14.385 Problems Set 3A

Fall 2007

These are two problems that will be included in problem set 3. There will be an additional one or two problems.

1. Consider a linear panel data model

$$Y_{it} = X_{it}'\beta + \alpha_i + \eta_{it}, \quad i = 1, ..., n, \quad t = 1, ..., T.$$

Suppose that conditional on $X_{i1}, ..., X_{iT}$ and α_i, the vector $(\eta_{i1}, ..., \eta_{iT})'$ is distributed as $N(0, \sigma^2 I_T)$.

a) What is the conditional density $f(Y_{i1}, ..., Y_{iT} \mid X_{i1}, ..., X_{iT}, \alpha_i; \beta, \sigma^2)$?

b) Calculate the plim of the estimator of σ^2 from maximizing the conditional log-likelihood over σ^2, β, *and* $\alpha_1, ..., \alpha_n$.

c) What is the conditional density $f(Y_{i1}, ..., Y_{iT} \mid X_{i1}, ..., X_{iT}, \alpha_i, \sum_{t=1}^{T} Y_{it}; \beta, \sigma^2)$?

d) For the conditional density in c), find the conditional MLE for β and σ^2, and show that the estimator of σ^2 is consistent.

2. Suppose that Y_{it} is count data with $Y_{it} \in \{0, 1, 2, ...\}$, and that $E[Y_{it}|X_i, \alpha_i] = \exp(X_{it}'\beta_0 + \alpha_i)$.

a. Consider an estimator of β that is obtained as

$$\tilde{\beta} = \arg \min_{\beta, \alpha_1, ..., \alpha_n} \sum_{i,t} \{Y_{it} - \exp(X_{it}'\beta_0 + \alpha_i)\}^2.$$

Is this estimator consistent? Why or why not.

b. Consider the function $\rho_{it}(\beta) = Y_{it} \exp(X_{i,t-1}'\beta) - Y_{i,t-1} \exp(X_{it}'\beta)$. Show that $E[\rho_i(\beta_0)|X_i] = 0$.

c. How could you use the result from b) to form an estimator of β?

d. (extra credit) Consider the conditional maximum likelihood estimator obtained when conditional on X_i and α_i, the variables $Y_{i1}, ..., Y_{iT}$ are independent Poisson with $E[Y_{it}|X_i, \alpha_i] = \exp(X_{it}'\beta_0 + \alpha_i)$ and we consdition on $S_i = \sum_{t=1}^{T} Y_{it}$ is consistent if only the conditional mean assumption is satisfied (i.e. they need not be Poisson). Hint: A sum of independent Poissions is Poisson.