12.864 Inference from Data and Models 10 March 2003
Problem Set No. 2 Due: 19 March 2003

1. You have a single vector $\mathbf{v}_1 = [1,1,1,1]^T$, but need three more vectors in order to have a complete spanning set. Starting with $\mathbf{v}_1$, construct a complete orthonormal basis set.

2. You have a set of equations,

$$1.1045x_1 + 1.2385x_2 + 1.2900x_3 = 4$$
$$1.2385x_1 + 1.3887x_2 + 1.4465x_3 = 5$$
$$1.2900x_1 + 1.4465x_2 + 1.5067x_3 = 4$$

(a) Using the eigenvector/eigenvalue formulation, find *all* possible solutions (if any) and explain any unusual structures in your answer.
(b) Using least-squares and the method of Lagrange multipliers, find at least one solution.

3. An engineer has a material slab as shown in cross-section in the figure and is concerned that in its manufacture a defect occurred so that instead of being homogeneous in properties, a defect has occurred. But he is unable to drill into it without destroying it. He therefore obtains an ultrasound source and receiver, and measures the time of travel between the source and receiver along the dashed paths as shown. Based upon his previous experience, he suspects that defects will lead to anomalies in travel time by causing deviations of the reciprocal soundspeed $S = 1/c$ from uniformity and to be spatially roughly the size of the 9 interior blocks depicted. For convenience, the size of each block can be taken as 1 unit × 1 unit. Assume, unrealistically, that all measurements are perfect. So for example, measured travel-time $T_1 = S_1 + S_4 + S_7$.

Suppose there is indeed an anomaly, confined wholly to box 5, of 2 inverse soundspeed units relative to a uniform background. What are the resulting observation equations? Using the singular vector expansion, can you recover the "true" solution? Suppose the anomaly is confined to any one of the 9 boxes—can you recover it? To what degree?