Problem Set #2 (Solutions)
14.41 Public Economics

DUE: October 1, 2010

Question 1

ⓐ State Provision Only

1. Maine maximizes utility by solving
$$\max_{\{G,E\}} U = \alpha \ln(G) + (1-\alpha)\ln(E) \quad \text{s.t. } G + p_E E \le W$$

From utility maximization theory, we know utility is maximized when the marginal rate of substitution is equal to the price ratio, and when all money in the budget is spent. I.e.,

$$(1) \quad \frac{1}{p_E} = \frac{\partial U/\partial G}{\partial U/\partial E} = \frac{\alpha/G^*}{(1-\alpha)/E^*} \Rightarrow (1-\alpha)G^* = \alpha p_E E^*$$

$$(2) \quad G^* + p_E E^* = W \Rightarrow (1-\alpha)G^* + (1-\alpha)p_E E^* = (1-\alpha)W$$

$$\Rightarrow \boxed{E^* = (1-\alpha)\frac{W}{p_E} \qquad G^* = \alpha W}$$

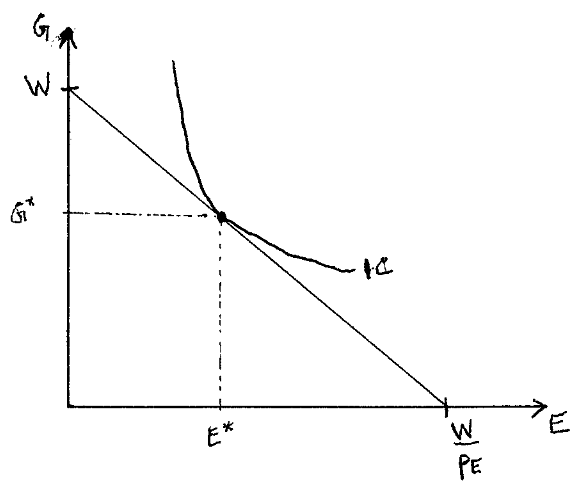
2. From E^* and G^* calculated above in part 1,

$$\left\{\begin{array}{l}\text{fraction of } W \\ \text{spent on education}\end{array}\right\} = \frac{p_E E^*}{W} = \frac{p_E}{W}(1-\alpha)\frac{W}{p_E} = (1-\alpha)$$

$$\left\{\begin{array}{l}\text{fraction of } W \\ \text{spent on other public goods}\end{array}\right\} = \frac{G^*}{W} = \frac{\alpha W}{W} = \alpha$$

Neither of these ratios depends on W or p_E. Note that this is an artifact of log utility, and not a general principal.

3.



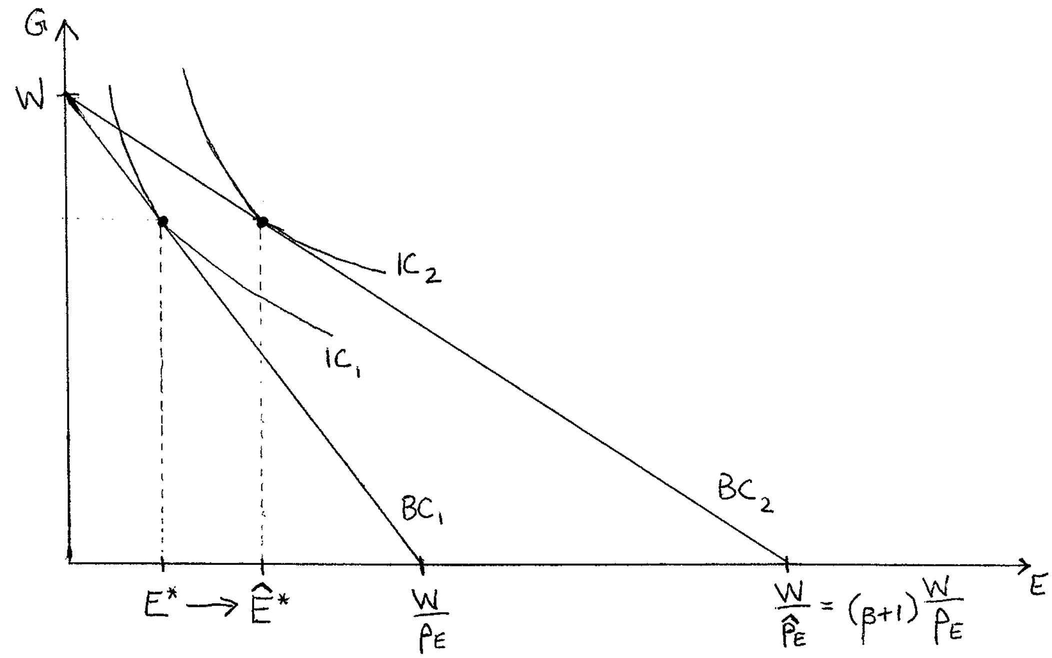
Question 1 (continued)

(b) Federal Matching Grant

1. Under this grant, Maine's effective price per unit of E is $\boxed{\hat{p}_E = \frac{p_E}{1+\beta}}$.

2. We expect education quality provided to increase under this proposal, through both an income effect and a substitution effect. The lower effective price makes education relatively cheaper, resulting in more education provided through the substitution effect. The federal grant makes Maine wealthier, and since education is a normal good here, more education is provided through the income effect.

3.



4. This is the same problem Maine faced in part (a), except now facing price $\hat{p}_E$ for education. So the amount of education provided is

$$\hat{E}^* = (1-\alpha)\frac{W}{\hat{p}_E} = (1+\beta)\underbrace{\left[(1-\alpha)\frac{W}{p_E}\right]}_{=E^*} = (1+\beta)E^*$$

So education provision increases by a factor of $(1+\beta)$.

Question 1 (continued)

(b) 4. (continued)

Maine and the federal government split the bill for the cost of $\hat{E}^*$: for every \$(1+β) spent, Maine pays \$1 and the federal gov't pays \$β.

Thus, Maine pays $\frac{p_E \hat{E}^*}{1+\beta} = p_E E^* = (1-\alpha)W$

and the federal gov't pays $\beta \frac{p_E \hat{E}^*}{1+\beta} = \beta p_E E^* = \boxed{\beta(1-\alpha)W}$ ← size of the federal grant

5. Maine still spends $(1-\alpha)W$ of its own resources on E, hence the grant does not crowd out state spending. Note that this is, again, an artifact of log preferences, and not a general point.

(c) Federal Block Grant of size $B = \beta(1-\alpha)W$, from part (b) above.

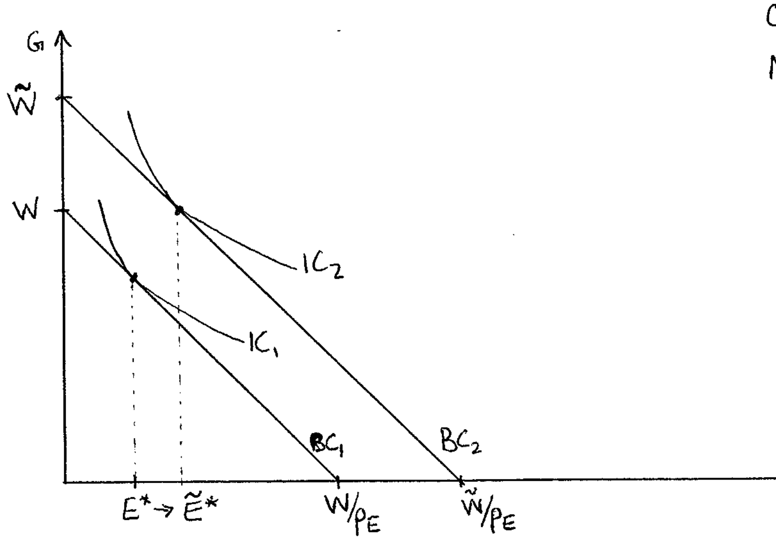
1. Block grants are simply an infusion of cash, and do not distort prices. So Maine ~~p~~ faces the same price p_E as in (a).

2. Because block grants only increase wealth but do not distort prices, provided education will rise only through an income effect.

Old wealth $= W$

New wealth $= W + B = (1+\beta(1-\alpha))W \equiv \tilde{W}$

3.



Question 1 (continued)

(c) 4. From (a), $\tilde{E}^* = \frac{(1-\alpha)\tilde{W}}{p_E} = \underbrace{\frac{1-\alpha}{p_E}}_{=E^*} W[1+\beta(1-\alpha)] = E^*[1+\beta(1-\alpha)]$

5. Now we compare $\tilde{E}^*$ from (c) and $\hat{E}^*$ from (b):

$\tilde{E}^* = E^*(1+\beta(1-\alpha))$ ← income effect only

$\hat{E}^* = E^*(1+\beta)$ ← income & substitution effects

since $\alpha \in (0,1)$, $\hat{E}^* > \tilde{E}^*$

(d) Federal Conditional Block Grant of size $B = \beta(1-\alpha)W$, from (b)

1. Maine's utility max problem under the conditional grant is the same as the unconditional grant, except now there is an additional constraint $E \geq \frac{B}{p_E}$, which says all grant funds must be spent on E.

If the solution $\tilde{E}^*$ to the unconditional grant satisfies $\tilde{E}^* \geq \frac{B}{p_E}$, we can ignore this constraint, and the resulting solution is the same as in (c). But if $\tilde{E}^* < \frac{B}{p_E}$, then the constraint binds, and all grant money is spent on E, and all state resources W are spent on G.

For what parameter values is $\tilde{E}^* \geq \frac{B}{p_E}$?

$$\tilde{E}^* \geq \frac{B}{p_E} \iff E^*[1+\beta(1-\alpha)] \geq \beta\frac{(1-\alpha)W}{p_E} = \beta E^*$$

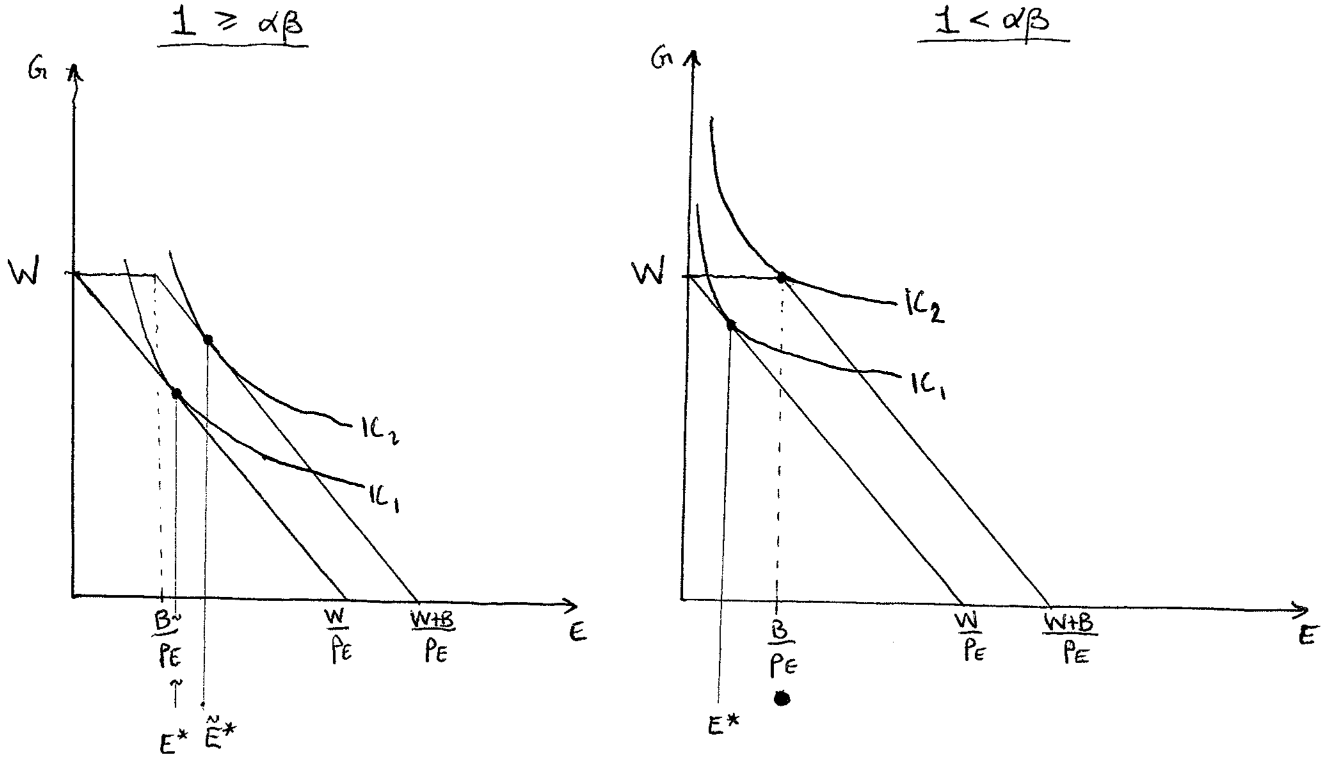
$$\iff 1+\beta(1-\alpha) \geq \beta$$

$$\iff 1 \geq \alpha\beta$$

So provision of education under a conditional grant is <u>the same</u> as under the unconditional grant if <u>$1 \geq \alpha\beta$</u>. If <u>$1 < \alpha\beta$</u>, the provision of education under the conditional grant is <u>larger</u> than under the unconditional grant.

Question 1 (continued)

d) 2.



3. Per the discussion in d.1.,

if $1 \geq \alpha\beta$, $\breve{E}^* = \tilde{E}^*$ units provided, as in c

if $1 < \alpha\beta$, $\breve{E}^* = \frac{B}{P_E}$ ~~units~~ units provided, and

$$\breve{E}^* > \tilde{E}^*$$

4. Consider the case where $1 \geq \alpha\beta$, so the theoretical prediction is that the same amount of education will be provided as in part c.
The "flypaper effect" is an empirical observation that directed money (e.g. a conditional grant) sticks to its target, with realized crowd-out less than predicted. In this case, $\breve{E}^* > \tilde{E}^*$ even if $1 \geq \alpha\beta$.

Question 1 (continued)

(e) • The Matching Grant in (b) yields the highest provision of education $\hat{E}^* = (1+\beta)E^*$. This is clearly bigger than the amount of education supplied under the unconditional block grant ($\tilde{E}^* = (1+\beta(1-\alpha))E^*$) or under the conditional block grant ($\check{E}^* = \tilde{E}^*$, or $\check{E}^* = \frac{B}{p_E} = \frac{\beta(1-\alpha)W}{p_E} = \beta E^*$).

The intuition here is that the Matching Grant encourages a larger supply of education both through a wealth effect, as well as by a price effect (distorting down Maine's effective price of education).

• The unconditional block grant in (c) gives Maine the highest utility. You could check this by plugging allocations into the state's utility function and seeing which is biggest, but there is an easier way. We know from maximization theory that adding addition constraints to a maximization problem can only lead to lower maximum values of the objective function. For grants of the same dollar value (all three grants above satisfy this), the grant with the fewest constraints will give Maine the highest utility. Hence, the unconditional block grant is the best for Maine from an overall welfare perspective.

Question 2

1. Pure public goods must be both non-rival and non-excludable. As long as citizens are free to walk on all paths in Wheredat, then clearing weeds is non-excludable. As long as path congestion is not a problem, it is also non-rival.

2. (a) The earl solves

$$\max_{X_e, A_e} U_e = \log(X_e) + 2\log(A_e + A_p) \quad \text{s.t.} \quad X_e + A_e \leq 15$$

The pleb solves

$$\max_{X_p, A_p} U_p = \log(X_p) + \log(A_p + A_e) \quad \text{s.t.} \quad X_p + A_p \leq 10$$

We will have $A_e > A_p$ because ① the earl is richer than the pleb (an income effect) and ② because the earl has a larger MRS, coming from the larger coefficient on the $\log(A_p + A_e)$ term.

(b) We know that budget constraints will bind in the solution, so

$$X_e + A_e = 15 \quad \text{and} \quad X_p + A_p = 10$$

Plugging into the objective function gives

earl

$$\max_{A_e} U_e = \log(15 - A_e) + 2\log(A_e + A_p)$$

FOC $\Rightarrow$ $$-\frac{1}{15 - A_e} + \frac{2}{A_e + A_p} = 0$$

pleb

$$\max_{A_p} U_p = \log(10 - A_p) + \log(A_e + A_p)$$

FOC $\Rightarrow$ $$-\frac{1}{10 - A_p} + \frac{1}{A_e + A_p} = 0$$

This gives us two equations in two unknowns. We solve to find the solution to be

$$\boxed{\begin{aligned} A_e^* &= 10 \\ A_p^* &= 0 \end{aligned}}.$$

So the pleb doesn't buy any weed clearing, while the earl provides 10 units.

Resulting utilities:

$$U_e^* = \log(15 - 10) + 2\log(10) = \ln(500) \approx 6.21$$
$$U_p^* = \log(10 - 0) + \log(10) = \ln(100) \approx 4.61$$

Social surplus = 10.82

(c) This is like the mansion owner (discussed in lecture) who plows the drive even though the shack-dweller sharing the drive does not contribute.

Question 2 (continued)

3. ⓐ The government chooses A to solve: $\max_A U_p + U_e = \log(15 - A/2) + 2\log(A) + \log(10 - A/2) + \log(A)$.

$$\text{FOC} \Rightarrow \frac{-\frac{1}{2}}{15 - A/2} + \frac{2}{A} + \frac{-\frac{1}{2}}{10 - \frac{A}{2}} + \frac{1}{A} = 0$$

$$\Rightarrow \frac{3}{A} = \frac{1}{30-A} + \frac{1}{20-A} = 0 \quad \text{do some rearranging}$$

$$\Rightarrow 1800 - 200A + 5A^2 = 0$$

$$\Rightarrow A \in \{13.675, 26.32\} \quad \Rightarrow \boxed{A^* = 13.675}$$

this value gives both citizens $-\infty$ (actually, undefined) utility

Utilities here are
$$U_e^* = \log(15 - \tfrac{A^*}{2}) + 2\log(A^*) \cong 7.33$$
$$U_p^* = \log(10 - \tfrac{A^*}{2}) + \log(A^*) \cong 3.77$$
Social Surplus $\cong$ 11.10

The earl's utility increased from $6.21 \rightarrow 7.33$, the pleb's utility went down from $4.61 \rightarrow 3.77$. But total surplus increased from $10.82 \rightarrow 11.1$.

ⓑ
$$MRS_e = \frac{\partial U_e}{\partial A} \Big/ \frac{\partial U_e}{\partial x} = \frac{2X_e^*}{A^*} = \frac{2(15 - \frac{A^*}{2})}{A^*} \approx 1.19$$
$$MRS_p = \frac{\partial U_p}{\partial A} \Big/ \frac{\partial U_p}{\partial x} = \frac{X_p^*}{A^*} = \frac{10 - \frac{A^*}{2}}{A^*} \approx 0.23$$
$$\Rightarrow MRS_e + MRS_p > 1$$
the price ratio here.

The sums of the individuals' MRS do not equal the price ratio here because the ability of earls the pretend they are plebs adds a constraint to the government's maximization problem, keeping the government from obtaining the largest possible amount of social surplus. The solution here is the best the gov't can do under this constraint.

Question 2 (continued)

3. (c) The government chooses A_e and A_p to maximize

$$\max_{A_e, A_p} U_e + U_p = \log(15 - A_e) + 2\log(A_e + A_p) + \log(10 - A_p) + \log(A_e + A_p)$$

FOCs:

$$[A_e]: \quad -\frac{1}{15 - A_e} + \frac{3}{A_e + A_p} = 0$$

$$[A_p]: \quad -\frac{1}{10 - A_p} + \frac{3}{A_e + A_p} = 0$$

$$\Rightarrow \boxed{A_e^* = 10, \quad A_p^* = 5} \qquad \text{Total } A^* = A_e^* + A_p^* = 15$$

amount earl gets taxed (A_e^*), amount pleb gets taxed (A_p^*)

$$MRS_e = \frac{2x_e^*}{A^*} = \frac{2(15 - A_e^*)}{A^*} = \frac{10}{15}$$

$$MRS_p = \frac{x_p^*}{A^*} = \frac{10 - A_p^*}{A^*} = \frac{5}{15}$$

$MRS_e + MRS_p = 1$, So here the sums of the individuals' MRS equals the price ratio of 1.

$$U_e^* = \log(15 - A_e^*) + 2\log(A^*) = \log(5) + 2\log(15) \simeq 7.03$$

$$U_p^* = \log(10 - A_p^*) + \log(A^*) = \log(5) + \log(15) \simeq 4.32$$

social surplus $\simeq$ 11.35

4. (a) The High city should provide $A_H = 10$ units of weed clearing, and the Low city provides $A_L = 5$. The FOCs of the individuals' maximization problems are the same as in part 2, so the earl will want to live in High city and the pleb will want to live in Low city. Neither wishes the two would switch. Moreover, since $MRS_e = \frac{2(15 - A_H)}{A_H} = 1$ and $MRS_p = \frac{10 - A_L}{A_L} = 1$, neither wishes his city would provide a different level of weed control. Thus, surplus in each city, and hence in society, is maximized.

(b) Answered above in (a).

Question 3

1. (a) $\hat{TS} = Docs_{TX,2006} - Docs_{TX,2002} = 175 - 158 = 17$

The time-series estimator restricts attention to the treatment group (TX), comparing average Doctors before (2002) and after (2006) Bill 4 came into force in 2003.

(b) The key identifying assumption is that no other factor influenced the Δ in Docs in TX other than Bill 4 over the 2002 - 2006 period. I.e., $\Delta Docs = 0$ absent Bill 4.

(c) General economic fluctuations affect the rates at which new physicians are added to the physician pool. Good economic times over this period may have lead to hiring of more docs in TX, and also help existing doctors to pay the costs of litigating malpractice lawsuits, without going out of business.

2. (a) $\hat{CS} = Docs_{TX,2006} - Docs_{Neighbors,2006} = 175 - 180 = -5$

The cross-sectional estimator compares Doc levels between the treatment group (TX) and the control group (neighbors) after the Bill is in effect.

(b) The key identifying assumption is that there are no systematic differences between these two groups aside from Bill 4.

(c) If neighboring states had different malpractice laws from TX, (aside from Bill 4) this could cause differences in the # of doctors as well.

3. (a) $\hat{DD} = (Docs_{TX,2006} - Docs_{TX,2002}) - (Docs_{Neighbors,2006} - Docs_{Neighbors,2002})$

$= (175 - 158) - (180 - 189) = 26$

The difference-in-difference estimator may be considered either

(1) the TS estimator with explicit control for time effects bias, or

(2) the CS estimator with explicit control for pre-existing differences between the treatment and control groups.

Question 3 (continued)

3. (b) Validity of the DD estimator hinges on the "Parallel Trend Assumption." That is, absent Bill 4, $\Delta Docs_{TX} = \Delta Docs_{Neighbors}$.

(c) Suppose aggregate economic fluctuations differentially affected TX and its neighbors. This would violate the Parallel Trend Assumption.

(d) Reconstruct the DD estimator using periods 1998 and 2002. These were both years ~~where~~ before Bill 4 was enacted. This is a falsification test of the Parallel Trend Assumption.

$$\widehat{Falsification} = (Docs_{TX,2002} - Docs_{TX,1998}) - (Docs_{Neighbors,2002} - Docs_{Neighbors,1998})$$

$$= (158 - 152) - (189 - 196) = 13 \neq 0 \,!$$

The magnitude of the falsification test (not $= 0$) ~~indicated~~ indicates that the Parallel Trend Assumption is invalid.

Question 4

$X_A^* = 3/2$
$\Rightarrow X_B^* = 5/2$
$X_C^* = 9/2$

1. Household i $\max_X U_i(x) = \gamma_i X - X^2$. FOC $\Rightarrow X_i^* = \gamma_i/2$

2. ⓐ $X = 3/2$ vs. $\underline{X = 5/2}$ $\Rightarrow$ HH_A votes for $3/2 = X_A^*$
   HH_B votes for $5/2 = X_B^*$
   HH_C votes for $5/2$ $[U_C(3/2) < U_C(5/2)]$

   $\underline{X = 5/2}$ vs. $X = 9/2$ $\Rightarrow$ $5/2$ is selected by similar argument.

   $\Rightarrow$ option $X = 5/2$ beats any other option.

   ⓑ Regardless of the initial voting pair, option $X = 5/2$ will ultimately be selected, and it will consistently keep winning from then on.

3. ⓐ <u>Cycling</u> (lack of transitivity) $\Rightarrow$ no stable outcome.
   $X = 3$ vs. $X = 6$ $\Rightarrow$ option 6 selected
   $X = 6$ vs. $X = 8$ $\Rightarrow$ option 8 selected
   $X = 8$ vs. $X = 3$ $\Rightarrow$ option 3 selected

   ⓑ In (2), preferences were single-peaked (preference restriction). Here, HH_F does <u>not</u> have single-peaked preferences.

4.

Selectman's Preference	Sequence
Option $X = 3$	(1) $X=6$ vs. $X=8$ $\Rightarrow$ 8 wins ... go to round 2 (2) $X=8$ vs. $X=3$ $\Rightarrow$ $X=3$ is selected.
Option $X = 6$	(1) $X=8$ vs. $X=3$ $\Rightarrow$ 3 wins ... go to round 2 (2) $X=3$ vs. $X=6$ $\Rightarrow$ $X=6$ is selected.
Option $X = 8$	(1) $X=3$ vs. $X=6$ $\Rightarrow$ 6 wins ... go to round 2 (2) $X=6$ vs. $X=8$ $\Rightarrow$ $X=8$ is selected.

So, by appropriately choosing the initial pairwise election, the outcome is dictated.

5. Provided the (many) assumptions of the Tiebout framework are satisfied, then each household will eventually settle into a homogeneous town of like-minded residents (sorting), such that each household's optimal choice is provided ("voting with your feet.").