14.127 Behavioral Economics (Lecture 1)

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1 Overview

- **Instructor**: Xavier Gabaix

- **Time** 4-6:45/7pm, with 10 minute break.

- **Requirements:** 3 problem sets and
  Term paper due September 15, 2004 (meet Xavier in May to talk about it)

2 Some Psychology of Decision Making

2.1 Prospect Theory (Kahneman-Tversky, Econometrica 79)

Consider gambles with two outcomes: x with probability p, and y with probability $1-p$ where $x \geq 0 \geq y$.

- Expected utility (EU) theory says that if you start with wealth W then the (EU) value of the gamble is

$$V = pu(W + x) + (1 - p)\, u(W + y)$$

- Prospect theory (PT) says that the (PT) value of the game is

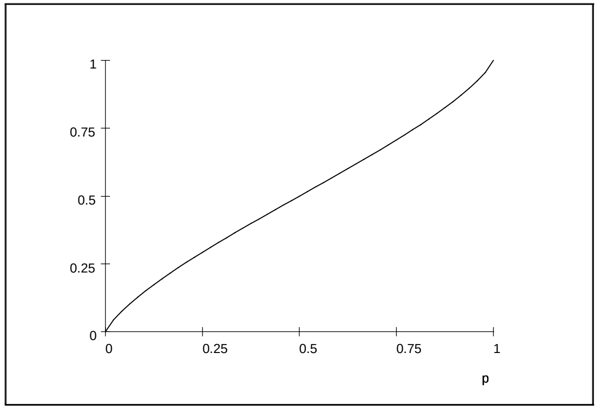
$$V = \pi(p)\, u(x) + \pi(1 - p)\, u(y)$$

where π is a probability weighing function. In standard theory π is linear.

- In prospect theory π is concave first and then convex, e.g.

$$\pi(p) = \frac{p^{\beta}}{p^{\beta} + (1-p)^{\beta}}$$

for some $\beta \in (0,1)$. The figure gives $\pi(p)$ for $\beta = .8$



2.1.1 What does the introduction of the weighing function π mean?

- $\pi(p) > p$ for small p. Small probabilities are overweighted, too salient. E.g. people play a lottery. Empirically, poor people and less educated people are more likely to play lottery. Extreme risk aversion.

- $\pi(p) < p$ for p close to 1. Large probabilities are underweight.

In applications in economics $\pi(p) = p$ is often used except for lotteries and insurance

2.1.2 Utility function u

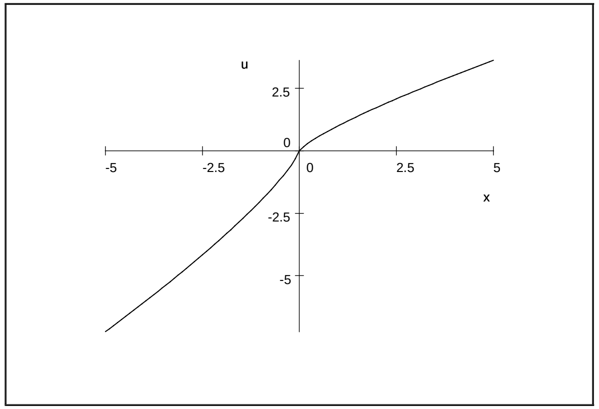
- We assume that $u(x)$ is increasing in x, convex for loses, concave for gains, and first order concave at 0 that is

$$\lim_{x \to 0+} \frac{-u(-x)}{u(x)} = \lambda > 1$$

- A useful parametrization

$$u(x) = x^{\beta} \text{ for } x \geq 0$$
$$u(x) = -\lambda |x|^{\beta} \text{ for } x \leq 0$$

- The graph of $u(x)$ for $\lambda = 2$ and $\beta = .8$ is given below



2.1.3 Meaning - Fourfold pattern of risk aversion u

- Risk aversion in the domain of likely gains

- Risk seeking in the domain of unlikely gains

- Risk seeking in the domain of likely losses

- Risk aversion in the domain of unlikely losses

2.1.4 How robust are the results?

- Very robust: loss aversion at the reference point, $\lambda > 1$
- Robust: convexity of u for $x < 0$
- Slightly robust: underweighting and overweighting of probabilities $\pi(p) \gtrless p$

2.1.5 In applications we often use a simplified PT (prospect theory):

$$\pi(p) = p$$

and

$$u(x) = x \text{ for } x \geq 0$$
$$u(x) = \lambda x \text{ for } x \leq 0$$

2.1.6 Second order risk aversion of EU

- Consider a gamble $x+\sigma$ and $x-\sigma$ with $50:50$ chances.

- Question: what risk premium π would people pay to avoid the small risk σ?

- We will show that as $\sigma \rightarrow 0$ this premium is $O\left(\sigma^2\right)$. This is called *second order risk aversion*.

- In fact we will show that for twice continuously differentiable utilities:

$$\pi(\sigma) \cong \frac{\rho}{2}\sigma^2,$$

where ρ is the curvature of u at 0 that is $\rho = -\frac{u''}{u'}$.

- The risk premium π makes the agent with utility function u indifferent between

$$u(x) \text{ and } \frac{1}{2}u(x+\sigma+\pi(\sigma))+\frac{1}{2}u(x-\sigma+\pi(\sigma))$$

- Assume that u is twice differentiable and take a look at the Taylor expansion of the above equality for small σ. .

$$u(x)=u(x)+\frac{1}{2}u'(x)\,2\pi(\sigma)+\frac{1}{4}u''(x)\,2\left[\sigma^2+\pi(\sigma)^2\right]+o\left(\sigma^2\right)$$

or

$$\pi(\sigma)=\frac{\rho}{2}\left[\sigma^2+\pi(\sigma)^2\right]+o\left(\sigma^2\right)$$

- Since $\pi(\sigma)$ is much smaller than σ, so the claimed approximation is true. Formally, conjecture the approximation, verify it, and use the implicit function theorem to obtain uniqueness of the function π defined implicitly be the above approximate equation.

2.1.7 First order risk aversion of PT

- Consider same gamble as for EU.

- We will show that in PT, as $\sigma \to 0$, the risk premium π is of the order of σ when reference wealth $x = 0$. This is called the *first order risk aversion*.

- Let's compute π for $u(x) = x^{\alpha}$ for $x \geq 0$ and $u(x) = -\lambda |x|^{\alpha}$ for $x \leq 0$.

- The premium π at $x = 0$ satisfies

$$0 = \frac{1}{2}(\sigma + \pi(\sigma))^{\alpha} + \frac{1}{2}(-\lambda)|-\sigma + \pi(\sigma)|^{\alpha}$$

or

$$\pi\left(\sigma\right)=\frac{\lambda^{\frac{1}{\alpha}}-1}{\lambda^{\frac{1}{\alpha}}+1}\sigma=k\sigma$$

where k is defined appropriately.

2.1.8 Calibration 1

- Take $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, i.e. a constant elasticity of substitution, CES, utility

- **Gamble 1**
  \$50,000 with probability $1/2$
  \$100,000 with probability $1/2$

- **Gamble 2.** $\$x$ for sure.

- Typical x that makes people indifferent belongs to $(60k, 75k)$ (though some people are risk loving and ask for higher x.

- Note the relation between x and the elasticity of substitution γ:

x	$70k$	$63k$	$58k$	$54k$	$51.9k$	$51.2k$
γ	1	3	5	10	20	30

  Right γ seems to be between 1 and 10.

- Evidence on financial markets calls for γ bigger than 10. This is the equity premium puzzle.

2.1.9 Calibration 2

- **Gamble 1**
  \$10.5 with probability 1/2
  \$-10 with probability 1/2

- **Gamble 2.** Get \$0 for sure.

- If someone prefers Gamble 2, she or he satisfies
$$u(w) > \frac{1}{2}u(w + \pi - \sigma) + \frac{1}{2}u(w + \pi + \sigma).$$
Here, $\pi = \$.5$ and $\sigma = \$10.25$. We know that in EU
$$\pi < \pi^*(\sigma) = \frac{\rho}{2}\sigma^2$$

And thus with CES utility

$$\frac{2W\pi}{\sigma^2} < \gamma$$

forces large γ as the wealth W is larger than 10^5 easily.

2.1.10 Calibration Conclusions

- In PT we have $\pi^* = k\sigma$. For $\gamma = 2$, and $\sigma = \$.25$ the risk premium is $\pi^* = k\sigma = \$.5$ while in EU $\pi^* = \$.001$.

- If we want to fit an EU parameter γ to a PT agent we get

$$\hat{\gamma} = \frac{2kW}{\sigma}$$

  and this explodes as $\sigma \to 0$.

- If someone is averse to 50-50 lose \$100/gain g for all wealth levels then he or she will turn down 50-50 lose L/gain G in the table

$L\backslash g$	\$101	\$105	\$110	\$125
\$400	\$400	\$420	\$550	\$1,250
\$800	\$800	\$1,050	\$2,090	∞
\$1000	\$1,010	\$1,570	∞	∞
\$2000	\$2,320	∞	∞	∞
\$10,000	∞	∞	∞	∞

2.2 What does it mean?

- EU is still good for modelling.

- Even behavioral economist stick to it when they are not interested in risk taking behavior, but in fairness for example.

- The reason is that EU is nice, simple, and parsimonious.

2.2.1 Two extensions of PT

- Both outcomes, x and y, are positive, $0 < x < y$. Then,

$$V = v(y) + \pi(p)(v(x) - v(y)).$$

Why not $V = \pi(p)v(x) + \pi(1-p)v(y)$? Because it becomes self-contradictory when $x = y$ and we stick to K-T calibration that puts $\pi(.5) < .5$.

- Continuous gambles, distribution $f(x)$
EU gives:

$$V = \int_{-\infty}^{+\infty} u(x) f(x) dx$$

PT gives:

$$V = \int_0^{+\infty} u(x) f(x) \pi' (P(g \geq x)) dx$$
$$+ \int_{-\infty}^{0} u(x) f(x) \pi' (P(g \leq x)) dx$$