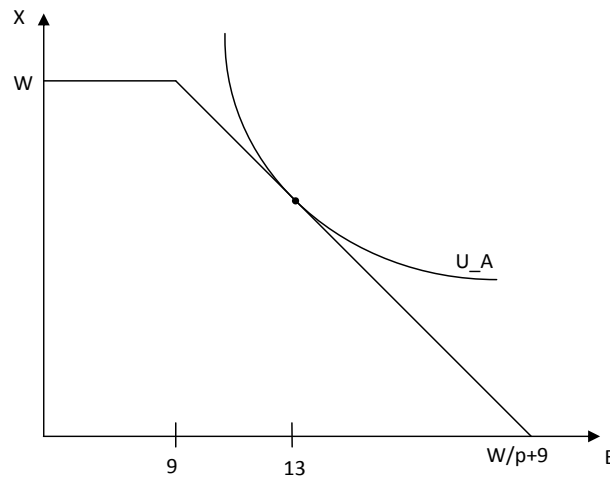
Compared to no government intervention, this intervention will increase education quality demanded by everyone. This is because you can get 9 units for free (so nobody will demand less than 9 units). And for families who already were demanding 9 or more units of quality, the voucher acts as a pure wealth transfer. Since education is a normal good, these folks will increase the number of education quality units demanded.

Thus, since Abigail's family chose $10 > 9$ units of quality in part 3, they now solve that same problem but with effective wealth $\tilde{W} = W + p * 9 = 390$. Thus, Abigail will choose

$$E^* = (1 - \alpha) \frac{\tilde{W}}{p} = \frac{1}{3} \frac{390}{10} = 13$$

There is no crowd-out if Abigail's family increases the education quality from part 3 (10 units) by the amount of the voucher (9 units). Since Abigail's family only chooses 13 units here, there is $19 - 13 = 6$ units of crowd-out. Thus,

$$\text{crowd-out} = p * 6 = \$60.$$



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14.41 Public Finance and Public Policy
Fall 2010

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