12.804 — Point Vortices

Point potential vortices — analysis

The simplest PV distribution to invert is a point vortex for which

$$q = \nabla^2 \psi = s_1 \delta(x - x_1)\delta(y - y_1)$$

where s_1 is the strength (the integrated PV anomaly) and x_1, y_1 is the position.

Find the velocity field.

Point potential vortices — inversions

The code can be run from the **point vortices** link. Start with a time length for the run of 0; this gives just the velocity field given the initial positions. Try

```
vort=[0,0,1];
vort=[0,1,1;0,-1,1];
vort=[0,1,1;-,-1,-1];
vort=[-1.4,1/2,1;-1.4,-1/2,-1;-1,1/2.1;-1,-1/2-1];
```

The local field of the vortex would mask the important non–azimuthal component of velocity which actually moves the vortex. Plotting of that part of the velocity field has been suppressed. Try and figure out how the vortices will move.

Motion

From your velocity expression, show that the velocity of the i^{th} vortex can be written

$$u_i + \imath v_i = \frac{\imath}{2\pi} \sum{}' \frac{s_j}{z_i^* - z_j^*}$$

where $z_i = x_i + \imath y_i$. The program `pv` calculates the evolution given the initial condition. Try this with various of the initial states above.

Things to try:

- What does a ring of N identical vortices do? How about if you perturb it slightly?
- What happens to a pair of vortices with different strengths?
- Study the movement/collisions of dipoles.
- Try some other initial conditions.
- Can you model the merger of two clumps of vortices?

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