12.510 Introduction to Seismology
Spring 2008

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Surface Waves (Ground Roll)

April 2, 2008

Today we will look at the interaction of an acoustic wave (ground roll) with a layer over half space. We will consider travel time curves for acoustic waves and describe them using higher modes of surface wave propagation. We will also use a propagation matrix as a reflectivity method to calculate synthetic seismograms.

Case 1: Layer over Source in a Half Space

Lets look at what happens when we have 2 interfaces. We are working with acoustic waves, so we will take P = pressure field. Note that the methods used here can be used for SH waves as well. The same principles can be applied to P-SV waves, but the algebra becomes more complicated.

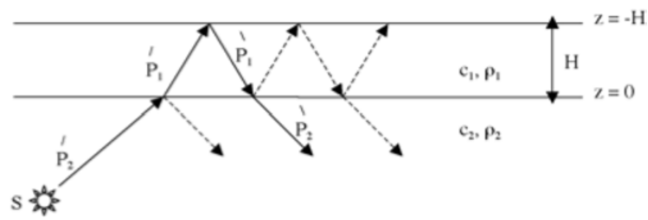

Figure 1: Diagram of a source in a half space under a layer

The wave equation for the acoustic case is

$$\ddot{P} = k\nabla \cdot (\frac{1}{\rho}\nabla P) \tag{1}$$

with the displacement given by

$$\mathbf{u} = \frac{1}{\rho\omega^2}\nabla P \tag{2}$$

This case becomes a bit more complicated than the simple reflection previously discussed. The analytical methods used to decribe the simple reflection begin to break down when multiple layers are introduced.

We will renormalize the incoming wave to $\acute{P}_2 = 1$ and define $\grave{P}_2 = R$. Using the plane wave description,

$P = \acute{P}e^{i(k_x x - k_z z - \omega t)} + \grave{P}e^{i(k_x x - k_z z - \omega t)}$

Notice the positive and negative $k_z z$ terms to describe the vertical slowness, as well as the different amplitudes $\acute{P}$ and $\grave{P}$. We want to know what is happening with respect to crossing the interface, so we will ignore the x-direction, leaving the $e^{ik_z z}$ terms and giving

$P = \acute{P}e^{-i\nu z} + \grave{P}e^{-i\nu z}$

where the vertical wave number is given by

$\nu_z = k_z = \frac{\omega}{c_z} = \frac{\omega \cos i}{c} = \omega\eta$

$P = \acute{P}e^{-i\nu z} + \grave{P}e^{-i\nu z}$ can be set up for each layer. We can solve for the pressure by taking the gradient in the z-direction:

$u_z = \frac{\delta P}{\delta z} = \frac{1}{\rho\omega^2}(i\nu\grave{P}e^{i\nu z} - i\nu\acute{P}e^{i\nu z}) = \frac{i\nu}{\rho\omega^2}(\grave{P}e^{i\nu z} - \acute{P}e^{i\nu z})$

To solve this system we will follow the steps:

1. Look at Potentials (Pressure Field)
2. Kinematic and Dynamic Boundary Conditions
3. Zoeppritz equations

4. Solve for R and T

At z=0, the welded interface, the stress is continuous so
$\acute{P}_2 + \grave{P}_2 = 1 + R = \acute{P}_1 + \grave{P}_1$

The displacement at $z = 0$ is given by
$u(0) = \frac{i\nu}{\rho\omega^2}(\grave{P}_1 - \acute{P}_1)$

which implies that at the free surface:
$\grave{P}_2 - \acute{P}_2 = \frac{\nu_2\rho_1}{\nu_1\rho_2}(\grave{P}_1 - \acute{P}_1) = \frac{\nu_2}{\rho_2}(R - 1)$

At the free surface $z = -H$
$\acute{P}_1 e^{-i\nu_1 H} + \grave{P}_1 e^{i\nu_1 H} = 0$ where ν_1 is the vertical wave number $k_z = \frac{\omega}{c_z}$.

The three equations above give the Zoeppritz matrix and we solve for R

$$\begin{bmatrix} 1 & 1 & -1 \\ \frac{-\nu_1}{\rho_1} & \frac{\nu_1}{\rho_1} & \frac{-\nu_2}{\rho_2} \\ e^{-i\nu_1 H} & e^{i\nu_1 H} & 0 \end{bmatrix} \begin{bmatrix} \acute{P}_1 \\ \grave{P}_1 \\ R \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$R = \frac{(1-\eta)+(1+\eta)e^{2i\nu_1 H}}{(1+\eta)+(1-\eta)e^{2i\nu_1 H}}$

where $\nu = \frac{\rho_1\nu_2}{\rho_2\nu_1}$

Some remarks on the reflection at the free surface:
- The reflection coefficient is a complex number. In the cases we studied in previous lectures, R became complex when $i = i_c$.
- $|R| = 1$, which means that the energy is not stored in the upper layer but is eventually all — reflected back. E.g., if the input is a single spike, the output is a series of reverberations (see homework 2, problem 3). This occurs because of conservation of energy; if there is no source in the upper layer, there can be no residual trapped energy in the upper layer. The energy is reflected from $z =?H$ and both reflected and partially transmitted from $z = 0$ until it fully dissipates back into the half space.
- There is a frequency?dependent phase shift $\frac{\gamma}{\omega}$ in the waveform, ie. $R = e^{2i\gamma}e^{i(kx-\omega t+\frac{\gamma}{\omega})}$.

Case 2: Source Within a Layer Over a Half Space

The source within the layer is analagous to a source in a weathered layer, e.g. an induced source in exploration seismology or surface waves in earthquake seismology. This scenario is similar to the Love waves case discussed in previous lectures. In the case of angles such that i¿ic (evanescent waves), no energy will be transmitted into the half space.

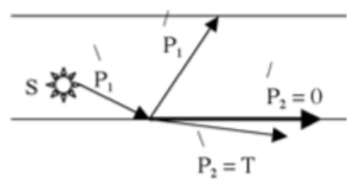

Figure 2: Diagram of a source in a layer over a half space

To solve this system we will follow the steps as before:

1. Look at Potentials (Pressure Field)
2. Kinematic and Dynamic Boundary Conditions
3. Zoeppritz equations
4. Solve for R and T

Because stress must be continuous,
$P = \acute{P}_1 + \grave{P}_1 = \acute{P}_2 + \grave{P}_2 = 0 + T \Rightarrow \acute{P}_1 + \grave{P}_1 - T = 0$

The total displacement must be continuous at the interface, so
$u = \frac{1}{\rho\omega^2}\nabla P = \frac{i\nu}{\rho\omega}(\grave{P}e^{i\nu z} + \acute{P}e^{-i\nu z})$
$\Rightarrow \frac{\nu_1}{\rho_1}(\grave{P}_1 - \acute{P}_1) = \frac{\nu_2}{\rho_2}T$
$\Rightarrow (\grave{P}_1 - \acute{P}_1) = \frac{\rho_1\nu_2}{\rho_2\nu_1}T = \eta T$
$\Rightarrow \grave{P}_1 - \acute{P}_1 - \eta T = 0$ where $\eta = \frac{\rho_1\nu_2}{\rho_2\nu_1} = \frac{\rho_1 c_1}{\rho_2 c_2} = \frac{z_1}{z_2}$ and $\rho_1 c_1$=acoustic impedance.

At $z = -H$,
$\grave{P}_1 + e^{2i\nu_1 H}\acute{P}_1 = 0$

Notice that the T in these equations is not the conventional T as before because it is evanescent. In contrast to previous cases, we can now form a homogeneous set of Zoeppritz equations:

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & -\zeta \\ 1 & e^{2i\nu_1\eta} & 0 \end{bmatrix} \begin{bmatrix} \grave{P}_1 \\ \acute{P}_1 \\ T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The solution of this system depends on the layer thickness (H), the vertical wave number (ν), the direction of the rays ($\frac{\omega \cos i}{c}$), and the impedence contrast ($\eta = \frac{z_1}{z_2}$). To obtain the nontrivial solution to this system, we must set the determinant of the 3x3 term to zero, leading to

$$\det \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & -\eta \\ 1 & e^{2i\nu_1 H} & 0 \end{bmatrix} = 0$$

$$\Rightarrow \eta e^{2i\nu_1 H} - e^{2i\nu_1 H} - \eta - 1 = 0$$

$$\Rightarrow (\eta - 1)e^{2i\nu_1 H} = \eta + 1$$

$$\Rightarrow e^{2i\nu_1 H} = \frac{\eta+1}{\eta-1}$$

We know that in the complex plane, generally, $e^{-2i\theta} = \frac{a-b}{a+b}$, giving $\tan\theta = \frac{-ib}{a}$. Using this, we can rewrite our nontrivial solution as $\tan(\nu_1 H) = \frac{1}{i\eta} = \frac{\rho_2 \nu_1}{i\rho_1 \nu_2}$

$$\Rightarrow tan(\nu_1 H) = \frac{1}{i\eta} = \frac{\rho_2 \nu_1}{i\rho_1 \nu_2}$$

where $\nu_1 = \omega\sqrt{(\frac{1}{c_1})^2 - p^2}$ and $\nu_2 = \omega\sqrt{(\frac{1}{c_2})^2 - p^2}$.

The **dispersion relationship for ground roll** (which is the same as the dispersion relationship for Love waves) is thus given by

$$\tan\left(\omega H\sqrt{(\frac{1}{c_1})^2 - p^2}\right) = \frac{\rho_2\sqrt{(\frac{1}{c_1})^2 - p^2}}{\rho_1\sqrt{(\frac{1}{c_2})^2 - p^2}}$$

For a given frequency and layer thickness H, for certain directions given by $p = \frac{\sin i_1}{c_1} = \frac{\sin i_2}{c_2}$ and known c_1, c_2, we can solve this system.

Given fixed ω , ie. $i_1 = 90^o$ solves the dispersion equation, ie. the direct wave is a solution to the dispersion equation. See Figure 3.

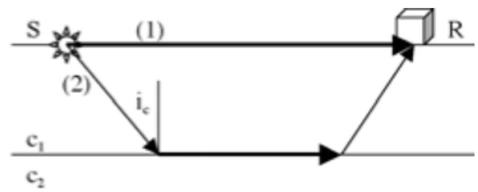

Figure 3: Diagram of (1) a direct wave and (2) a critical wave

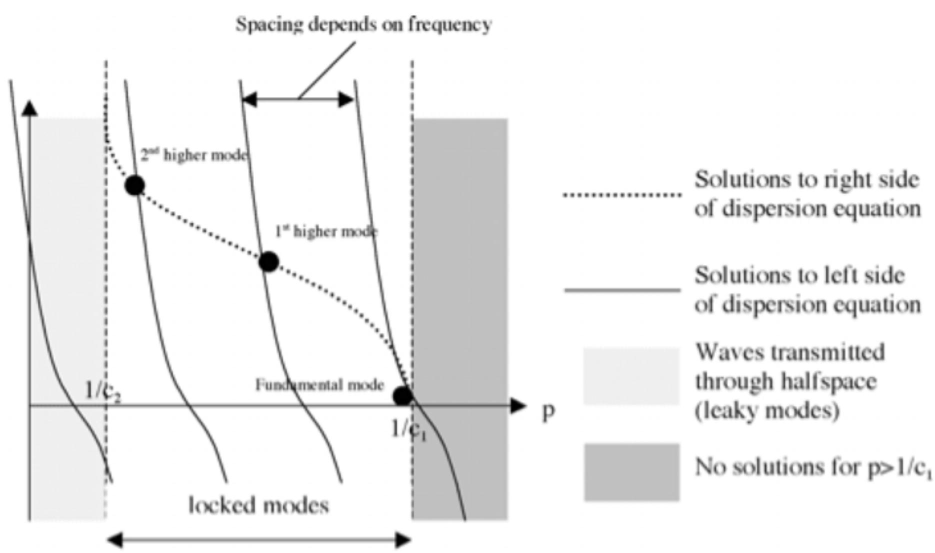

Figure 4: Graphical representation of the dispersion relationship for ground roll.

For all postcritical angles i, or $\frac{1}{c_2} < p < \frac{1}{c_1}$, we have locked modes, where all of the energy stays in the upper layer. The fundamental mode labeled at $\frac{1}{c_1}$ represents the p-value for a direct wave. The subsequent intersections of the left side solutions and the right side solutions give progressively higher modes (overtones) as p decreases to $\frac{1}{c_2}$. For precritical angles i, or $p < \frac{1}{c_2}$, energy is lost due to wave transmission into the half space, so we get leaky modes.

Also, because the spacing between the asymptotes of the left side solutions is dependent on ω, for any given frequency, there is a finite set of solutions (i.e. modes) to the dispersion relation. Each mode has a specific horizontal slowness and take?off angle because the waves can only propagate in a way that creates constructive interference.

As the frequency decreases and wavelength increases, the spacing between the left side solution curves increases, thus decreasing the number of overtones. Similarly, as the frequency increases, the number of overtones also increases. A plot of these overtones as frequency approaches infinity will approach a continuous graph along the plot of the right side solution. Another way to think about overtones is to visualize the direct wave on Figure 3 with a wavelength equal to the distance between the source and the receiver. This is a high wavelength with a low frequency, and in this case a plot of the dispersion relation would show that the left side solutions are spaced so far apart that few, if any, overtones exist, i.e. given the particular long wavelength and H-value, there are few other incident angles that will result in a coincidence between the path of the wave and the receiver (constructive interference). As the frequency increases and wavelength decreases, there will be more incident angles such that the path of the wave coincides with the receiver and results in constructive interference.

We begin RAY THEORY briefly...
As $\omega \to \infty$ infinite continuous spectrum of solutions with phase velocity $c_n = \frac{\omega}{k_n}, k_n = \frac{\omega}{c_n}$
$k_n = \frac{1}{c_n} = \frac{\cos i_n}{c}$
So we can simply use Snell's Law. It holds now that there is no concept of interference.