14.30 PROBLEM SET 4

TA: Tonja Bowen Bishop

Due: Tuesday, March 21, by 4:30 p.m.

Note: The first three problems are required, and the remaining two are practice problems. If you choose to do the practice problems now, you will receive feedback from the grader. Alternatively, you may use them later in the course to study for exams. Credit may not be awarded for solutions that do not use methods discussed in class.

Problem 1

Suppose that X, Y, and ε are random variables with $Y = \alpha + \beta X + \varepsilon$. Assume $E(X) = 0$, $Var(X) = \sigma^2$, ε has a uniform distribution over $\left[\frac{-1}{2}, \frac{1}{2}\right]$, and $Cov(X, \epsilon) = 0$.

a. What is the distribution (pdf) of $Y|X$?

b. What is $E(Y|X)$? What is $Var(Y|X)$?

c. What is $E(Y)$? What is $Var(Y)$?

Problem 2

a. Assume that X is distributed uniformly over the interval $[0, 4]$. Calculate the moment generating function of X. Use the MGF to find the mean and variance of X.

b. Use the Chebyshev inequality to calculate an upper bound on the probability that X is outside the interval $[0.5, 3.5]$.

c. Now use the fact that X is distributed uniformly over the interval $[0, 4]$ to calculate the probability that X is outside the interval $[0.5, 3.5]$. Is it higher or lower than the answer you got in part b?

d. Based on your answers in parts b and c, comment on the usefulness of the Chebyshev inequality.

Problem 3

Let X be distributed uniformly on the interval $[0, 1]$.

a. Let $Y = X^2$. Use the 1-step and 2-step methods to find the pdf of Y.

b. Let $Z = -\lambda \log(X)$, for $\lambda > 0$. Calculate the pdf of Z.

Problem 4

Let X have the following distribution.

$$\begin{aligned} f(x) &= \frac{1}{3}\left(\frac{2}{3}\right)^{x} \\ x &\in \{0,1,2,...\} \end{aligned}$$

Find the pdf of $Y = \frac{X}{X+1}$.

Problem 5

Let X have the pdf $f(x) = \frac{1}{2}(x+1)$ for $-1 \leq x \leq 1$.

a. Let $Y = X^2$. Use the 1-step and 2-step methods to find the pdf of Y.

b. Find the MGF of Y.

c. What is $E(Y)$? What is $Var(Y)$?

d. What is $E\left(Y|X \geq \frac{1}{2}\right)$?