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**PROFESSOR:** Hi, and welcome back to the 14.01 problem solving videos. Today, we're going to work on Fall 2010, problem set one, problem number four. And in this problem, we're going to be working with elasticities. But instead of starting with a demand function, and starting with a supply function, and calculating the elasticity given those functions, we're going to be given the elasticity of demand and the elasticity of supply. And we're going to have to back out what the demand functions and the supply functions should have looked like. So we're basically just working in reverse from what we did in lecture.

Let's go ahead and read the full problem up through part A. You have been asked to analyze the market for steel. From public sources, you are able to find that last year's price for steel was \$20 per ton. At this price, 100 million tons were sold on the world market.

From trade association data, you are able to obtain estimates for their own price elasticities of demand and supply on the world markets as negative 0.25 for demand and 0.5 for supply. Assume the steel has linear demand and supply curves throughout. Part A asks us to solve for the equations of demand and supply in this market, and to sketch the demand and supply curves.

So looking at the formal definition of elasticity of demand and elasticity of supply, we basically are going to have three different parts to it. We have the derivative of either demand or supply function with respect to  $P$ , in this case the own price of  $P$ , or the price of steel. And we also have the equilibrium price, or any price at the point on the curve, and a quantity. In this case, it's going to be the equilibrium quantity.

So basically, what we have now is we are given-- for the elasticity of demand, we're given three variables. We're given the price, the quantity, and the elasticity of demand. And that means the only thing that we don't know is the derivative of the demand curve with respect to  $P$ .

So if we can isolate this derivative, then we can integrate the number that we're able to solve

through for. And then we can solve out for what our demand curve is going to look like. So let's go ahead and walk through that process together.

Substituting in for the elasticity of demand  $P$  and  $Q$ , we're gonna have this equation. And the one thing that I want you to notice is since the derivative of the demand curve with respect to  $P$  is negative 0.25, in this case, we know that it's linear. But just because it's linear at the point where price is 20 and quantity is 100, that doesn't necessarily mean it's gonna be linear throughout. So it's useful to know that at any point on this line, it's always going to have the derivative equal to negative 0.25. So that's useful. We know we can integrate and have the correct answer.

Solving for  $dQ/dP$ , we're gonna have negative 1.25. And we're just going to integrate this with respect to  $P$ . And after we integrate, we're going to be left with a constant.

In this case, we're going to call the constant  $a$ . This is how much the demand curve has actually shifted up to begin with, shifted up or down. And to solve for  $a$ , all we have to do is we can just plug back in for the \$20 and the 100 quantity, and we can solve through for what  $a$  is going to be equal. When you solve through plugging in  $Q$  and  $P$ , you're going to find that  $a$  is equal to 125. So we're gonna have that our final demand function is gonna be negative 1.25 $P$  plus 125.

Now we can go through this exact same process with the elasticity of supply now. And all we have to do now is use the number 0.5 instead, and we can solve through. We can integrate. And then we're going to solve for the other constant  $b$ , again, get our supply curve.

Substituting in the information we have, we're going to be left with this equation. And we're gonna go ahead and isolate the derivative that we have. And when we integrate again, we have to remember that we are going to have a constant that we're gonna have to solve for. And I'm gonna just call this constant  $c$ . Again, plug in the price of 20 and the quantity equal to 100 and you're gonna find that the supply curve is gonna be equal to 2.5 $P$  plus 50.

And we can do a quick sketch of this on our axes here. We're just gonna go ahead and draw our upward-sloping supply curve, our downward-sloping demand curve. And we're gonna mark the equilibrium point and label the equilibrium quantities and the equilibrium prices, as well.

And before we move on to the second part of this problem, we can pause here. And we can

think about what did the elasticities that we started with actually mean. Well, if we were to look at this point of intersection at the equilibrium of the demand curve, we're looking at the percentage change at this point in quantity per percentage change in price. So we're basically just saying, for that tiny change, an infinitesimally small change at this point for the demand curve, how much does quantity change, percentage-wise, relative to price? And that's also what we're looking at with the supply curve.

So when you're given a elasticity, if you have an elasticity of supply, it makes sense that it's gonna be positive, in this case 0.5, because when price increases, suppliers are willing to supply more. And it makes sense that the demand elasticity that we're given is negative, or negative 0.25, because when price begins to increase, the consumers are gonna want less of the product.

Now, the second part of this problem is going to give us new elasticities of demand and supply. And I'm gonna just quickly run through the actual calculation, because it's gonna be the same as our calculation that we just did. And instead, we're gonna think about possible causes for the shifts that we see in the supply and the demand curve.

Part B says, suppose that you discover that the current price of steel is \$15 per ton and the current level of worldwide sales of steel is 150 million tons. The most recent elasticity estimates from the trade association this year are negative 0.125 for demand and 0.25 for supply. Describe the change in the supply and the demand curves over the past year using your diagram from part A. What sort of events might explain the change?

Now, I've given us the information for this part of the problem on this board. And you'll notice that our inputs, or our variables, have changed now. The price has dropped from \$20. Now it's gonna be down to \$15. You're gonna notice that the quantity has actually increased from 100 up to 150. And our elasticities of demand and elasticities of supply have changed, because the price and the quantity are different, and we're at a different point on our graph.

The process we're gonna do to solve for our demand curves and our supply curves are going to be exactly identical. And when you follow the same process-- I'll just do the first step up here-- you're gonna substitute in for the information that's given in the problem. And all you're going to do is you're, again, gonna solve through for the derivative. You're gonna integrate. And you're gonna find the constants.

After you do that entire process, you're gonna find that the demand curve is given by this

equation. And you're gonna find that the supply curve is given by this new equation. Now, if we look at this new demand curve and this new supply curve, we'll actually notice that the slope, with respect to  $P$ , is going to be identical in both of the cases that we solved for, both the beginning case and the case in the end of the problem.

The only thing that's shifted between our quantities demanded and our quantities supplied, or the curves, is there's been a shift. And the shift for the demand curve-- it went from an intercept of 125 now to an intercept of 168.75. So our demand curve is shifting up and out.

So we can represent this shift in demand like this. Notice that the slope is going to be exactly identical. I'm going to write a small  $db$  for part B.

And then we can do the same sort of interpretation for our supply curve. Looking at our supply curve, the intercepts, now, is 112.5. But before, it was only at 50. And what this means, this means that the supply curve is going to shift in and down.

And so my graph with the equilibrium price that I've drawn-- it's a little bit off, but what you should see-- you should see that the new equilibrium price has fallen. In this case, it's fallen to 15. And the equilibrium quantity has increased from 100 to 150. So since we had both a shift in supply and a shift in demand, necessarily we see that quantity is going to increase.

But if the demand curve had shifted way up here, we could see that price could have increased. So the effect on the price in this market is ambiguous. We can say that, necessarily, the effect on quantity is going to be clearly towards an increase.

So to wrap up this problem, we saw that changes in elasticities can also represent changes in the underlying demand and supply functions. Let's wrap up by just thinking about what could have caused the demand shift that we've seen. And what could have caused the supply shift that we saw?

Now, there are a couple of ideas that we can have for demand. The first idea that we could have is we could just have had an increase in the income of a consumer. If a consumer has more income, then they might be willing to spend more on steel.

A second idea that we have, we could have that the price of a substitute-- perhaps you're considering building a bridge out of iron instead of steel-- if the price of the substitute has increased, then perhaps the consumers are going to be willing to pay more to get the steel

since the iron is more expensive. A third possible idea is that the number of goods that you need to make from steel is increasing. So if you suddenly find new uses for steel, then the price that you're willing to pay at any given point is going to be higher.

Basically, to affect the demand curve, you have to think about why would people be more willing to pay more for a fixed quantity. And I just listed off a couple of ideas there. We can also think about reasons about why the supply curve could be shifting in. In this case, why is it-- why are sellers willing to offer a cheaper price at any fixed quantity?

And one idea that we could have for this is just that there are more firms in this market. If this market isn't perfectly competitive to start off with, then increasing the number of firms is gonna increase competition, and the producers are gonna have to drop their prices. A second idea for why we've seen the supply curve shift out and down could be the fact that input price for steel has dropped. Perhaps the way of manufacturing or getting the raw material is cheaper because the machine they're using to get the steel is cheaper. Basically, when you're thinking about the shift that's making it cheaper for suppliers to produce the good, all you need to think about is what could make it so that they're more willing to produce at a lower price.

So again, with this problem, we went through working with elasticities and demands. We've seen that we can go from a demand curve or supply curve to elasticities, or we can go from elasticities to demands. And then, once we've had the supply and the demand curves, we looked at how do we interpret the shifts and shocks? And we looked at possible explanations for those shift and shocks. I hope you found this problem helpful.