14.123 Microeconomics III Final Exam SOLUTIONS

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Instructions. This is an open-book exam. You can use the results in the notes and the answers to the problem sets without proof, but you need to invoke them explicitly. You have 80 minutes. Each question is 25 points. Good Luck!

1. Consider the following game

	x	y	z
a	0,0	0,0	6,1
b	0,0	3,3	0,0
c	1,6	0,0	5,5

(a) Compute the set of correlated equilibrium distributions. (It suffices to write down the necessary and sufficient conditions; do not waste your time in trying to simplify the conditions.)

Answer: Write $p(s)$ for the probability of strategy profile s. The inequalities for player 1 are

$$
\begin{aligned}
6p(a,z) &\geq 3p(a,y) \\
6p(a,z) &\geq p(a,x) + 5p(a,z) \\
3p(b,y) &\geq 6p(b,z) \\
3p(b,y) &\geq p(b,x) + 5p(b,z) \\
p(c,x) + 5p(c,z) &\geq 6p(c,z) \\
p(c,x) + 5p(c,z) &\geq 3p(c,y),
\end{aligned}
$$

which simplify to

$$
\begin{aligned}
2p(a,z) &\geq p(a,y) \\
p(a,z) &\geq p(a,x) \\
p(b,y) &\geq 2p(b,z) \\
3p(b,y) &\geq p(b,x) + 5p(b,z) \\
p(c,x) &\geq 2p(c,z) \\
p(c,x) + 5p(c,z) &\geq 3p(c,y).
\end{aligned}
$$

Similarly, the conditions for player 2 are

$$
\begin{aligned}
2p(c,x) &\geq p(b,x) \\
p(c,x) &\geq p(a,x) \\
p(b,y) &\geq 2p(c,y) \\
3p(b,y) &\geq p(a,y) + 5p(c,y) \\
p(a,z) &\geq 2p(c,z) \\
p(c,x) + 5p(c,z) &\geq 3p(b,z).
\end{aligned}
$$

(b) Identify a correlated equilibrium payoff vector that is not in the convex hull of Nash equilibrium payoff vectors.

Answer: Take $p(a,z) = p(c,x) = p(c,z) = 1/3$, which yields the payoff vector of $(4,4)$. This is clearly outside of the convex hull of Nash equilibria, as the sum of the payoffs are less than 7 in every Nash equilibrium. [To see the latter note that in order for sum to be greater than 7 in any Nash equilibrium σ, it must be that $\sigma_1(c)\sigma_2(z) > 0$. For c to be a best response, we must have $\sigma_2(x) + 5\sigma_2(z) \geq 6\sigma_2(z)$, i.e., $\sigma_2(x) \geq \sigma_2(z)$. Similarly, we must have $\sigma_1(a) \geq \sigma_1(c)$.

Hence, $\sigma_1(a)\sigma_2(x) > \sigma_1(c)\sigma_2(z)$. Thus, total payoff from (a,x) and (a,y) cannot be higher than $10/2 = 5$. Since the sum in all other profiles is less than or equal to 7, the sum of Nash equilibrium payoffs is less than or equal to 7.]

2. Alice has M dollars and has a constant absolute risk aversion α (i.e. $u(x) = -e^{-\alpha x}$) for some $\alpha > 0$. With some probability $\pi \in (0,1)$ she may get sick, in which case she would need to spend L dollars on her health. There is a health-insurance policy that fully covers her health care expenses in case of sickness and costs P to her. (If she buys the policy, she needs to pay P regardless of whether she gets sick.)

   (a) Find the set of prices P that she is willing to pay for the policy. How does the maximum price $\bar{P}$ she is willing to pay varies with the parameters M, L, α, and π?

   Answer: If uninsured, her expected payoff is

   $$-(1-\pi)e^{-\alpha M} - \pi e^{-\alpha(M-L)}.$$

   If she is insured, her expected payoff is $-e^{-\alpha(M-P)}$. Hence, she is willing to buy iff

   $$-(1-\pi)e^{-\alpha M} - \pi e^{-\alpha(M-L)} \le -e^{-\alpha(M-P)}.$$

   This simplifies to

   $$(1-\pi) + \pi e^{\alpha L} \ge e^{\alpha P}.$$

   That is,

   $$P \le \bar{P}(\alpha, \pi, L) \equiv \frac{1}{\alpha}\ln\left(1 - \pi + \pi e^{\alpha L}\right).$$

   The maximum price is clearly independent of M and increasing in π and L. It is also increasing in α because $e^{\bar{P}} = \left(1 - \pi + \pi e^{\alpha L}\right)^{1/\alpha}$ and

   $$\partial e^{\bar{P}}/\partial\alpha = \frac{1}{\alpha}\pi L e^{\alpha L}\left(1 - \pi + \pi e^{\alpha L}\right)^{(1-\alpha)/\alpha} > 0.$$

   (b) Suppose now that there is a test $t \in \{-1,+1\}$ that she can take before she makes her decision on buying the insurance policy. If she takes the test and the test t is positive, her posterior probability of getting sick jumps to $\pi^+ > \pi$ and if the test is negative, then her posterior probability of getting sick becomes 0. What is the maximum price c she is willing to pay in order to take the test? (Take $P \le \bar{P}$.)

   Answer: By Bayes rule, $\Pr(t=1) = \pi/\pi^+$. If she takes the test and $t = -1$, she does not buy insurance and will have M dollars. If $t = +1$, she will definitely buy the insurance (because $P < \bar{P}(\alpha, \pi, L) < \bar{P}(\alpha^+, \pi, L)$), and she will have $M - P$. Therefore, her expected payoff is

   $$U_{test} = -\frac{\pi}{\pi^+}e^{-\alpha(M-P-c)} - \left(1 - \frac{\pi}{\pi^+}\right)e^{-\alpha(M-c)},$$

   where c is the cost of the test. If she does not take the test, since $P < \alpha\bar{P}$, she will buy insurance, and her payoff is $-e^{-\alpha(M-P)}$. Therefore, she buys the test iff

   $$e^{\alpha C} \le \frac{e^{\alpha P}}{\frac{\pi}{\pi^+}e^{\alpha P} + \left(1 - \frac{\pi}{\pi^+}\right)},$$

   i.e.,

   $$c \le \bar{c} \equiv P - \frac{1}{\alpha}\ln\left[\frac{\pi}{\pi^+}e^{\alpha P} + \left(1 - \frac{\pi}{\pi^+}\right)\right].$$

   Note that $\bar{c} > 0$.

3. In Question 2, assume that the insurance is provided by Bob, who is risk neutral. Assume also that $P \in (\pi L, \bar{P})$, $\pi^+ = 1$, and that the test is free. Consider the following game between Alice and Bob:

   • First, Alice decides whether to take the test. If she takes the test she learns the value of t (the test result).

- Then, she decided whether to apply to the insurance policy. If she does not apply, then the game ends; she remains uninsured and Bob gets 0.

- If she applies, then Bob learns whether she took the test without learning the test result. Then, Bob decides whether to accept her application.

- If he accepts, then he insures Alice (for price P); if he rejects, then the game ends with Alice uninsured and Bob with zero payoff.

(a) Compute a sequential equilibrium of this game.

Answer: The unique sequential equilibrium in pure strategies is as follows:

- Alice does not take the test;
- She applies to insurance if her test result is positive or if she does not take the test; she does not apply if her test result is negative;
- Bob assigns probability 1 on positive test result if she takes the test and applies;
- Bob accepts the application if she did not take the test and rejects it if she did take the test.

Sequential rationality is obvious. To check consistency, note that under any perturbation, $\Pr(t = 1|Test, App)$ approach 1 (so any perturbation would do). There are also mixed strategy sequential equilibria, but they all need to the same outcome as the above equilibrium. The only variation is $\Pr(App|t = 1)$ and $\Pr(App|t = -1)$ can vary within the constraint that

$$\Pr(App|t = 1) \geq \frac{1 - \pi}{\pi} \frac{P}{L - P} \Pr(App|t = -1)$$

ensuring that $\Pr(t = 1|App) \geq P/L$, in which case Bob rejects the application after test rationally.

(b) Comparing your answers to the Questions 2b and 3a, briefly discuss the value of information in strategic and non-strategic environments.

Answer: In a non-strategic environment, information has a non-negative value, as one can always ignore her information in her decision making. It has often strictly positive value because one can condition her decision on a relevant information. Indeed, in Problem 2b, Alice is willing to pay positive amount for a test before deciding on buying an insurance. In a strategic environment, however, information may have negative value of the others know that one has (or can get) information, as such access to information may change the other players' behavior adversely. Indeed, in 3b, Alice does not want to take the free test. She could even pay to avoid the test. This is because she cannot get the insurance after the test from Bob.

Bonus [10 points]: What happens in sequential equilibrium if Bob cannot observe whether Alice has taken the test?

Answer: In any sequential equilibrium, Bob rejects Alice's application with probability 1. If he accepts it with positive probability, Alice would take the test and apply if and only if the test result is positive as a unique best response. Then, Bob must assign probability 1 on $t = 1$ in his information set, and he must reject the application, a contradiction. Of course, since Bob rejects the application, Alice has multiple best responses, leading to multiple sequential equilibria in mixed strategies as in part (a), but they all lead to the same outcome of no insurance. (This shows that access the information may be detrimental. In fact, the situation is worse here because Alice cannot prove that she doesn't have the information.)

4. Under the assumptions P1-P5 of Savage, for any two partitions $\{A_1, \ldots, A_n\}$ and $\{B_1, \ldots, B_n\}$ of the state space S in which $A_1 \mathbin{\dot\sim} \cdots \mathbin{\dot\sim} A_n$ and $B_1 \mathbin{\dot\sim} \cdots \mathbin{\dot\sim} B_n$, show that $A_1 \mathbin{\dot\sim} B_1$. (Recall that $\{A_1, \ldots, A_n\}$ is a partition of S means that $A_1 \cup \cdots \cup A_n = S$ and $A_i \cap A_j = \varnothing$ for any distinct i and j.)

Answer: I will use the following **fact**: For disjoint sets A_1 and A_2, if $A_1 \mathbin{\dot\succ} B_1$ and $A_2 \mathbin{\dot\succ} B_2$, then $A_1 \cup A_2 \mathbin{\dot\succ} B_1 \cup B_2$. (You have been asked to prove this in a problems set; the proof for weak preference applies also for the strict preference.) Towards a contradiction, suppose that $A_1 \mathbin{\dot\succ} B_1$. Then, for any i, $A_i \mathbin{\dot\sim} A_1 \mathbin{\dot\succ} B_1 \mathbin{\dot\sim} B_i$, showing that $A_i \mathbin{\dot\succ} B_i$ for all i by transitivity (P1). Then, an inductive application of the fact above implies that

$$S = A_1 \cup \cdots \cup A_n \mathbin{\dot\succ} B_1 \cup \cdots \cup B_n = S,$$

a clear contradiction. (Here, the equalities are due to the fact that $\{A_1, \ldots, A_n\}$ and $\{B_1, \ldots, B_n\}$ are partitions of S, and $\succdot$ is due to the inductive application of the above fact and the fact that $A_i \cap A_j = \varnothing$ for any distinct i and j.)

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