14.01 Principles of Economics Fall 2018
Midterm Exam Solutions

1 True/False Question (20 minutes)

For each of the following statements, write whether it is True or False, and justify your answer. Points will be given based on your explanation.

1. John has an income of $1,000. He spends $100 on potatoes and $900 on everything else. The government is planning to assist low income people like John. In particular, they are considering giving him either $200 in cash or giving him $200 in stamps that can only be used to buy potatoes.

    (a) If potatoes are a normal good, then John will be strictly better off with $200 cash than with $200 in stamps.
    Solution: False, if John wants to spend more than $200 in potatoes when he receives a $200 cash transfer, then he can also buy that same bundle when he receives the $200 in stamps. Therefore, he could be indifferent between cash or stamps.

    (b) If potatoes are an inferior good, then John will be strictly better off with $200 cash than with $200 in stamps.
    Solution: True, since potatoes are an inferior good, we know that when he receives $200 in cash he will spend less than $100 in potatoes. However, that bundle is not affordable when he receives $200 in stamps, so he will have to choose another bundle on a lower indifference curve.

2. Mary consumes only two goods, x and y. When the price of x increases, she consumes less of good y. Claim: y can't be a Giffen good.
Solution: True, when p_x increases the substitution effect leads to more consumption of y, so the income effect must be negative if the total effect on y is negative. This implies that, y is a normal good, so it can't be Giffen (recall that Giffen goods have to be inferior).

3. Consider the market for cars. All firms have the same production technology, and we know that the long-run average cost curve has a minimum at a cost of $10,000 per car. Therefore, we can never observe a market equilibrium in which the equilibrium price is below $10,000.
Solution: False, we could have a short-run equilibrium with a price below $10,000 because the firms may not want to shut down even if they are making negative profits as long as they can cover their variable costs.

4. Consider a firm that produces computers. Suppose that the market price of a computer is $\bar{p}$ (both in the short-run and in the long-run). If in the short-run this firm decides to shut-down (i.e. produces $q = 0$), then it must be that in the long-run it decides to exit the market.
Solution: False, it is possible that the firm shuts down in the short run, but if it were able to choose another level of capital it could produce more efficiently and stay in the market.

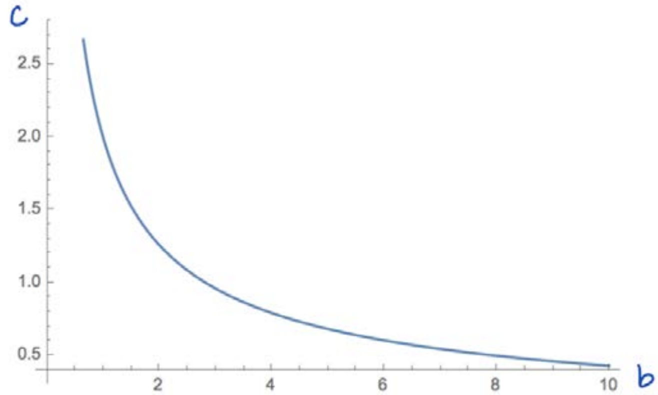
2 Long Question: Consumer Theory

For this question, it is okay to have non-integer answers
Rebecca likes eating bagels (b) and drinking coffee (c) for breakfast. Her preferences can be represented by the function

$$U(c, b) = b^2 c^3$$

1. Graph Rebecca's indifference curve with bagels on the x-axis

Answer.



2. Suppose that Rebecca budgets m dollars for breakfast every week, and the price of bagels and coffee at the cafe are p_b and p_c respectively.

   (a) Write down Rebecca's optimization problem when she is deciding how many coffees and bagels to consume each week.

   Answer. Rebecca's problem is given by

$$\max_{b,c} \quad b^2c^3 \quad \text{s.t.} \quad p_b b + p_c c = m$$

   (b) Solve for how many coffee and bagels Rebecca would consume as a function of p_b, p_c, and m.

   Answer. She would choose to consume

$$b = \frac{2m}{5p_b}, \quad c = \frac{3m}{5p_c}$$

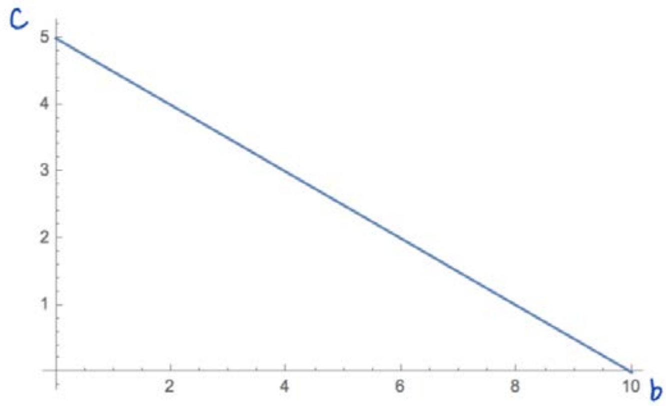
   (c) Are coffee and bagel normal goods or inferior goods? Why?

   Answer. Both are normal goods since consumption of both increases with income.

3. Now suppose that Rebecca budgets 10 dollars for breakfast every day and only eats at Flour Bakery, where the price of a bagel is 1 dollar and the price of a cup of coffee is 4 dollars. However, because Rebecca is part of the loyalty program at the cafe, Rebecca receives a 50 percent discount on coffee.

   (a) Graph Rebecca's budget set with bagels on the x-axis

Answer.



(b) Find the number of bagels and cups of coffee that Rebecca chooses to buy. (You can use the demand functions you found in a previous part)

Answer. From before, we have

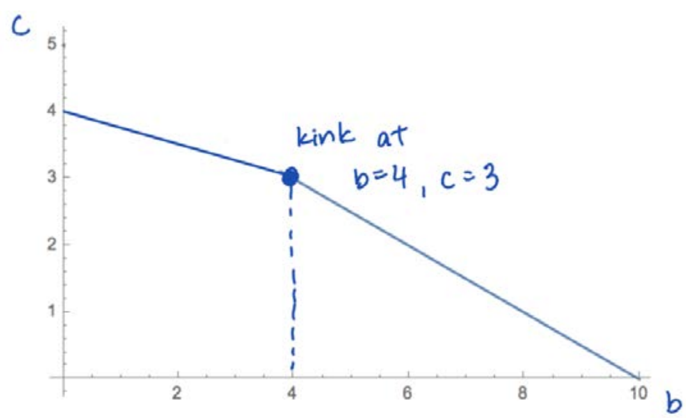
$$b = \frac{2m}{5p_b}, \quad c = \frac{3m}{5p_c}$$

Plugging in $m = 10$, $p_b = 1$ and $p_c = 2$, we get $b = 4$ and $c = 3$

4. After a while, Flour Bakery realizes it is losing money because too many people are using its loyalty program to buy coffee. Flour Bakery decides that the 50 percent discount only apples to the first three cups of coffee.

(a) Graph Rebecca's new budget set (carefully label important points on the graph)

Answer.



(b) What quantity of coffee and bagels would Rebecca choose to consume given the new budget set? How does this answer compare with the previous result? Explain.

Answer. Note that Rebecca's old bundle of 4 bagels and 3 coffees is still affordable at this new budget set. Therefore, she will choose to consumer her old bundle.

5. Flour Bakery is still losing money. It decides to get rid of its loyalty program altogether so that no one gets any discounts on coffee.

(a) Find Rebecca's consumption of coffee and bagels without the loyalty program.

Answer. From before, we have

$$b = \frac{2m}{5p_b}, \quad c = \frac{3m}{5p_c}$$

Plugging in $m = 10$, $p_b = 1$ and $p_c = 4$, we get $b = 4$ and $c = 1.5$

(b) Comparing your answer to the answer you got in part 3b: how much of the change in Rebecca's consumption of coffee is due to the substitution effect? How much is due to the income effect?

Answer. To find the substitution effect, we use the optimality condition at the new prices and the old utility level.

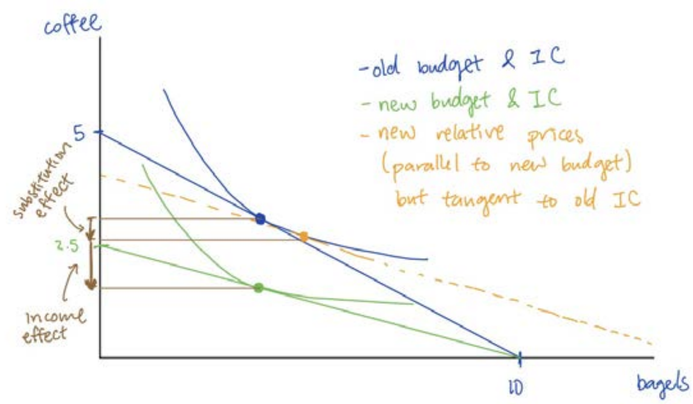
$$\frac{2c}{3b} = \frac{1}{4}, \qquad b^2c^3 = 16 \cdot 27 = 432$$

Solving the above, we get

$$b = 4 \cdot 2^{\frac{1}{5}} \approx 4.59, \quad c = \sqrt[5]{\frac{3^5}{4}} \approx 2.3$$

The substitution effect causes consumption of coffee to decrease by $3 - 2.3 = 0.7$, and the income effect causes consumption to decrease by $2.3 - 1.5 = 0.8$

Graphically, we can also see the income and substitution effect



6. Disappointed that the loyalty program has been canceled, Rebecca goes searching for another cafe and discovers that Tatte sells coffee for 2 dollars and bagels for 2 dollars. The quality of coffee and bagels is the same between Tatte and Flour Bakery.

(a) How many bagels and coffee does Rebecca buy at Tatte?

Answer. From before, Rebecca's optimization is given by

$$b = \frac{2m}{5p_b}, \quad c = \frac{3m}{5p_c}$$

so at Tatte, she would buy

$$b = \frac{20}{10} = 2, \qquad c = \frac{30}{10} = 3$$

(b) Rebecca only has time to go to one coffee shop before class. Which one does she go to? Why?

Answer. The utility the Rebecca gets from going to Tatte is equal to $\left(2^2\right)\left(3^3\right) = 108$. Rebecca's maximum utility from going to Flour is $\left(4^2\right)\left(1.5^3\right) = 54$. Since Rebecca gets higher utility from going to Tatte, she would prefer to go there.

(c) Now suppose that Rebecca's utility function for coffee and bagels is given by $U(c,b) = \sqrt{b+c}$. Which coffee shop does she go to and how much of each good does she consume at that coffee shop? (Hint: it is possible to arrive at the solution for this problem without doing any math. If you do so, please explain the reasoning you used to arrive at your answer.)

Answer. Note that bagels and coffee are perfect substitutes. At Flour, Rebecca would therefore choose to consume 10 bagels, whereas at Tatte, she can consume 5 bagels. Therefore, Rebecca would prefer going to Flour.

7. Now suppose that there are 30 consumers, all with utility for coffee and bagels $U(c,b) = b^2c^3$. Suppose that ten of the consumers budget five dollars for breakfast, ten of the consumers budget ten dollars for breakfast, and ten budget fifteen dollars for breakfast

   (a) Find the aggregate demand curve for coffee.

   Answer.
   - For the 10 customers who budget 5 dollars, they each demand

   $$c = \frac{3m}{5p_c} = \frac{15}{5p_c} = \frac{3}{p_c}$$

   - For the 10 customers who budget 5 dollars, they each demand

   $$c = \frac{3m}{5p_c} = \frac{30}{5p_c} = \frac{6}{p_c}$$

   - For the 10 customers who budget 15 dollars, they each demand

   $$c = \frac{3m}{5p_c} = \frac{45}{5p_c} = \frac{9}{p_c}$$

   To find the aggregate demand, we sum quantity across the different consumers. This means that at any price of coffee P, the total demand is

   $$Q^d(P) = 10 \cdot \frac{3}{P} + 10 \cdot \frac{6}{P} + 10 \cdot \frac{9}{P} = \frac{180}{P}$$

   (b) Suppose that the supply curve for coffee is given by $Q^s(P) = 5P$. What is the equilibrium price and quantity in the market for coffee?

   Answer. To find the equilibrium, we set the quantity supplied equal to the quantity demanded and solve

   $$Q^s(P) = Q^d(P) \implies 5P = \frac{180}{P} \implies P^* = 6$$

   We can then plug this price back into either the supply or demand curve to find equilibrium quantity: $Q^* = 5 \cdot 6 = 30$

3 Producer theory

Suppose we are studying the market for clam chowder in the Boston area, which is perfectly competitive and there is free entry. All the stores in the Boston area face the same long-run cost function, $C^B(q) = 49 + 5q + q^2$. If they decide not to produce, the total cost is zero –i.e. $C^B(0) = 0$. Stores can sell the product at the same price p.

1. Compute the marginal cost and the average total cost for one of the stores.
   Solution:

$$MC^B(q) = 5 + 2q$$
$$ATC^B(q) = \frac{49}{q} + 5 + q$$

2. Find this store's supply function, expressing the quantity supplied by the store as a function of the price.
Solution:
The firm maximizes profits where $p = MC^B(q_s)$. *Therefore,* $q^B = \frac{p-5}{2}$. *However, the store will decide not to produce if revenues are less than costs, that is, if* $pq^B < C^B(q^B)$. *Alternatively,* $p = MC^B(q^B) < ATC^B(q^B)$:

$$5 + 2q^B < \frac{49}{q^B} + 5 + q^B \quad \Leftrightarrow \quad q^B < 7$$

The firm will not produce if $q^B < 5$, *that is, when* $p < 19$. *The supply curve is:*

$$q_s^B = \begin{cases} \frac{p-5}{2} & \text{if } p \geq 19 \\ 0 & \text{otherwise} \end{cases}$$

3. Recall that all the clam chowder stores in the Boston area share the same cost function above. Also, a study has shown that the demand for clam chowder takes the following form, $Q_D = 235 - 5p$. What is the long-run market price and the long-run number of stores? What is the quantity produced by each store? What are their profits?
Solution:
In the long-run and with free entry, stores will enter into the market as long as they can make a positive profit. Hence, stores will enter up to the point were they make zero profits, $p = AC^B(q_s)$. *Since* $p = MC^B(q_s)$, *we can find the quantity they will produce by equating marginal cost to average total cost,*

$$5 + 2q = \frac{49}{q} + 5 + q \quad \Leftrightarrow \quad q^* = 7$$

which implies that the long-run price is $p^* = 19$. *To find the total quantity demanded, we plug this price into the demand function, so* $Q_D^* = 235 - 95 = 140$. *To find the number of stores in equilibrium, we just have to divide the total quantity over each store's production,* $N^* = 140/q^* = 140/7 = 20$. *There will be 20 stores making 0 profits.*

4. Suppose that 15 out of the clam chowder stores that decided to operate in the long-run in the previous question are in Cambridge –one of the cities in the Boston area. In addition, suppose that Cambridge introduces a constant subsidy of \$33 for each of these 15 clam chowder stores if $q > 0$, that is, each of the 15 stores gets a \$33 check from the city government if they decide to produce a positive quantity of clam chowder.

   (a) What is the new cost function for one of Cambridge's clam chowder stores (denote it by $C^C(q)$)?

   (b) Compute the marginal cost and average total cost for this store, as well as the supply curve. Does the marginal cost change? Why or why not?

   (c) Solve for the new supply function for one of the stores.

   Solution:

   (a) The new cost function can be expressed as $C^C(q) = 16 + 5q + q^2$.

(b) *The new marginal cost and average cost curves are:*

$$MC^C(q) = 5 + 2q$$
$$ATC^C(q) = \frac{16}{q} + 5 + q$$

The marginal cost does not change because the subsidy does not affect the cost of producing one more unit –it is a fixed quantity that the store receives no matter the production level.

(c) *The firm maximizes profits where $p = MC^C(q_s)$. Therefore, $q^C = \frac{p-5}{2}$. The firm will decide to shut down only if revenues are less than costs, that is, if $pq^C < C^C(q^C)$. Alternatively, $p = MC^C(q^C) < ATC^C(q^C)$:*

$$5 + 2q^C < \frac{16}{q^C} + 5 + q^C \quad \Leftrightarrow \quad q^C < 4$$

The firm will shut down if it finds optimal to produce $q^C < 4$, that is, when $p < 13$. The supply curve is:

$$q_s^C = \begin{cases} \frac{p-5}{2} & \textit{if } p \geq 13 \\ 0 & \textit{otherwise} \end{cases}$$

5. Consider the market after the introduction of the subsidy in Cambridge for those 15 stores (that is, other clam chowder stores that open in Cambridge do not get the subsidy).

   (a) What is the long-run market price? And the number of stores of each type? Explain the intuition.

   (b) (6 minutes) How many clam chowder units does each subsidized store sell? What about unsubsidized stores? What are the profits of each of the subsidized stores? And the profits of the unsubsidized stores? Explain why.

Solution:

(a) *The long-run price and number of stores is the same as in question (3), $p^* = 19$ and $N^* = 20$, since the threat of entry still makes the 5 unsubsidized firms price at the average cost level, i.e. they still produce $q_B^* = 7$ and make zero profits.*

(b) *Subsidized stores will produce the same quantity, $q_C^* = 7$ –remember that the marginal cost is the same for both types of stores. However, these stores will make positive profits in equilibrium,*

$$\pi^C = pq^* - C^C(q^*) = 19^*7 - 16 - 5 \cdot 7 - 7^2 = 150 - 16 - 37.5 - 56.25 = 33$$

which is equal to the subsidy level. The intuition is the following. In the long run, the price is equal to the minimum average cost for the unsubsidized stores (which are higher cost, since they do not have the subsidy), and they make zero profits. Since both these and subsidized stores share the same supply curve when they produce, they sell the same quantity at the same price and have the same costs, but with the difference that Cambridge gives a fixed amount of money to the 15 stores in Cambridge, which will be their profit.

6. (6 minutes) Demand for clam chowder in the Boston area drops after the subsidy policy and becomes $Q_D = 207 - 5p$. What is the long-run price and number of firms? How many stores will there be in the Boston area and how many of them will be subsidized? How many clam chowder units do subsidized stores sell? What about unsubsidized stores?

   Solution:

   As said above, the pricing decisions do not change because marginal costs don't change, so firms will always produce $q^* = 7$ and the price will be $p^* = 19$. However, the number of firms will change, because the total quantity demanded in the market is $Q_D^* = 207 - 5 \cdot 19 = 112$, so $N^* = 112/7 = 16$.

   This implies that 4 firms will exit the market. Since the subsidized firms in Cambridge are making positive profits, they will not exit, and it will be 4 of the remaining Boston area stores who exit the market (note that since they were making no profits they are indifferent between staying and exiting). Consequently, there will be 15 clam chowder stores in Cambridge and 1 clam chowder store in the remaining Boston area.