14.123 Microeconomics III—Problem Set 1

Muhamet Yildiz

Instructions. You are encouraged to work in groups, but everybody must write their own solutions. Each question is 25 points. Good Luck!

1. Consider a monopolist with n buyers. At each date $t = 1, \ldots, n$, simultaneously, monopolist chooses quality $q \in \{H, L\}$ and the buyer t decides whether to buy (choosing $b \in \{B, N\}$). The stage payoffs are as in the following table

	B	N
H	$2-c, 1$	$-c, 0$
L	$2, -1$	$0, 0$

   where $c \in \{-1, 1\}$ is the cost of producing a high quality product, privately known by the monopolist. The ex-ante probability of $c = 1$ is $1 - \varepsilon$ for some $\varepsilon \in (0, 1/2)$; the cost is the same at all dates, and all the previous moves are publicly observable. Find a sequential equilibrium. For any n, find the largest ε under which monoplist chooses high quality in the first date regardless of his cost.

2. Let P be the set of lotteries over $\{a, b, c\} \times \{L, M, R\}$. In which of the following pairs of games the players' preferences over P are the same?

   (a)

	L	M	R
a	2,-2	1,1	-3,7
b	1,10	0,4	0,4
c	-2,1	1,7	-1,-5

	L	M	R
a	6,-1	4,0	-4,2
b	4,3	2,1	2,1
c	-2,0	4,2	0,-2

   (b)

	L	M	R
a	1,2	7,0	4,-1
b	6,1	2,2	8,4
c	3,-1	9,2	5,0

	L	M	R
a	1,5	7,1	4,-1
b	6,3	2,4	8,8
c	3,-1	9,5	5,1

3. Consider the set of lotteries (p_x, p_y, p_z) on the set of outcomes $\{x, y, z\}$ where p_x, p_y, and p_z are the probabilities of x, y, and z, respectively.

   (a) For each (partial) preference below, determine whether it is consistent with expected utility maximization. (If yes, find a utility function; if so, show that it cannot come from an expected utility maximizer.)

      1. $(1/4, 1/4, 1/2) \sim (3/4, 0, 1/4) \succ (1/4, 1/2, 1/4) \succ (3/4, 1/4, 0)$
      2. $(1/4, 1/4, 1/2) \succ (3/4, 0, 1/4) \succ (5/6, 1/6, 0) \succ (1/2, 1/3, 1/6)$

   (b) Find a complete and transitive preference relation on the above lotteries that satisfies the independence axiom but cannot have an expected utility representation.

(c) Find a complete, continuous, and transitive preference on the above lotteries such that

- whenever there is an indifference between (p_x, p_y, p_z) and (q_x, q_y, q_z), there is an indifference between (p_x, p_y, p_z) and $\alpha (p_x, p_y, p_z) + (1 - \alpha) (q_x, q_y, q_z)$ for every $\alpha \in [0, 1]$, and yet
- the preference relation does not have an expected utility representation.

4. Exercise 9 in Chapter 5 of the Lecture Notes.

14.123 Microeconomic Theory III

Spring 2015