Fall 2018 14.01 Problem Set 8 - *Solutions*

Problem 1: True or False (20 points)

Determine whether the following statements are True or False. Explain your answer.

1. *(4 points)* Raising the minimum wage may improve the welfare of workers, but will always lead to an increase in the deadweight loss.
   Solution: False, if employers have market power, then raising the minimum wage could decrease the DWL.

2. *(4 points)* An increase in the interest rate has an ambiguous effect on the savings of a utility maximizing household.
   Solution: True, depending on how strong the income and substitution effects are, savings could either increase or decrease when the interest rate increases.

3. *(4 points)* A student starting a 3 year long post-doc at MIT is considering two alternative car rental programs to use during those 3 years. Program 1 charges a $1200 initial membership fee and then $120 per year. Program 2 charges a $360 initial membership fee and then $240 per year. The student will always strictly prefer program 2. Assume $r > 0$.
   Solution:True. The PDV of the student's expenses under program 1 will be larger as long as:

$$1200 + 120 + \frac{120}{1+r} + \frac{120}{(1+r)^2} > 360 + 120 + \frac{240}{1+r} + \frac{240}{(1+r)^2}$$

   which simplifies to $6 > \frac{2+r}{(1+r)^2}$ or $(2r+1)(3r+4) > 0$, which is always the case for $r > 0$. (Note to the grader: Any other algebraic justification that gives the right answer is fine.)

4. *(4 points)* Allen is working, consuming, and saving rationally for retirement. A rise in real interest rates will definitely lead Allen to optimally decide to save more for retirement and consume less today.
   Solution: False. The interest rate reflects the tradeoff between consumption now and consumption later. So a rise in interest rates does not change the amount

that is feasible to consume today, but allows us to consume more later if we save. However, this does not mean we necessarily consume more later, because, most importantly, **while the substitution effect means we should buy more of consumption later, the income effect can offset it and make us consume more today***. It is possible that a rise in interest rates means that the money Allen is already saving will be worth more when he retires, and thus he does not necessarily need to save more.*

5. *(4 points)* Suppose that interest rates are at 2 percent and a firm is considering a project with strictly positive net present value. If interest rates increase to 4 percent, the firm will still decide to make the investment to start that project.
Solution: False or Uncertain. This depends on the how the costs and earnings are distributed over the time horizon. For example, a one-time cost that doesn't yield benefits until 10 years in the future could have a positive NPV when interest rates are 2 percent but a negative NPV when interest rates are 4 percent.

Problem 2: Monopsony and the labor market (30 points)

Suppose that a logging company in Northern Carolina faces a perfectly competitive market for the lumber it produces (that is the company takes the price of lumber p as given). However, the logging company is the only employer in the area and has a monopsony.

1. *(8 points)* Suppose that workers in the area (employed by the logging company) are identical and have utility over consumption and labor given by

$$u(c, \ell) = \frac{5}{3}c^{\frac{3}{5}} - \frac{2}{3}\ell^{\frac{3}{2}}$$

and earn w for each unit of labor supplied. The price of one unit of consumption is equal to 1, and the only income a worker has is his labor income. Find the amount of labor that each worker will supply as a function of w.
Solution: The worker's budget constraint is given by $c = w\ell$. Then, the worker's utility maximization problem becomes

$$\max_{\ell} \quad \frac{5}{3}(w\ell)^{\frac{3}{5}} - \frac{2}{3}\ell^{\frac{3}{2}}$$

Taking the derivative with respect to ℓ yields

$$(w\ell)^{\frac{-2}{5}}w = \ell^{\frac{1}{2}} \implies w^{\frac{3}{5}} = \ell^{\frac{9}{10}} \implies \ell = w^{\frac{2}{3}}$$

Then, consumption would be given by $c = \ell w = w^{\frac{5}{3}}$

2. *(2 points)* Suppose that there are 81 workers in the area. Find the market labor supply as a function of wage.
   Solution: Since all workers are have the same utility function, we can simply add up their individual labor supply to find that

   $$L^s(w) = 81w^{\frac{2}{3}}$$

3. *(8 points)* Suppose that the firm's production of lumber is given by

   $$F(L) = 240L^{\frac{1}{4}}$$

   and the price of lumber is equal to $p = 9$ per unit. Recall that the firm has a monopsony over the labor market (they are the only employers in the area). Write the firm's total revenue and total cost as a function of w.
   Solution: Note that in equilibrium, the labor supplied must be equal to the labor demanded by the firm. Therefore, the firm's revenue is given by

   $$R = p \cdot q = 240pL^{\frac{1}{4}} = 240p \left(81w^{\frac{2}{3}}\right)^{\frac{1}{4}} = 6480w^{\frac{1}{6}}$$

   Likewise, the total cost is given by

   $$TC = w \cdot L = w \left(81w^{\frac{2}{3}}\right) = 81w^{\frac{5}{3}}$$

4. *(4 points)* Find the wage that the firm sets and the amount of labor the firm demands.
   Solution: The firm's optimality condition is given by $MR = MC$. Taking the derivatives with respect to w, we get

   $$MR(w) = 1080w^{\frac{-5}{6}}, \qquad MC(w) = 135w^{\frac{2}{3}}$$

   Setting these equal we get

   $$1080w^{\frac{-5}{6}} = 135w^{\frac{2}{3}} \implies 8 = w^{\frac{3}{2}} \implies w = 4$$

   To find labor demanded in equlibrium, we plug the wage back into the labor supply function to get $L = 81(4)^{\frac{2}{3}} \approx 204.1$.

5. *(8 points)* Suppose that the government wants to improve total surplus by imposing a minimum wage in North Carolina. What wage should the government set?
   Solution: The socially optimal level of wages satisfies

   $$w = p \cdot MPL \implies w = 540L^{\frac{-3}{4}}$$

where L is the amount of labor demanded by the firm. Rewriting the above, we get

$$\frac{w}{540} = L^{\frac{-3}{4}} \implies L = \left(\frac{540}{w}\right)^{\frac{4}{3}}$$

Setting this equal to the labor supply, we get

$$\left(\frac{540}{w}\right)^{\frac{4}{3}} = 81w^{\frac{2}{3}} \implies \frac{1}{81}(540)^{\frac{4}{3}} = w^2 \implies w = \frac{1}{9}(540)^{\frac{2}{3}} = (20)^{\frac{2}{3}}$$

Thus, the minimum wage the government should set is equal to $w = (20)^{\frac{2}{3}}$

Problem 3: Intertemporal Choice (20 points)

A household has to decide how much to consume during their working age and how much to save for retirement. We will model this as if there were two periods: period 1 is the working age, while period 2 is retirement. Suppose that we can represent the preferences of this household with the utility function

$$U(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma}$$

where c_1 is consumption in period 1, and c_2 is consumption in period 2 and $\sigma > 0$. Buying 1 unit of consumption costs \$1 in both periods. Income in period 1 is W dollars and zero during retirement. The household can save at the market interest rate r. Assume that the household gets no utility from leaving any money behind after death.

1. *(5 points)* What is the price today of one unit of consumption during retirement? Why?
   Solution: The present value of one unit of consumption during retirement is $\frac{1}{1+r}$. To spend one more dollar tomorrow, I only need to forgo $\frac{1}{1+r}$ dollars of consumption today because when I this amount in the bank I will get paid interest so that tomorrow I will have $(1+r) \times \frac{1}{1+r} = 1$.

2. *(5 points)* Write an expression for the household's budget constraint in terms of today's value of consumption and income.
   Solution: The budget constraint is

$$c_1 + \frac{c_2}{1+r} = W$$

3. *(5 points)* How much of its income will the household consume and how much will it save given the interest rate r?
   Solution: From the tangency condition, we have

$$c_2^{\sigma} = (1+r)c_1^{\sigma} \implies c_2 = (1+r)^{\frac{1}{\sigma}}c_1$$

Replacing in the budget constraint

$$c_1 + (1+r)^{\frac{1}{\sigma}-1} c_1 = W \implies c_1 = \frac{W}{1 + (1+r)^{\frac{1-\sigma}{\sigma}}}$$

and this implies that

$$c_2 = \frac{W(1+r)^{\frac{1}{\sigma}}}{1 + (1+r)^{\frac{1-\sigma}{\sigma}}}$$

Savings is equal to

$$s = W - c_1 = W\left(1 - \frac{1}{1 + (1+r)^{\frac{1-\sigma}{\sigma}}}\right) = \frac{W(1+r)^{\frac{1-\sigma}{\sigma}}}{1 + (1+r)^{\frac{1-\sigma}{\sigma}}}$$

4. *(5 points)* First consider the case when $\sigma = \frac{1}{2}$. Does increasing interest rates increase or decrease savings? Next consider the case when $\sigma = 2$. Does increasing interest rates increase or decrease savings in these two cases? If the qualitative effect of interest rates on savings are different in these two cases, provide some intuition for why this might happen.
   Solution: When $\sigma = \frac{1}{2}$, we have that savings is equal to

$$s = \frac{W(1+r)}{1 + (1+r)} \implies \frac{\partial s}{\partial(1+r)} = \frac{W}{(1 + (1+r))^2} > 0$$

   So increasing interest rates increases savings and decreases consumption in period 1. When $\sigma = 2$, we have that savings is equal to

$$s = \frac{W(1+r)^{\frac{-1}{2}}}{1 + (1+r)^{\frac{-1}{2}}} \implies \frac{\partial s}{\partial(1+r)} = \frac{-\frac{1}{2}W(1+r)^{\frac{-3}{2}}}{\left(1 + (1+r)^{\frac{-1}{2}}\right)^2} < 0$$

   So increasing interest rates decreases savings and increases consumption in period 1.
   In the case when $\sigma = \frac{1}{2}$, the substitution effect dominates the income effect so savings increases. In the case when $\sigma = 2$, the income effect dominates, so consumption in period 1 increases and savings decreases.

Problem 4: Equilibrium interest rate (30 points)

Suppose that agents live for two periods $t = 1, 2$ and die. Agents have wealth in the first period $W_1 > 0$ but no wealth in the second period ($W_2 = 0$). Additionally agents have utility over consumption in the first and second period, given by

$$u(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma}$$

where $\sigma > 0$. The interest rate in the economy is r. For every dollar an agent saves, the agent gets $1 + r$ in period 2. Buying one unit of consumption costs \$1 in both periods.

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1. *(6 points)* Write down the budget constraints of the agents.
   Solution: For the agent
   $$c_1 + \frac{c_2}{1+r} = W_1$$

2. *(6 points)* Find the optimal level of savings for the agent as a function of r.
   Solution: The tangency (optimality) condition gives us
   $$c_2^\sigma = c_1^\sigma(1+r) \implies c_2 = c_1(1+r)^{\frac{1}{\sigma}}$$
   Combining with the budget constraint, we get
   $$c_1 + (1+r)^{\frac{1}{\sigma}-1}c_1 = W \implies c_1 = \frac{W}{1+(1+r)^{\frac{1-\sigma}{\sigma}}}$$
   and this implies that
   $$c_2 = \frac{W(1+r)^{\frac{1}{\sigma}}}{1+(1+r)^{\frac{1-\sigma}{\sigma}}}$$
   For agent an agent who has wealth at time 1, we have
   $$s_A = W_1 - c_1 = \frac{W_1(1+r)^{\frac{1-\sigma}{\sigma}}}{1+(1+r)^{\frac{1-\sigma}{\sigma}}}$$

3. *(6 points)* Suppose that there are N agents who all have wealth W_1 in time 1 and 0 wealth at time 2, with the same utility function as above. What is the supply of savings at time 1? *Solution: The supply is given by the agents who want to save and is equal to*
   $$S = \frac{NW_1(1+r)^{\frac{1-\sigma}{\sigma}}}{1+(1+r)^{\frac{1-\sigma}{\sigma}}}$$

4. *(6 points)* Suppose that demand for savings as a function of the interest rate is given by
   $$D = \frac{\kappa}{(1+r)\ \ 1+(1+r)^{\frac{1-\sigma}{\sigma}}}$$
   where κ is a constant. Find the equilibrium interest rate in this market (solve for r as a function of N, W_1 and κ.
   Solution: We set supply equal to demand to get
   $$\frac{NW_1(1+r)^{\frac{1-\sigma}{\sigma}}}{1+(1+r)^{\frac{1-\sigma}{\sigma}}} = \frac{\kappa}{(1+r)\left[1+(1+r)^{\frac{1-\sigma}{\sigma}}\right]} \implies (1+r)^{\frac{1}{\sigma}} = \frac{\kappa}{NW_1}$$
   $$r = \left(\frac{\kappa}{NW_1}\right)^\sigma - 1$$

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5. *(6 points)* How do changes in N, W_1, κ affect the equilibrium interest rate? Explain the intuition.

   Solution: Increasing either κ increases the equilibrium interest rate. Increasing N or W_1 decreases the equilibrium interest rate. This is intuitive because when κ increases, there is a higher demand for savings (at any interest rate) which pushes up the interest rate. Conversely, increasing the number of agents who want to save increases the supply of savings which decreases the interest rate. If type agents have more money in period 1, this also increases their desire to save so that they can have higher consumption in period 2.

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