Fall 2018 14.01 Problem Set 5 - *Solutions*

Problem 1: True or False (24 points).

Determine whether the following statements are True or False. Explain your answer.

1. *(4 points)* A government sets a price ceiling for widgets that is below the equilibrium price. This intervention will always decrease the producer surplus, increase consumer surplus and decrease total surplus.
   Solution: False, it might increase or decrease consumer surplus. Price decreases but also quantity decreases. The firm(s) might also exit the market if the price is below average cost which would result in zero consumer surplus.

2. *(4 points)* A non-discriminating monopoly will optimally choose a price and quantity where the market demand curve is inelastic (i.e. elasticity is lower than one in absolute value).
   Solution: False, the monopolist will always choose a price where the demand is elastic.

3. *(4 points)* If a monopolist charging a uniform price begins to practice perfect price discrimination, some consumers may become strictly better off.
   Solution: False, the perfectly discriminating monopolist will extract all the surplus from consumers, so no consumer can be strictly better off.

4. *(4 points)* Transferring a monopolist's profit to consumers eliminates the inefficiency associated with monopoly.
   Solution: False. The total surplus will not change. Because this policy would increase consumer surplus and reduce producer surplus by the same amount. There is still a deadweight loss associated with a monopolist producing a quantity that is lower than the socially optimal given by $p = MC$.

5. *(4 points)* Martin knows that if he sells 500 widgets, his revenue will be $1000 and that if he sells 800 widgets, his revenue will be $1500. The market for widgets is perfectly competitive.
   Solution: False. Martin's output affects the market price which indicates that he is not a price taker.

6. *(4 points)* Suppose that a monopolist initially charging a uniform price, now moves to charging a different price to young and old consumers, but the total quantity sold by the monopolist doesn't change. Claim: total welfare must be lower after the monopolist starts discriminating.

   Solution: True, if the quantity sold is the same then total costs are the same, but because with price discrimination the goods might not be going to those who value them the most, the allocation becomes more inefficient. Note: don't take down points if students answer that the monopolist might choose not to discriminate when she is allowed to, so total welfare is unchanged (that is, the monopolist might choose to sell at a uniform price even if she can discriminate between old and young consumers).

Problem 2 (41 points)

Consider the perfectly competitive market for gasoline. The aggregate demand for gasoline is

$$D(p) = 100 - p$$

while the aggregate supply is

$$S(p) = 3p$$

1. *(5 points)* Calculate the equilibrium price and quantity. At this equilibrium, compute the consumer surplus, producer surplus and total surplus.
   Solution:

$$p = 25 \tag{1}$$
$$q = 75 \tag{2}$$

$$CS = \frac{75^2}{2} = 2812.5 \tag{3}$$
$$PS = \frac{25 \times 75}{2} = 937.5 \tag{4}$$
$$TS = 2812.5 + 937.5 = 3750 \tag{5}$$

2. *(5 points)* Suppose now that the government is concerned because many gas stations are going out of business, so it decides to set a minimum price of $\bar{p} = 30$ to help them. What will be the new equilibrium price and quantity with this intervention? Compute the consumer surplus and producer surplus; who gains and who loses from this regulation? How is the total surplus affected? Briefly explain

the intuition.

Solution: The price will be $p = 30$, and the quantity $q = 70$.

$$CS = \frac{70^2}{2} = 2450 \tag{6}$$

$$PS = \frac{\frac{70}{3} 70}{2} + \left(30 - \frac{70}{3}\right) \times 70 = 1283.3 \tag{7}$$

$$TS = 2450 + 1283.3 = 3733.3 \tag{8}$$

Consumers are worse off, producers are better off, and the total surplus decreases.

3. *(5 points)* Suppose now that instead of regulating prices, the government decides it is better to help gas stations by setting quantity regulations. In particular the government sets a quota of $\bar{q} = 70$ (this means that aggregate quantity supplied can't exceed 70 units). What will be the new equilibrium price and quantity with this regulation? How does it compare to the results obtained with the minimum price?

   Solution: We will obtain the same equilibrium as with the minimum price, so both policies are equivalent.

4. *(6 points)* Suppose now that because of a conflict in the Middle East prices of oil increase, which increases the cost of supplying each unit of gas by $\$x$. Find the minimum value of x such that the regulations discussed above become irrelevant to determine the market equilibrium.

   Solution: The new supply function shifts up by x : $S = 3(p - x)$. I want to find x such that the equilbrium price is 30:

$$100 - 30 = 3(30 - x) \tag{9}$$

$$70 = 90 - 3x \tag{10}$$

$$x = \frac{20}{3} \tag{11}$$

5. *(5 points)* From this question onward assume that instead of perfect competition, there is a monopolist in this market. Its cost curve is $C(q) = q^2/6$. Assume it sets a uniform price. What price and quantity does the monopolist set? Compute the consumer surplus, producer surplus, total surplus and the deadweight loss in this case. Compare your results to the perfectly competitive case and explain.

   Solution: The monopolist maximizes profits by setting $MR = MC$.

$$TR = p(q) \times q = (100 - q)q \tag{12}$$

$$MR = 100 - 2q \tag{13}$$

$$MC = q/3 \tag{14}$$

$$\tag{15}$$

So setting $MR = MC$ gives:

$$100 - 2q = q/3 \tag{16}$$
$$q = 300/7 \approx 42.86 \tag{17}$$
$$\tag{18}$$

Substituting back into the demand function, we find the price

$$p = 100 - 300/7 = 400/7 \approx 57.14 \tag{19}$$
$$\tag{20}$$

$$CS = \frac{(300/7)^2}{2} \approx 918.37 \tag{21}$$
$$PS = \frac{(100/7) \times (300/7)}{2} + (300/7)^2 \approx 2142.12 \tag{22}$$
$$TS = 918.37 + 2142.12 \approx 3060 \tag{23}$$
$$\tag{24}$$

Compared to perfect competition, consumers are worse off and producers are better off. This is due to the market power that the monopolist has. Total welfare decreases because the monopolist charges one price. It does not want to lower the price because it would lose money on all existing sales and this results in deadweight loss.

The deadweight loss is the difference between the total surplus under perfect competition and the total surplus under monopoly i.e.

$$DWL = 3750 - 3060 = 690 \tag{25}$$
$$\tag{26}$$

6. *(5 points)* Suppose that the government would like to help the monopolist by setting a minimum price. What price should it set?
   Solution: The monopolist is choosing its price without any constraints in order to maximize profits, hence no government intervention can result in a (strictly) better outcome for the firm.

7. *(5 points)* Now suppose that the government wants to set a price ceiling in order to maximize welfare (total surplus). What price should it set and what would be the resulting total surplus?

Solution: The competitive equilibrium maximizes total welfare so the government should set $p = 25$ which would force the monopolist to become a price taker and produce the socially optimal quantity of $q = 75$. The total surplus would be equal to $TS = 3750$ as computed in question 1.

8. *(5 points)* If the monopolist were able to practice perfect price discrimination, what quantity would it produce? Compute the consumer surplus, producer surplus, total surplus and the deadweight loss in this case.

Solution: Under perfect price discrimination, each consumer is charged their marginal willingness to pay for all units up until $p = MC$. Hence the monopolist would produce the competitive equilibrium quantity $q = 75$.

In this case

$$
\begin{align}
CS &= 0 \tag{27}\\
PS = TS &= 3750 \tag{28}\\
DWL &= 0 \tag{29}\\
 & \tag{30}
\end{align}
$$

we have maximum welfare but it all goes to the producer.

Problem 3 (20 points)

A uniform pricing monopolist has a cost function $C(q) = \frac{1}{2}q^2$. It faces a market demand of

$$D(p) = p^{-\mu}, \ \mu > 1$$

1. *(5 points)* Calculate the price elasticity of demand.
   Solution: Elasticity is

$$
\begin{align}
\frac{d\ln(x)}{d\ln(p)} &= \frac{\partial D(p)}{\partial p}\frac{p}{D(p)} = (-\mu)\,p^{-\mu-1}\frac{p}{p^{-\mu}} \tag{31}\\
&= -\mu \tag{32}
\end{align}
$$

2. *(5 points)* Write down the monopolist's profit maximization problem as a function of p. Differentiate with respect to p to find the optimal price.
   Solution: The monopolist's problem is

$$\max_{p} p \times p^{-\mu} - \frac{1}{2}\left(p^{-\mu}\right)^2 = p^{1-\mu} - \frac{1}{2}p^{-2\mu}$$

so the first order condition is

$$(1 - \mu) p^{-\mu} - (-\mu) p^{-2\mu-1} = 0 \tag{33}$$

$$p^* = \left(\frac{\mu}{\mu - 1}\right)^{\frac{1}{1+\mu}} \tag{34}$$

3. *(5 points)* What is the markup at the optimal price? What is the optimal price when $\mu = 2$? And when $\mu = 3$? Explain the intuition.
   Solution: The markup is

$$\frac{p^* - MC\,(q^*)}{p^*} = \frac{p^* - (p^*)^{-\mu}}{p^*} \tag{35}$$

$$= 1 - (p^*)^{-\mu-1} \tag{36}$$

$$= 1 - \frac{\mu - 1}{\mu} \tag{37}$$

$$= \frac{1}{\mu} \tag{38}$$

When $\mu = 2$, we have $p = 2^{\frac{1}{3}} \approx 1.26$. When $\mu = 3$, we have $p = \left(\frac{3}{2}\right)^{\frac{1}{4}} \approx 1.11$. When the elasticity of demand increases, the monopolist sets a lower price.

4. *(5 points)* Graph the marginal revenue, marginal cost, and demand curves, and show the deadweight loss from monopoly pricing in the graph (you don't need to assume any particular value of μ, you can do a qualitative graph).

Problem 4 (15 points)

There are two types of widget consumers in Boston. Type 1 consumers have (aggregate for all type 1) demand given by $Q_1 = 100 - p$. Type 2 consumers have (aggregate) demand given by $Q_2 = 110 - \frac{1}{2}p$. The cost of producing widgets is $TC = \frac{1}{2}Q^2$, where Q is the total number of widgets produced. Assume that the producer of widgets behaves as a monopolist.

1. (7 points) Suppose that the producer cannot distinguish between the two types of consumers. What price will it charge? What is the elasticity of demand at that point?

   Solution: We add up horizontally to find aggregate demand. However, for $p > 100$ only one group of consumers demands the good, so

$$Q = Q_1 + Q_2 = \begin{cases} 210 - \frac{3}{2}p, & if\, p \le 100 \\ 110 - \frac{1}{2}p, & if\, 100 < p \le 220 \\ 0, & if\, p > 220 \end{cases} \tag{39}$$

Figure 1:
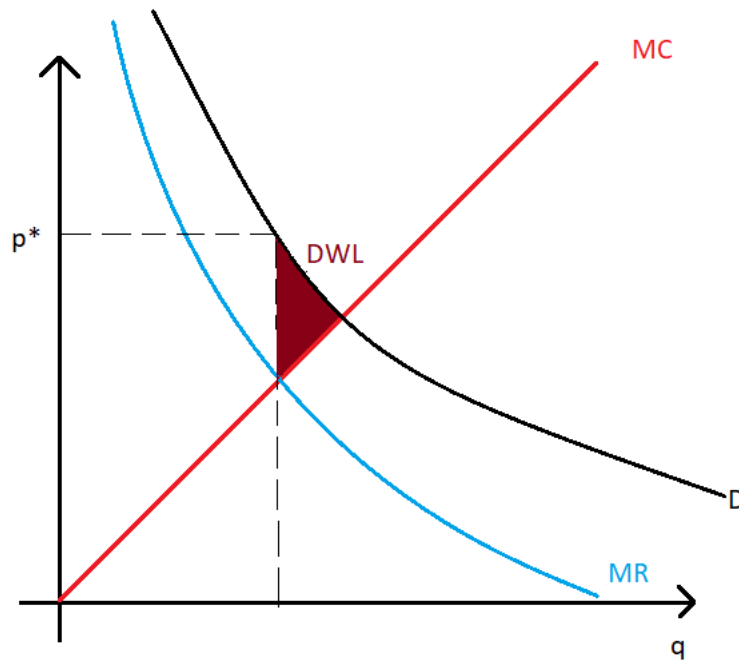
The monopolist will choose to sell at the part of demand which yields the highest profits.

If it chooses to sell at the large part of the market then the inverse demand is:

$$p = 140 - \frac{2}{3}Q \tag{40}$$

$$\tag{41}$$

The monopolist maximizes profits by setting $MR = MC$.

$$TR = p(Q) \times Q = 140Q - \frac{2}{3}Q^2 \tag{42}$$

$$MR = 140 - \frac{4}{3}Q \tag{43}$$

$$MC = Q \tag{44}$$

$$\tag{45}$$

So setting $MR = MC$ gives:

$$Q = 60 \tag{46}$$
$$\tag{47}$$

Substituting back into the demand function, we find the price

$$p = 140 - \frac{2}{3}60 = 100 \tag{48}$$
$$\tag{49}$$

which is in the required range and is an acceptable solution. The profits at that point are:

$$\pi = TR - TC = 60 \times 100 - \frac{1}{2}60^2 = 4200 \tag{50}$$
$$\tag{51}$$

If it chooses to sell at the higher price range $220 \geq p > 100$, then only one group of consumers is willing to buy. The inverse demand is:

$$p = 220 - 2Q \tag{52}$$
$$\tag{53}$$

: So:

$$TR = p(Q) \times Q = 220Q - 2Q^2 \tag{54}$$
$$MR = 220 - 4Q \tag{55}$$
$$MC = Q \tag{56}$$
$$\tag{57}$$

So setting $MR = MC$ gives:

$$Q = 44 \tag{58}$$
$$\tag{59}$$

Substituting back into the demand function, we find the price

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$$p = 220 - 2 \times 44 = 132 \tag{60}$$
$$\tag{61}$$

which is in the required range and is an acceptable solution. The profits at that point are:

$$\pi = TR - TC = 132 \times 44 - \frac{1}{2}44^2 = 4840 \tag{62}$$
$$\tag{63}$$

Thus, the firm will choose to sell only to one group of consumers at the higher price since profits are larger.

The kinked demand curve generates a discontinuous marginal revenue curve (MR) which intersects the marginal cost curve (MC) twice. This gives two local maxima and them we compare the values of those local maxima to find the global maximum

Elasticity at that point is

$$\epsilon = \frac{\partial Q}{\partial p}\frac{p}{Q} = -1/2\frac{132}{44} = -\frac{3}{2} \tag{64}$$
$$\tag{65}$$

2. (8 points) Now assume that the monopolist can distinguish between the two types of consumers.

   (a) (5 points) How many widgets will the monopolist sell to each group of consumers and how much will it charge them?

   Solution: The monopolist will maximize total profits

$$\pi = TR_1(Q_1) + TR_2(Q_2) - TC(Q_1 + Q_2) \tag{66}$$
$$\tag{67}$$

   The inverse demand functions are:

$$p_1 = 100 - Q_1 \tag{68}$$
$$p_2 = 220 - 2Q_2 \tag{69}$$
$$\tag{70}$$

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and

$$TR_1 = 100Q_1 - Q_1^2 \tag{71}$$
$$TR_2 = 220Q_2 - 2Q_2^2 \tag{72}$$
$$\tag{73}$$

It will thus set

$$MR_1(Q_1) = MR_2(Q_2) = MC(Q_1 + Q_2) \tag{74}$$
$$\tag{75}$$

Thus, we have the following two equations with two unknowns

$$100 - 2Q_1 = (Q_1 + Q_2)$$

and

$$220 - 4Q_2 = (Q_1 + Q_2)$$

Solving this system of equations yields

$$Q_1^* = 20 \quad and \quad Q_2^* = 40$$

Prices are determined by plugging the quantities into the respective inverse demand curves:

$$P_1^* = 100 - Q_1^* = 80 \quad and \quad P_2^* = 220 - 2Q_2^* = 140$$

(b) (3 points) What are the elasticities of demand at the optimal points for the two types of consumer. Explain.

Solution: Elasticities for the two markets are

$$\epsilon_1 = \frac{\partial Q_1}{\partial p_1} \frac{p_1}{Q_1} = -1\frac{80}{20} = -4 \tag{76}$$
$$\tag{77}$$

$$\epsilon_2 = \frac{\partial Q_2}{\partial p_2} \frac{p_2}{Q_2} = -1/2\frac{140}{40} = -7/4 \tag{78}$$
$$\tag{79}$$

The consumers with less elastic demand are charged a higher price.

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