Quasi-linear Model of the Equatorial Atmosphere
Model Description and Operating Instructions

This model solves numerically the equations developed in class, in an equatorially-centered channel with a uniform Cartesian grid. The equations are phrased on a β plane. The model differs slightly from the linear model developed in class, in that several key nonlinearities are retained. These include, most importantly, a switch that shuts off moist convection when and where the atmosphere becomes stable to convection. Also, the fully nonlinear surface stress and surface enthalpy exchange terms are retained. The model's thermodynamic variables are the *free atmosphere saturation entropy*, denoted s, the *boundary layer entropy*, denoted s_b, and the *lower tropospheric entropy*, denoted s_m. These are multiplied by the difference between the surface temperature and the mass-weighted average temperature along the mean state moist adiabat, $\overline{T}$. Thus

$$s \equiv c_p \Delta T ln \left(\theta_e^* \right),$$

$$s_b \equiv c_p \Delta T ln \left(\theta_{eb} \right),$$

and

$$s_m \equiv c_p \Delta T ln \left(\theta_{em} \right).$$

Here $\Delta T \equiv T_s - \overline{T}$, c_p is the heat capacity of air at constant pressure, θ_e^* is the saturation equivalent potential temperature of the troposphere, θ_{eb} is the boundary layer equivalent potential temperature, and θ_{em} is the equivalent potential temperature of the lower to middle troposphere. In addition, we define a saturation entropy of the sea surface, which is a function of the sea surface temperature alone. It is defined

$$s_0 \equiv c_p \Delta T ln \left(\theta_{es}^* \right),$$

where θ_{es}^* is the saturation equivalent potential temperature of the sea surface. The sea surface entropy, s_0, may be a function of x and y.

The conservation equations for the perturbation zonal, meridional and vertical wind, s, and s_b are:

$$\frac{\partial u}{\partial t} = \frac{\partial s}{\partial x} + \beta y v - C_D |\mathbf{V}| u / H, \tag{1}$$

$$\frac{\partial v}{\partial t} = \frac{\partial s}{\partial y} - \beta y u - C_D |\mathbf{V}| v / H, \tag{2}$$

$$w = -H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right), \tag{3}$$

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$$\frac{\partial s}{\partial t} = N^2 \left(-w + \epsilon M\right) - \dot{R}, \tag{4}$$

and

$$H \; \frac{\partial s_b}{\partial t} = C_k |\mathbf{V}| \left(s_0 - s_b\right) + MAX \left(0, \left(w - M\right)\right) \left(s_b - s_m\right). \tag{5}$$

Here, C_D is the surface drag coefficient, C_k is the surface enthalpy exchange coefficient, H is a characteristic depth of the boundary layer, $|\mathbf{V}|$ is the total surface wind speed, defined

$$|\mathbf{V}| = \sqrt{\left((u + \overline{U})^2 + v^2 + u_*^2\right)}, \tag{6}$$

N^2 is a dry static stability, defined

$$N^2 \equiv c_p \Delta T \frac{\Gamma_d}{\Gamma_m} \frac{\partial ln(\theta)}{\partial z},$$

M is the convective updraft mass flux, ϵ is the precipitation efficiency, and $\dot{R}$ is the radiative cooling. In (6), $\overline{U}$ is the specified mean background zonal wind, and u_* is a gustiness factor that allows there to be some flux even when the total wind speed is zero.

The convective mass fluxes are defined using boundary layer quasi-equilibrium. First we define an updraft mass flux that keeps the boundary layer entropy constant. Using (5), this is

$$M_{eq} = w + C_k |\mathbf{V}| \frac{s_0 - s_b}{s_b - s_m}, \tag{7}$$

with $M_{eq} = 0$ if $s_b < s_m$, so that in equilibrium there is no convection of the atmosphere is stable. The actual updraft mass flux is relaxed to this equilibrium value over a finite time:

$$\frac{\partial M}{\partial t} = \frac{M_{eq} - M}{\tau_c}, \tag{8}$$

where τ_c is a convective equilibration time scale.

The radiative cooling may either be a fixed constant, or may depend on the magnitude of subsidence drying, as formulated by Sandrine Bony:

$$\dot{R}_{eq} = \dot{R}_0 - FN^2 \left(w - \epsilon(M - \frac{\dot{R}_0}{\epsilon N^2}) \right) - \alpha \left(1 - \frac{s_b - s_m}{s_{b0} - s_{m0}} \right) \tag{9}$$

Here $\dot{R}_0$ is the background, constant rate of radiative cooling. The middle term on the right is the *perturbation* radiative cooling owing to perturbation high cloudiness, which is taken to be proportional to the drying by subsidence in between clouds. This is in turn proportional to the difference between the large-scale ascent rate, w, and the *perturbation* convective mass flux. The last term on the right is the perturbation of the radiative cooling owing to moistening of the middle troposphere: a moister middle troposphere gives less radiative cooling. (Here s_{m0} is the initial value of s_m if the latter is variable; see below.) The magnitude of the cloud radiative feedback is controlled by F, while that of the middle troposphere moistening is controlled by α. As with the convective fluxes, the actual rate of radiative cooling is relaxed to the equilibrium value given by (9) over a finite time scale:

$$\frac{\partial \dot{R}}{\partial t} = \frac{\dot{R}_{eq} - \dot{R}}{\tau_R}, \tag{10}$$

where τ_R is a time scale for the production of high cloud.

We also include, optionally, a rate equation for the middle troposphere entropy. This is allowed to vary according to the perturbation convective mass flux, advecting boundary layer entropy upward:

$$\frac{\partial s_m}{\partial t} = \gamma(s_b - s_m) \left(M - \frac{\dot{R}_0}{\epsilon N^2} \right) / H - \frac{s_b - s_m - s_{b0} + s_{m0}}{\tau_m}, \tag{11}$$

where γ controls the rate of moistening and the last term on the right relaxes the middle troposphere entropy back toward its initial value.

Finally, the precipitation efficiency may be a specified constant or may vary with the relative humidity of the middle troposphere, according to

$$\epsilon = 1 - (1 - \epsilon_0) \frac{s_b - s_m}{s_{b0} - s_{m0}}, \tag{12}$$

where ϵ_0 is the background precipitation efficiency. This equation increases the precipitation efficiency in regions of high humidity.

The model, then, consists of (1)-(12). The linear model is set up in a re-entrant, equatorially centered channel. One may specify the distribution of sea surface temperature (SST), which is fixed in time.

The FORTRAN routine *tropics2* reads control parameters from the ASCII text file *input.txt*. This file can be edited to control the program. The file consists of three columns: The first is the parameter value actually read by *tropics2*; the second is the "normal" value of the parameter, and the third is just a terse description of the parameter. Edit the first column.

The first two blocks of values in *input.txt* contain the various model parameters as described above.

The next block of text contains parameters controlling the geometry of the channel and the distribution of SST. The SST has a bell-shaped distribution in y, peaking at a specified maximum SST and tailing off exponentially to a specified minimum SST. (By taking the maximum SST equal to the minimum SST, the SST is specified as a constant.) A zonal wavenumber 1 SST anomaly may also be specified if desired.

The final block of text contains parameters that control the integration. If y (for "yes") is specified in the last line, the program reads its initial condition from the files produced at the end of the most recent run of *tropics2*.

All output is written to the subdirectory *output* which must be created before the program can run.

The MATLAB routines *tropics.m* and mmov.m display output and run a movie of the convective mass flux, respectively.

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