

Fall 2018 14.01 Problem Set 10 - *Solutions*

Problem 1: True or False (12 points)

Determine whether the following statements are True or False. Explain your answer.

1. (3 points) Consider a gamble that pays \$10 with probability 50% and \$0 with probability 50%. No expected-utility maximizing individual will strictly prefer this gamble over a certain payment of 5.

Solution: FALSE. The risk-loving individuals, or individuals with convex utility curves will prefer this gamble.

2. (3 points) Consider a gamble with which you gain \$50 with probability 20%, you gain \$130 with probability 30%, or you lose \$95 with probability 50%. A risk-neutral individual would take this gamble.

Solution: TRUE. The expected value of the gamble is strictly positive

3. (3 points) Consider an economy with two people, Alan and Brian, and a single good: potatoes. Both Alan and Brian share the same utility function $u = \sqrt{p}$, where p is the number of potatoes they possess. Now consider a following transfer: Alan gives some potatoes to Brian, but half of which are lost in transportation. A government who maximizes the utilitarian social welfare function will never prefer such a transfer.

Solution: FALSE. For example, if Alan has 9 potatoes and Brian has 0 potatoes. Alan now gives 2 potatoes to Brian but 1 is lost in transportation, so now Alan has 7 potatoes and Brian has 1. Before the transfer the social welfare is $\sqrt{3} + \sqrt{0} = 3$, after the transfer the social welfare is $\sqrt{7} + \sqrt{1} = 3.646 > 3$, so the government will prefer such a transfer.

4. (3 points) A country that is a net exporter of a good, in the absence of any government subsidies or interventions, must always have advanced technology in producing that good

Solution: FALSE. Comparative advantage can also come from differences in factor endowments—the country could also have more of the resources required for making the good, rather than better technology.

Problem 2: Diversification (30 Points)

Oliver has an endowment of \$10,000 that he wants to invest. He can either invest in

- a bond, which yields 1%,
- the stock market, which consist of two firms, Amazon and Toys R Us

Each firm's stock is costs \$100 today, and will be worth \$400 in one year with probability $\frac{1}{2}$ or will drop to \$0 with probability $\frac{1}{2}$. Assume that the evolution of both stocks is independent: that is, the probability that stock of Amazon rises in value does not vary or depend on what has happened to stock of Toys R Us, and vice-versa.

Finally, assume that Oliver's utility function is $U(w) = \sqrt{w}$, and there is no inflation.

1. (2 points) Suppose that due to institutional regulations, Oliver can invest only in bonds, or only in Amazon. He cannot buy Toys R Us stock and he cannot buy both Amazon stock and bonds. What is Oliver's utility of buying bonds? What if he invests only in Amazon's stock? What does he prefer?

Solution: If Oliver does not invest, his utility is $\sqrt{10,000} \approx 100.5$. As for investing in Amazon, the expected utility is $\frac{1}{2}\sqrt{40,000} + \frac{1}{2}\sqrt{0} = 100$. Oliver thus prefers to invest in bonds.

2. (8 points) Now suppose there is a change in regulations. Oliver can invest in either the stock market or in bonds, but not both. If Oliver decides to invest in the stock market, he can choose how much he wants to invest in each company. Let x denote the fraction of wealth that Oliver puts into Amazon and $1 - x$ denote the fraction of wealth Oliver puts into Toys R Us. If Oliver maximizes his expected utility, what should be the value of x ?

Solution: Oliver should put exactly half of his money into Toys R Us and half into Amazon. To see this, let x be the fraction that Oliver puts into amazon so that $(1 - x)$ is the fraction into Toys R Us. Then, the number of Amazon stocks held by Oliver is $\frac{10,000x}{100}$ and the number of Toys R Us stocks is $\frac{10,000(1-x)}{100}$. The four possible outcomes tomorrow are:

- Both stocks quadruple in value
- Amazon quadruples, Toys R Us is worth 0
- Toys R Us quadruples, Amazon is worth 0
- Both stocks are worth 0.

Oliver's expected utility from investing a fraction x into amazon and $1 - x$ into

Toys R Us is given by

$$\frac{1}{4}\sqrt{\left(\frac{10000x}{100}\right)400 + \left(\frac{10000(1-x)}{100}\right)400} \\ + \frac{1}{4}\sqrt{\left(\frac{10000x}{100}\right)400} + \frac{1}{4}\sqrt{\left(\frac{10000(1-x)}{100}\right)400} + \frac{1}{4}\sqrt{0}$$

We can take the derivative of this with respect to x and set equal to zero. This yields $x = \frac{1}{2}$

3. (5 points) What is the expected value of the strategy in the previous question, and Oliver's expected utility? Does Oliver prefer to use this strategy or to invest in bonds? What is the effect at work here? (3 points)

Solution: The expected value is still always \$20,000. Oliver's expected utility is now $\frac{1}{4}\sqrt{40000} + \frac{1}{2}\sqrt{20000} + \frac{1}{4}\sqrt{0} \approx 120.71$, which is larger than the utility of investing in bonds. Oliver is better off by diversifying his investments, because by doing so he reduced his risk without reducing his expected payoffs. Since he is risk-averse, that makes him strictly better off.

4. (5 points) Suppose again that Oliver can only invest in stocks or only invest in bonds. Additionally, now Toys R Us has gone bankrupt, but Oliver could invest in a third company, Walmart, whose stock today is worth \$100. Interestingly, the value of the stocks for Amazon and Walmart is negatively correlated, so that with probability 1/2 Amazon goes bankrupt and Walmart's stocks are worth \$400, and with probability 1/2 the opposite happens. What is Oliver's optimal investment? Is correlation hurting or helping Oliver?

Solution: Note that the expected value of any combination of Amazon and Walmart is the same, namely \$20000. However, Oliver's utility is maximized when he fully diversifies and buys 1/2 of A and 1/2 of C, in which case his utility is $\sqrt{20000} = 141$. Correlation helps Oliver, because he is able to obtain \$20000 with probability 1, and because he is risk-averse he values certainty.

5. (5 points) Suppose Oliver works for Amazon and he is investing for his retirement. Is the investment decision you found in last question still optimal? Why, or why not?

Solution: If Oliver works for A, then the investment we found is not optimal, because there is a new source of correlation: namely when A goes bankrupt Oliver also loses his job. Because Oliver is risk-averse, he would like to hedge against this event, by investing more than half of his savings in C, so that when A goes bankrupt Oliver's returns to investment are high.

Problem 3: European Potatoes (subsidy vs. welfare) (25 points)

Assume that the EU subsidizes the exports of potatoes to support employment in agriculture.

Supply and demand of potatoes of European producers and consumers are equal to $p = 400 + 10q_s$ and $p = 1000 - 2q_d$ respectively (where prices are in dollars and quantities in tons).

1. (3 points) First consider the case of Europe under autarky. What is the equilibrium price and quantity of potatoes when there is no subsidy?

Solution: When there is no government intervention and no trade, the equilibrium is given by

$$400 + 10q = 1000 - 2q \implies 12q = 600 \implies q = 50 \implies p = 400 + 10(50) = 900$$

2. (3 points) Assume that under free trade, the global price is \$800 and European consumers can import however many tons of potatoes at that price per ton. Does Europe import or export potatoes at this price? How much does the EU import or export?

Solution: Under free trade, equilibrium price is 800. The quantity demanded is 100 tons. The quantity supplied by domestic producers is 40, so the EU imports 60 tons of potatoes to meet demand.

3. (5 points) Now, to support employment in agriculture, the EU starts subsidizing potatoes by 6.25% such that the price European producers get from selling potatoes rises to 850 (consumers are still paying the old price, but the government gives the producers an additional 50 dollars per ton of potato). How does this change the supply of potatoes for European producers? What is the new equilibrium and how does the subsidy affect potato exports or imports?

Solution: Since producers can get 850, they produce 45 tons of potatoes. The price paid by consumers does not change so quantity demanded is still 100 tons and the EU imports 55 tons of potatoes.

4. (4 points) How much does EU consumer surplus change?

Solution: Consumer surplus does not change because the consumer price and quantities do not change.

5. (5 points) How much does EU producer surplus change?

Solution: The old producer surplus is given by $\frac{1}{2}(800 - 400)(40) = 8000$. The new producer surplus with the subsidy is given by $\frac{1}{2}(850 - 400)(45) = 10125$. The change in surplus with the subsidy is 2125.

6. (5 points) How much does this subsidy program cost the EU? What is the change in total surplus from this program? Explain why total surplus decreases

*Solution: For each ton sold, government must spend 50. So in total, the program costs: $50 * 45 = 2,250$. Total surplus takes into account the cost of the subsidy program. Total surplus decreases by 125 with the subsidy because the market was perfectly competitive before government intervention. Intervention in this case is inefficient in that it produces deadweight loss*

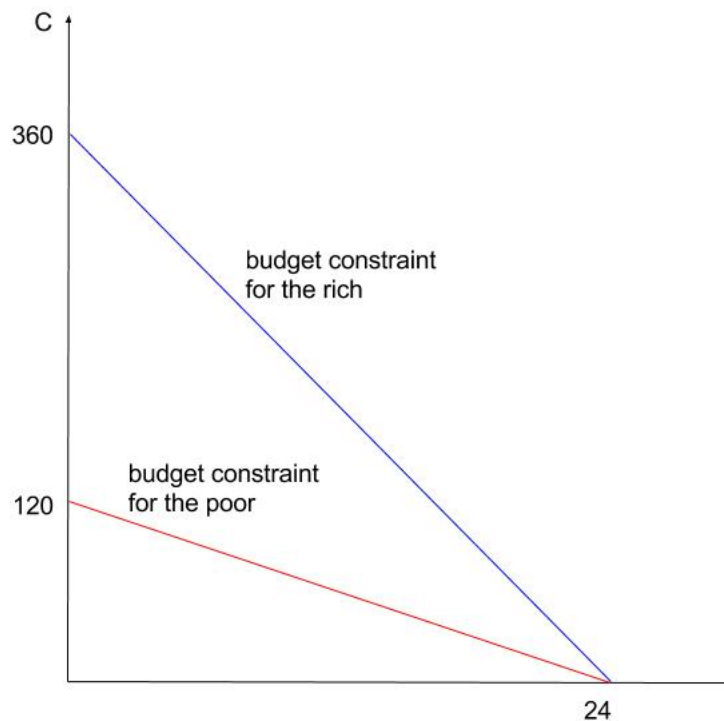
Problem 4: Taxes and Redistribution (33 points)

Consider an economy with two types of individuals: the skilled and the unskilled. The only difference between the two is that the skilled have a higher wage $w_s = 15$ than the unskilled do, $w_u = 5$. Suppose $1/7$ of the population is skilled and $6/7$ are unskilled. Suppose that each individual has a utility function over consumption (c) and labor (L) of the following form:

$$U(c, L) = c - \frac{1}{2}L^2$$

where $0 \leq L \leq 24$ Each individual's income is then $w_j L$, where $j = s, u$.

- (a) (3 points) Write down the individual's budget constraint in terms of consumption and leisure. Draw the different budget constraints for the skilled and unskilled *Solution: Individual j 's budget constraint is: $c + w_j \ell = 24w_j$ where $j = s, u$. The budget constraint is drawn below.*



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- (b) (4 points) Solve for each individual's optimal leisure, labor, and consumption choice.

Solution: We can solve for the optimum by setting the marginal rate of substitution between c and ℓ equal to their price ratio:

$$\frac{\partial U/\partial \ell}{\partial U/\partial c} = 24 - \ell = w$$

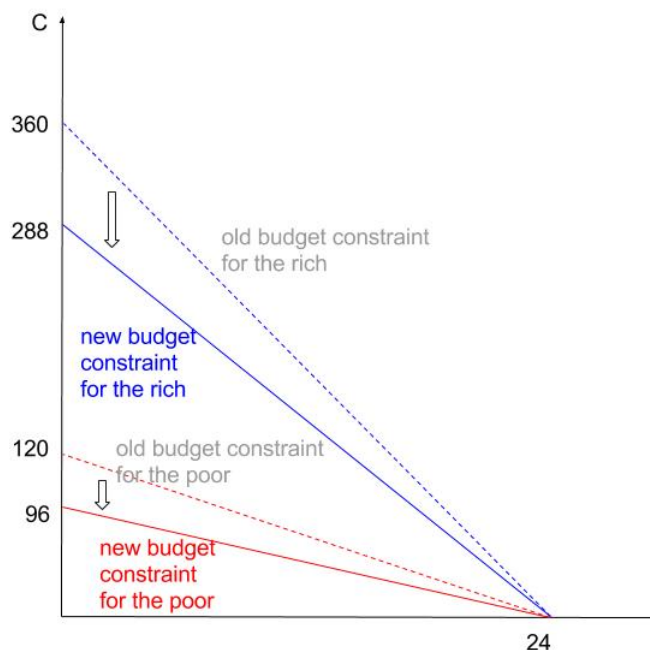
This implies directly implies $\ell^* = 24 - w$. If we then plug this back into the budget constraint and we rearrange, we find and $c^* = w \times w = w^2$.

unskilled individuals have wage $w_u = 5$ and thus enjoy $\ell_u^* = 24 - 5 = 19$ hours of leisure, work $h_u^* = 5$ hours, and consume $c_u^* = 25$. skilled individuals have wage $w_s = 15$ and thus enjoy $\ell_s^* = 24 - 15 = 9$ hours of leisure, work $h_s^* = 9$ hours, and consume $c_s^* = 225$.

2. Now suppose the government decides to levy an income tax. This tax collects $\frac{1}{5}$ of each individual's earnings. For the moment, assume it takes this tax but doesn't do anything with it.

- (a) (3 points) On the same set of axes, draw the new budget constraints faced by the two types of individuals. Again, label them clearly, with leisure on the x-axis.

Solution: The presence of the tax modifies the budget constraint to $c = \frac{4}{5}w(24 - \ell)$. This is the same budget constraint as an individual with wage $\frac{4}{5}w$ would face. In other words, imposing this tax is "like" reducing each individual's wage by $1/5$. The budget constraint is drawn below.



- (b) (4 points) Solve for each individual's new optimal leisure, labor, and consumption.

Solution: We can use the same solution from above. Instead of the wages $w_u = 3$ and $w_s = 9$, though, we'll have the (after tax) wages $w'_u = 4$ and $w'_s = 12$. unskilled individuals thus enjoy $\ell_u^{**} = 24 - 4 = 20$ hours of leisure, work $h_u^{**} = 4$ hours, and consume $c_u^{**} = 16$. skilled individuals enjoy $\ell_s^{**} = 24 - 12 = 12$ hours of leisure, work $h_s^{**} = 12$ hours, and consume $c_s^{**} = 144$.

- (c) (4 points) Compute the total per-capita tax collected by the government.

Solution: A simple way to compute the per-capital tax collected by the government is as follows. Observe that $c = 4/5y$, i.e., individuals get to consume $4/5$ of their income. Tax revenue is $1/5y$ (the rest of the income). So c is four times as big as tax revenue, or taxes are $1/4$ of consumption. This is 36 per skilled person and 4 per unskilled person. The per-person collection is thus $T = \frac{1}{7} \times 36 + \frac{6}{7} \times 4 = \frac{60}{7} = 8.57$.

3. Now suppose that the government uses the tax revenue to give an equal amount T to each resident at the beginning (this is called a lump-sum transfer).

- (a) (4 points) Show that the lump sum transfer T will not affect the optimal labor supply of either type of individual.

Solution: Adding in the lump-sum transfer changes the budget constraint to $c = T + \frac{4}{5}w(24 - \ell)$. Since T is a constant, it drops out when we take the first order conditions, and we still get $24 - \ell = \frac{4}{5}w$ at the optimum. So hours worked and hours of leisure enjoyed are not changed. This "unchanged leisure" is a special case: it only happens because the utility function is linear in c . This means that the two first order conditions don't have a c in them. If they did, we'd have to jointly solve the first order conditions with the budget constraint to find ℓ and c . But T would be in the budget constraint, so we'd find ℓ depends on T .

- (b) (6 points) Suppose the government sets T so that it spends what it collects (the government balances its budget). Show that this tax-plus-transfer policy is good for the unskilled and bad for the skilled (relative to the no-tax regime from part 1).

Solution: To see that the unskilled are better off, let's compute the utility before and after the tax and transfer were introduced. Before any tax was put in place, the unskilled had utility $25 - \frac{1}{2}(5)^2 = 12.5$ and the skilled had utility $225 - \frac{1}{2}(15)^2 = 112.5$. After the tax policy without transfers was in place, the unskilled had utility: $16 - \frac{1}{2}(4)^2 = 8$ and the skilled had utility $144 - \frac{1}{2}(12)^2 = 72$. The lump sum transfer T added to 24 to consumption without affecting ℓ , so the additional utility from this transfer was just 12. So the final utility of the unskilled was $8 + \frac{60}{7} = \frac{116}{7} = 16.57$. This is (much) greater than 12.5, so the unskilled are better off. But since $72 + \frac{60}{7} = \frac{564}{7} =$

80.571 < 112.5, the skilled are (much) worse off.

Note that it is not enough to show that the unskilled receive more in transfers than they pay in taxes. For example, if you put in a 99% tax rate and used the revenue for transfers, you'd find (a) the unskilled get more in transfers than they pay in taxes but (b) they end up strictly worse off than before the policy was introduced.

4. (3 points) Now suppose the government wants to maximize a utilitarian social welfare function. Would it prefer setting no taxes, as in part 1 of this section, or would it prefer to set an income tax of $\frac{1}{5}$ and redistribute it through an equal lump-sum transfer T , as in part 3 of this question?

Solution: A utilitarian social planner tries to maximize the sum of the utilities of unskilled and skilled individuals in society. As shown in the last section, when there are no taxes the utility of unskilled individuals is 12.5 and the utility of skilled individuals is 112.5. This implies that the utilitarian welfare function when there are no taxes yields the following value:

$$W(\text{No Taxes}) = \frac{6}{7} \times 12.5 + \frac{1}{7} \times 112.5 = \frac{375}{14} = 26.786.$$

Similarly, when there is an income tax of $\frac{1}{5}$ and the government makes an equal lump-sum transfer T to each individual, unskilled individuals get a utility of $8 + \frac{60}{7} = \frac{116}{7} = 16.571$ and skilled individuals get a utility of $72 + \frac{60}{7} = \frac{564}{7} = 80.571$, which leads to a utilitarian welfare function with the following value:

$$W(\text{Taxes}) = \frac{6}{7} \times \frac{116}{7} + \frac{1}{7} \times \frac{564}{7} = \frac{180}{7} = 25.714.$$

This shows that a utilitarian social planner clearly prefers to have no taxes.

5. (2 points) Which tax scheme would a Rawlsian social planner prefer? (No taxes vs. setting an income tax of 20% and redistributing this wealth equally).

A rawlsian social planner cares only about the worst-off member of the society which here is the unskilled. Therefore, such a government would prefer to tax at 20% and redistribute

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14.01 Principles of Microeconomics
Fall 2018

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