12.864 Inference from Data and Models   3 May 2004
Problem Set No. 6   Due: 12 May 2004

1. Using any computer language you like, write your own code for a conventional determination of the coefficients of a discrete Fourier series (not an FFT). Use the rules in Eqs. 6.29-6.31 of the "Primer" or any other consistent system (e.g., you can use complex arithmetic and/or an odd number of points). Demonstrate for a pure cosine of amplitude 2, with a period of your choosing, and a sampling interval and record length of your choosing that
(a) you can deduce from the output of your code the amplitude of the cosine
(b) you can invert the Fourier series and recover the original cosine
(c) the Parseval relationship is satisfied.
(I'm not prepared to read code.)

2. Using a random number generator, generate 128 numbers at an interval $\Delta t = 10$. (a) Compute the Fourier series coefficients (you can use your own code, or a library one), and plot, as a function of dimensionally labeled frequency, the real and imaginary parts, and their sum of squares. (The plot of the sum of squares as a function of frequency is called the "periodogram".) (b) Form histograms of the real and imaginary parts and their sum of squares, and describe the outcome. (c) Average the periodogram in groups of 8 neighboring frequencies, and plot the result. (This result is called a power spectrum estimate.). (d) Now generate a new time series with 256 points, and redo (c). How does the result differ? (e) Show that the Parseval relation is applicable to the random time series.

3. Let $x_t = 0.999x_{t-1} - 0.1x_{t-2} + \theta_t$ where θ_t is a unit variance, zero-mean white noise process, and let $x_0 = 0, x_1 = 1$. Generate a 128-point realization of x_t. For this new time-series, redo part (c) of problem 2, and describe the difference.

4. Let $y_t = x_t + \eta_t$ where x_t is defined in problem 3, and η_t is another zero-mean white noise process such that $\langle \theta_t \eta_{t'} \rangle = 0$, $\langle \eta_t^2 \rangle = 2$. Using a coherence analysis, show that you could infer the relationship between x_t, y_t.

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If you are serious about this subject, on your own time, work through the exercises in the Primer.