# CHAPTER 9
# THRESHOLD OF MOVEMENT

## INTRODUCTION

**1** One of the classic problems in sediment transport is to predict the flow strength at which sediment movement first begins. This condition for incipient movement is usually expressed in terms of a ***critical shear stress*** or ***threshold shear stress***, which I will denote by $\tau_{oc}$. The problem can be viewed either as the minimum shear stress needed to move a given particle, or as the largest grain size that can be moved by a given shear stress. The latter is termed ***competence*** by geologists.

**2** Incipient movement should be one of the simplest problems in sediment transport, because at that point the flow has not yet become a two-phase flow, and all the principles and techniques of sediment-free flow—what is called *rigid-boundary hydraulics*—should still apply. Even in this simplest of problems, however, understanding is far from complete, which should put you on your guard about the great many approaches and formulas in the literature that are supposed to deal with established sediment movement.

## FORCES ON BED PARTICLES

### *Introduction*

**3** Look at a representative sediment particle resting on the surface of a cohesionless sediment bed at the bottom of a flowing fluid (Figure 9-1). If the fluid is not moving fast enough to move the particle, then the particle is motionless, therefore not accelerating, so all the forces acting on it must be in balance. What are those forces? They are of three kinds: *particle weight,* particle-to-particle *contact forces,* and *fluid forces.*

**4** The particle weight is easy to deal with: it is just the submerged weight per unit volume of the sediment material, $\gamma'$, times the volume of the particle. It acts vertically downward through the center of mass of the particle. The contact forces, exerted upward on the given particle by the underlying particles (usually three) on which it rests, become adjusted in light of the contact geometry, the particle weight, and the fluid forces so that the particle is motionless.

**5** The fluid forces are much more difficult to deal with. First we need a little more background on the nature of flow very near the bed in a turbulent shear flow.
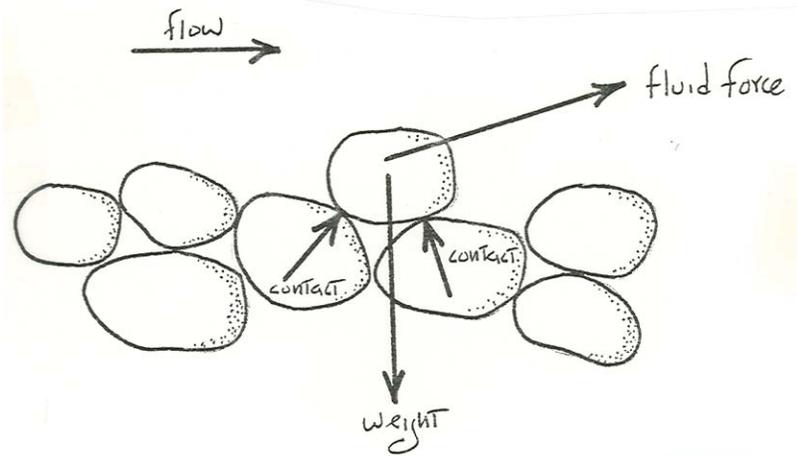
Figure 9-1.  Forces on a sediment particle resting on a bed of similar particles.
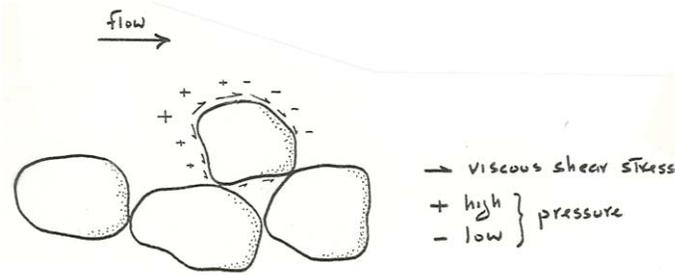


Figure 9-2.  Viscous shear stress and pressure acting at all points of a sediment particle resting on a bed of similar particles.

### *Fluid Forces*

**6**  Because there is locally a flow around the bed particle, you should expect there to be both viscous shear stresses and pressure acting at every point of the surface of the particle (Figure 9-2).  It is those viscous and pressure forces, summed over the entire surface of the particle, that give rise to the resultant fluid force on the particle.  This resultant force is specified by its *magnitude, direction, and line of action through the particle*.

**7**  Keep in mind that if the flow is turbulent, which is almost always the case in problems of sedimentological interest, the resultant force varies strongly with time, on time scales ranging from small fractions of a second to many minutes (in the case of very large-scale flows, in which the maximum eddy size can be very large), even if the flow is steady in the time-average sense.  And this is true even if the particle is within the viscous sublayer.  Remember that the viscous sublayer

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is not really nonturbulent: the local velocity fluctuates substantially within it. The important point is that the turbulence is unimportant in contributing turbulent shear stress to the total shear stress in the viscous sublayer. You can readily understand the existence of velocity fluctuations in the viscous sublayer in the context of the burst–sweep cycle described in Chapter 4: the viscous sublayer is thinned and accelerated as masses of high-speed fluid impinge on the bed, and then it thickens and decelerates again.

**8** What determines the magnitude, distribution, and relative importance of the viscous and pressure forces? Think about the variables that govern the fluid force on a bed particle. Some should come to mind readily: the diameter $D$ of the particle (which determines the surface area of the particle, and also how far up into the flow the particle projects), the fluid viscosity $\mu$ (viscous stresses are important); the fluid density $\rho$ (fluid is accelerating in the vicinity of the particle); the boundary shear stress $\tau_0$ (that is the variable that best characterizes the strength of the flow around the particle); and the geometry of the particle itself and its relationship to underlying particles. The geometry varies widely—it is different for each particle—but, for any given particle, the other variables lead naturally to a single dimensionless variable $\rho u_* D/\mu$, which you have already seen in Chapter 4 as the boundary Reynolds number.

**9** A further comment about the boundary shear stress is in order here. It is not the boundary shear stress, as carefully defined in Chapter 4 as the time-average force per unit area on the bed, averaged over an area of the bed that is large enough to guarantee a representative spatial average, that is directly relevant to threshold of movement; it is the picture of local fluid forces on individual bed particles and how those forces vary with time. Nonetheless, $\tau_0$ is a good *descriptor* of the threshold condition for movement, because it represents the average of the forces on the bed particles. There is an important qualification to that statement, however: the bed has to be planar in the large, without any features like bed forms that are much larger than the particles themselves, or otherwise much or most of the bed shear stress is associated with form drag rather than with the skin friction, which is what actually moves the particles.

**10** Figure 9-3 shows in a qualitative way how the fluid force on a given particle should vary with the boundary Reynolds number $\text{Re}_*$.

• For **small Re$_*$** (which corresponds to relatively small values of a Reynolds number based on local flow velocity around the particle) there is no well defined boundary layer along the top surface of the particle, and there is no flow separation behind the particle (Figure 9-3A). Both viscous forces and pressure forces are important. The line of action of the resultant force lies well above the center of mass of the particle, because the viscous forces are strongest on the uppermost surface of the particle.

• For **large Re$_*$** (which corresponds to relatively large values of a Reynolds number based on local flow velocity) there is a well defined local boundary layer on the surface of the particle, and pronounced flow separation, with a turbulent wake behind the particle (Figure 9-3B). Pressure forces far outweigh viscous

forces, and because the net pressure force comes about mainly by the difference in pressure from front to back the line of action of the resultant force is closer to the center of mass of the particle.

**lift component**

**total fluid force**

**C.M.**

**drag component**

**support forces**

**small Re$_*$ (< about 5)**

— particles in viscous sublayer
— form drag $\simeq$ skin friction
— lift unimportant(?)
— line of action high

**particle weight**

**separation**

— turbulent wake

**large Re$_*$ (> about 70)**

— no viscous sublayer
— form drag >> skin friction
— lift $\simeq$ drag
— line of action low
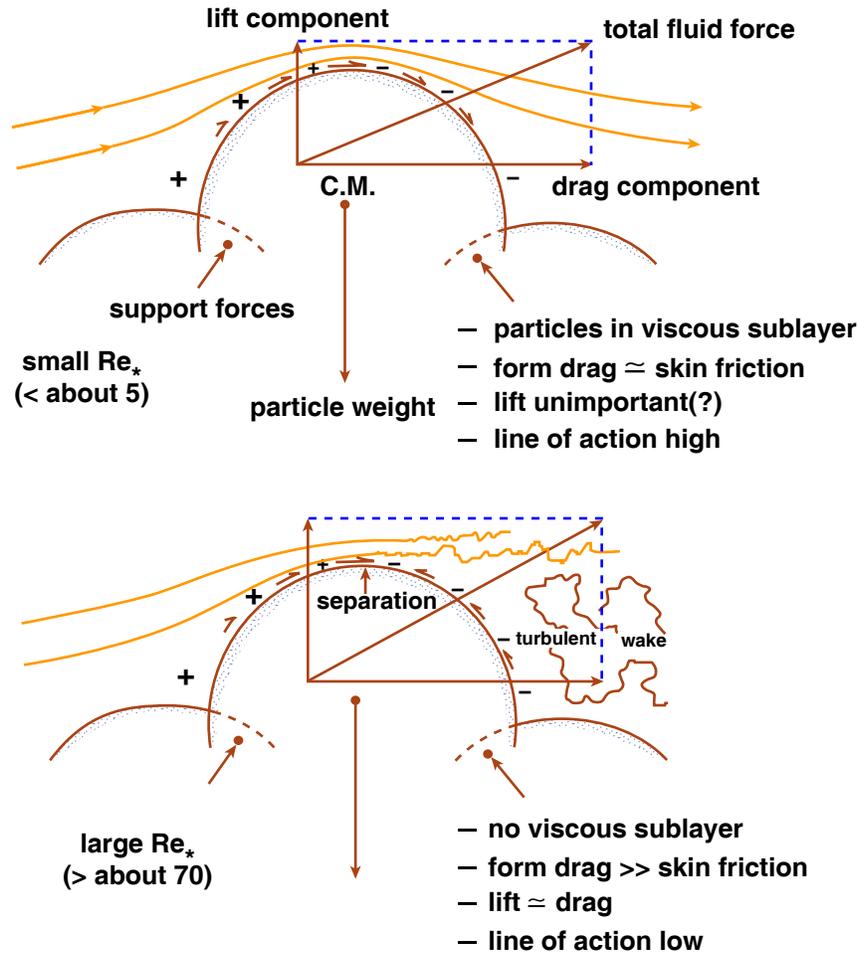
Figure 9-3. Fluid forces on a particle resting on a sediment bed, for **A)** small values and **B)** large values f the particle Reynolds number Re$_*$.

**11** There is no reason to expect the resultant force to be parallel to the overall surface of the bed; this leads to the idea of resolving the resultant force into a component parallel to the bed, called the **_drag_**, and a component normal to the bed, called the **_lift_** (Figure 9-4). Several investigators, using sometimes ingenious experimental methods, have attempted to make direct measurements of

the lift and drag forces on bed particles (or surrogates, like regularly spaced hemispheres on a planar boundary); see Einstein and El-Samni (1949), Chepil (1958, 1961), Coleman (1967), and Coleman and Ellis (1976).
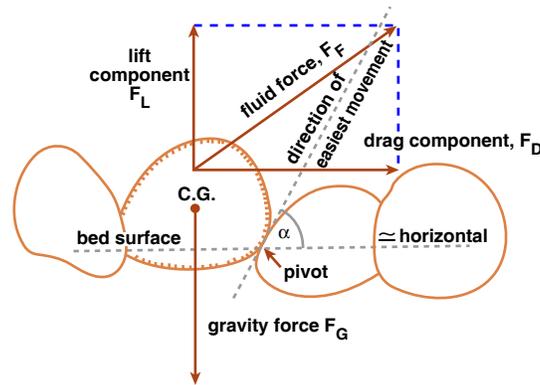
Figure 9-4.  Lift and drag on a bed sediment particle.  Figure by G.V. Middleton.)

**12**  A lift force on the particle is to be expected because the pressure is high around the base of the particle (the front stagnation point is located low on the front surface of the particle), and relatively low over the top surface of the particle, as well as in the rear; to see that, all you have to do is apply the Bernoulli equation qualitatively.  People have made ingenious experiments to measure the lift force, and the lift force has been found to be almost equal to the drag force at high boundary Reynolds numbers.  There is some evidence that at very low Reynolds numbers the lift force actually becomes weakly negative, for reasons no one seems to understand very well yet.

**13**  Another important thing you should expect about the fluid force is that it is extremely unsteady, because of fluctuations in velocity associated with the passage of fairly large eddies just outside the near-boundary layer of the flow. Actual measurements have shown fluctuations by as much as a factor of four in the instantaneous fluid forces on bed particles.

## BALANCE OF FORCES

**14**  We could take either of two approaches at this point:  make a dimensional analysis to develop a graphical framework for expressing and rationally organizing observational results, or try to develop an analytical solution.  First I will outline the classical analytical approach, to see where it leads.  That approach involves assuming that the particle is pivoted out of its

position, around its two downstream contact points, when the moment due to the fluid force finally becomes larger than the opposing moment due to the weight of the particle. Unfortunately this analytical approach turns out to be of little more help than a simple dimensional analysis, for two reasons: irregularity of geometry, and complexities of the fluid force itself.

**15** Particles begin to move on the bed when the combined lift and drag forces produced by the fluid become large enough to counteract the gravity and frictional forces that hold the particle in place. It is impossible to define the balance of forces or moments acting on particles uniquely for all grains: some particles lie in positions from which they are more easily lifted, slid, or rolled than others. It is equally impossible to define a single fluid force that applies to all particles: some particles are more exposed to the flow and subjected to larger fluid forces than other particles, and the fluid forces at the bed fluctuate with time because of turbulence in the flow.

**16** We will begin by considering an average particle, in an average position on the bed, subjected to an average fluid force; we will return later to the problem of an appropriate definition of these averages. To simplify matters further, assume that friction prevents sliding of one particle past another and that the moving particle simply pivots about an axis normal to the flow direction. The condition for the beginning of motion then is that the moments tending to rotate the particle downstream are just balanced by the moments (in the opposite sense) that tend to hold the grain in place (Figure 9-4).

**17** To determine the fluid-force moment exactly, we would have to sum all the products of the forces times their normal distances from the lines of action to the pivot axis. We can simplify further by assuming that the bed is horizontal and by considering, at first, only drag forces. Then it is convenient to consider only those components of the gravity and drag forces that act normal to the line joining the pivot to the center of gravity of the particle.

**18** The total moment produced by summing body forces (like the gravity forces acting on each element of volume making up the particle) is the same as the total force times the distance of the center of gravity from the pivot. You can readily see that if we divide the gravity force into two components, normal to and parallel to the line joining the pivot to the center of gravity, then the moment due to the second of these components must be equal to zero, because that component has a line of action passing through the pivot itself. So we can write the condition for the beginning of movement as

$$a_1(F_G \sin \alpha) = a_2(F_D \cos \alpha) \tag{9.1}$$

**19** The left side of Equation 9.1 is the total moment due to gravity, which tends to rotate the grain upstream about the pivot or to hold it in place against the moment due to fluid drag forces tending to rotate the particle downstream. The right side represents this fluid-drag moment in a purely conventional way. The

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drag moment must actually be calculated as the integral of all the products of the drag forces acting on each element of the surface, times the normal distance of each of these forces from the pivot. But because we do not know the distribution of the drag forces over the surface of the particle, there is no way we can actually evaluate that integral, so it is represented conventionally simply as a product of the total component of drag, $F_D \cos \alpha$, which opposes the total component of gravity, $F_G \sin \alpha$, times a normal distance $a_2$. The value of $a_2$ cannot be determined analytically, so $a_2$ is actually a "fudge factor" that is chosen to make the equation balance.

**20** The gravity force $F_G$ can be written

$$F_G = c_1 D^3 \gamma' \tag{9.2}$$

where $c_1$ is a coefficient that takes account of the particle shape. The fluid drag force $F_D$ can be assumed equal to the average boundary shear stress to times the area of the grain, and can be written

$$F_D = c_2 D^2 \tau_o \tag{9.3}$$

where the coefficient $c_2$ takes into account not only the geometry and packing of the grains (which determines the "area of the grain") but also the variation of the drag coefficient. Thus $c_2$ can be expected to vary with boundary Reynolds number. Substituting Equations 9.2 and 9.3 for $F_G$ and $F_D$ into Equation 9.1 and writing $\tau_o = \tau_c$ for the critical condition gives

$$a_1 c_1 D^3 \gamma' \sin \alpha = a_2 c_2 D^2 \tau_c \cos \alpha \tag{9.4}$$

or, solving for $\tau_c$,

$$\tau_c = \frac{a_1 c_1}{a_2 c_2} \gamma' D \tan \alpha \tag{9.5}$$

**21** Equation 2.5 can be made dimensionless by dividing both sides by $\gamma' D$:

$$\beta_c = \frac{\tau_c}{\gamma' D} = \frac{a_1 c_1}{a_2 c_2} \tan \alpha \tag{9.6}$$

where $\beta_c$, the critical value of a dimensionless variable $\tau_0/\gamma'D$, called ***Shields' $\beta$*** or the ***Shields parameter***, should be expected to be a function of grain geometry and boundary Reynolds number. (The Shields parameter is named after an American engineer who first put the study of incipient movement on a rational basis in the 1930s while working at a hydraulics laboratory in Germany.)

**22** What Equation 9.6 tells us is that the Shields parameter is a function of a term that itself depends on various effects, both geometrical and dynamical. The quantities $a_1$, $c_1$, and $\tan \alpha$ are geometrical, and depend on the grain shape and grain packing. The quantities $a_2$ and $c_2$ are partly also geometrical, but they also include a dependence on the details of flow around the grains and the resulting distributions of pressure forces and viscous forces, and they are therefore a function of the boundary Reynolds number. We cannot proceed any further than Equation 9.6 without knowing more about the details of this $\mathrm{Re}_*$ dependence, to say nothing of the problem of taking account of particle shape and packing.

**23** The foregoing analysis is not much different if lift is considered as well as drag, because there should be a proportionality between the two forces which also depends only on grain geometry and boundary Reynolds number.

**24** In deriving Equation 9.6 it was assumed that the bed slope is negligibly small. If this is not the case, then it is easily shown that $\sin \alpha$ in the Equation 2.6 should be replaced by $\sin(\alpha - \phi)$, where $\phi$ is the slope angle (positive in the downstream direction). So, if other conditions remain the same, increasing bed slope decreases the critical value of $\beta$.

**25** Many other theoretical approaches to incipient motion, along the same lines as that above but taking other effects, like lift forces and bed slope, into consideration as well, have appeared in the literature. None takes us much further than the foregoing simplified analysis.

## DIMENSIONAL ANALYSIS

**26** The list of variables that should describe the condition of incipient movement is fairly straightforward (Figure 9-5): $\tau_o$, $D$, $\rho$, $\mu$, $\rho_s$, and $\gamma'$. Flow depth should not be important, because the particles are in the inner layer of a turbulent boundary layer (see Chapter 4 of Part I), in which only the local structure of the flow governs the forces felt by the bed particles.



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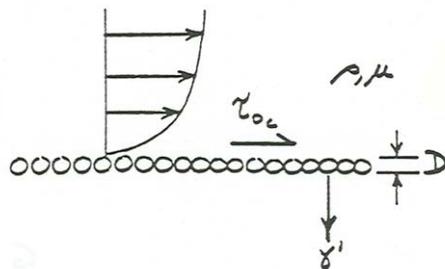
Figure 9-5. Variables that characterize the threshold of particle movement.

**27** You might think that the sediment density $\rho_s$ has no business here, because the sediment is not moving (by definition). In reality it might be important, however, because it affects the time scale of the response of the particle to a sudden acceleration of the flow: other things being equal, the denser the particle the less rapidly it would accelerate in response to a sudden increase in resultant fluid force to a value large enough to move the particle. And that is important for incipient movement, because the particle might be rocked out of its position of rest by a passing unusually strong eddy, only to fall back to it and undergo no permanent displacement.

**28** Two points about the list of variables above deserve further comment. The first has to do with the choice of $\tau_0$ as the variable that characterizes the strength of the flow. Because in the material in earlier chapters on flow around a sphere the drag force was related to a velocity, you might reasonably ask why not use a velocity rather than $\tau_0$. One answer is that, after all, what is moving the grains is basically a force acting on the bed, so the boundary shear stress is a more logical choice than any velocity. (You might reasonably respond that the force itself is caused by the local velocity of flow around the grains.) Another answer is that it is difficult to specify exactly which velocity should be used. The most easily measured velocities (the mean velocity of flow $U$ or the surface velocity $U_s$) are not, in any clear or straightforward way, related to the velocity measured near the bed, which is what determines the force that tends to move the grains. If we were to use the mean velocity we would have to add another variable, the depth of flow, because the same mean velocities may give rise to different near-bed velocities, or shear stresses, if the flow depth is different. To get around these problems it has always seemed most natural to use $\tau_0$ instead of a velocity—but remember that a graph or criterion for incipient movement in terms of $\tau_0$ (like the famous Shields diagram, introduced below) can always be recast into a form that involves flow velocity and flow depth, if it is velocity that you are most interested in.

**29** The second point is that in listing the variables I have chosen to combine the gravity $g$ and the sediment density $\rho_s$ into a single variable with the fluid density: $\gamma' = g(\rho_s - \rho)$. This is equivalent to assuming that the only important effect of both gravity and particle density is in controlling the submerged weight of the particle. We assume that surface gravity waves in the fluid are not important—which is equivalent to assuming that the flow is not shallow enough so that the motion of the fluid over the grains affects the free surface. This is clearly an invalid assumption for very shallow, gravel-bed rivers.

**30** So you should expect to deal with three independent dimensionless variables, and therefore to be able to express the condition for incipient movement as a surface in a three-dimensional graph. One of these can be the density ratio

$\rho_S/\rho$. The other two then have to involve $\tau_0$, $D$, $\mu$, and $\gamma'$. The traditional variables have been the boundary Reynolds number $\rho u_* D/\mu$ and the Shields parameter $\tau_0/\gamma' D$, already introduced above.

$$\text{threshold} = f(\rho, \mu, \gamma', D, \tau_0) \tag{9.7}$$

and, nondimensionalizing,

$$\frac{\tau_c}{\gamma' D} = f\left(\frac{\rho u_* D}{\mu}\right) \tag{9.8}$$

where $\tau_c$ is the threshold value of the bed shear stress.

**31** You already know about the hydraulic significance of the boundary Reynolds number: it characterizes the nature or structure of the flow near the bed. And recall from Chapter 8 that the Shields parameter also has a real physical meaning: by multiplying the top and bottom of the Shields parameter by $D^2$ you can see that it is proportional to the ratio of fluid force on the particle to the weight of the particle. The effect of the density ratio $\rho_S/\rho$ is still unclear, but is known not to be large, and anyway most sediment problems involve quartz-density sediment in water.

**32** So just by looking at the dimensional structure of the problem of incipient movement, we have arrived at the same conclusion as from the force-balance analysis, expressed by Equation 2.6, in the preceding section.

### HOW IS THE THRESHOLD FOR MOVEMENT IDENTIFIED?

**33** At this point, as a prelude to looking at the various diagrams that are in common use for incipient movement, it seems appropriate to pose the following fundamental question: *How is the condition of incipient movement identified?* An untutored outside observer might naturally assume that the answer is to watch the sediment bed to determine when, under conditions of slowly increasing flow strength, the sediment begins to move. But there is a serious problem with such a procedure, as can easily be demonstrated by a simple flume experiment: even for bed shear stresses (or flow velocities) that are well below what would conventionally be considered to represent the threshold or critical condition for sediment movement, some bed particles are moved by the flow. It is easy to understand why this is so. Recall from the material on turbulence in Chapter 3 that because of the impingement of turbulent eddies on the sediment bed the instantaneous fluid forces on sediment particles varies widely. The consequence is that even in weak flows a particularly strong turbulent eddy would occasionally cause one or more particles to move. There is thus a wide range of flow conditions that cause weak sediment movement. Put another way, the question becomes: How long should one wait to detect movement of a particle on the sediment bed? A minute? An hour? A day?

**34** The wide range of bed shear stresses for which there is weak particle motion brings forth an additional question: *Does a threshold bed shear stresses for incipient movement really exist?* Some would contend that the range of bed shear stresses for weak particle motion is *indefinitely* wide, and that because of that there is a conceptual flaw in the assumption that a definite threshold condition can be defined. It is true that the weaker the flow, the smaller the number of bed particles that are moved by the flow (per unit time and per unit area of the bed), but the lower limit for any particle motion is indefinite. For a cogent exposition of the impossibility of assigning a definite threshold condition, see the paper by Lavelle and Mofjeld (1987).

**35** This difficulty in defining the condition of incipient movement is largely because incipient movement is stochastic, in that the instantaneous resultant force on a bed particle varies irregularly through time just as does, say, a turbulent velocity component. One conceptually satisfying way of looking at the threshold of sediment movement is in terms of the relationship between two different probability frequency distributions: the distribution of instantaneous local $\tau_0$ needed to move the set of particles occupying some area of the bed surface, and the distribution of instantaneous local $\tau_0$ that acts on any small area of the bed, of about the size of the particles, through time (Figure 9-6; after Grass 1970). When the two distributions do not overlap (Figure 9-7A), the flow is never strong enough to move any of the particles on the bed, whereas when the two distributions overlap somewhat (Figure 9-6B), there is a subset of particles on the bed surface which can be, and therefore are, moved by the flow. With increasing flow strength the distributions come to overlap entirely (Figure 9-6C), meaning that all the particles on the bed surface are susceptible to movement. When the condition for movement threshold is viewed in this way, it is easy to identify the basis for the skeptics' view that it is not possible to define the condition of incipient movement: they would argue that the right-hand (high flow strength) tail of the frequency distribution of instantaneous local $\tau o$ extends indefinitely far to the right, toward higher bed shear stresses. One could argue, of course, that if the time average of the instantaneous shear stress is sufficiently small, none of the bed particles would ever move, but that condition is so far from the conventional view of the movement threshold as to be irrelevant to the problem, in a practical sense.
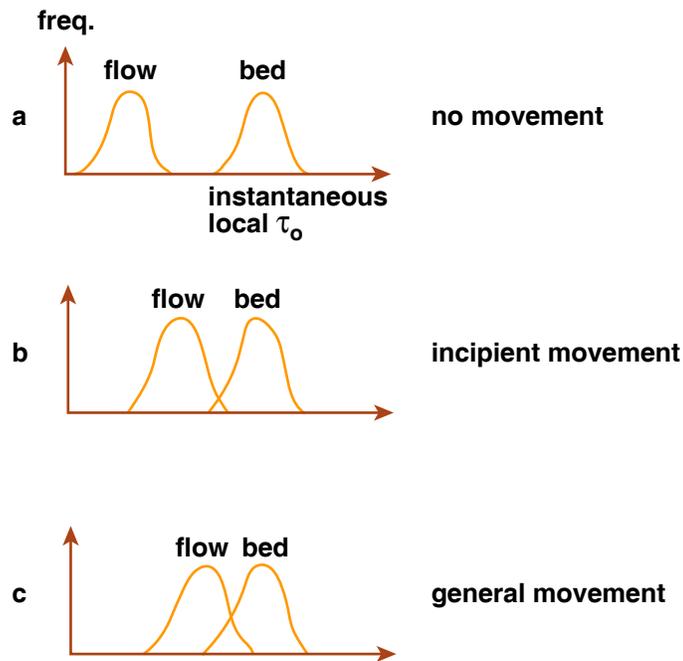
Figure 9-6. Threshold of movement, viewed in terms of the relationship between frequency distribution of instantaneous fluid forces exerted on bed particles and frequency distribution of fluid forces needed to move the particles. (Schematic; modified from Grass, 1970.)

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**36** Here we take the approach, in common with most sedimentationists, that the concept of a threshold for sediment movement has a certain physical reality, despite the uncertainty described above, and that techniques must be available to identify the threshold condition. There are two ways to attempt to identify the threshold condition, which might be termed, unofficially, the watch-the-bed technique and the reference-transport-rate method.

**37** *The watch-the-bed method:* as mentioned at the beginning of this section, this is in a sense the most natural way of defining the threshold condition. The problem of the wide range of conditions of weak sediment movement might be circumvented by general agreement, by convention, about where in this range of weak movement the threshold condition is situated. As you can imagine, much of the scatter in data on movement threshold is the result of differing views in this respect.

**38** There have been attempts at quantifying the conditions for incipient movement. Neill (1968) and Yalin (1977) argued that kinematic similarity of movement of grains implies identity of the dimensionless parameter $N = nD^3/u_*$, where $n$ is the number of grains in motion per unit area and unit time. They suggested adopting $10^{-6}$ as a practical critical value of $N$, and pointed out that, for equal values of the Shields parameter must be 30 times greater in air than in water, so that for equal $N$, $n$ must be 30 times greater. Such attempts at quantification have never come into general use.

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**39** *The reference-transport-rate method:* A second way of defining the threshold condition is to circumvent the uncertainty about when movement begins by defining some small value of the dimensionless unit sediment transport rate that seems to correspond most closely to what the consensus view of the threshold condition is, and assume, for practical purposes, that that value represents the threshold condition. There will be more on the reference-transport-rate method in the later chapter on mixed-size sediments.

**40** In practice, what has to be done is to make a fairly large number of measurements of the dimensionless unit sediment transport rate at a number of flow strengths above the threshold, plot the results, fit a curve to the results in some way, either as an analytical function of just "by eye", and then extrapolate (or interpolate, if at least one of the data points lies below the reference condition) to find the value of bed shear stress associated with the reference transport rate.

**41** There is a potential inconsistency between the two methods described in the preceding paragraphs. In the watch-the-bed method, a flume run is usually set up with an initially planar bed, and the bed is watched for signs of particle movement on that planar bed. In the reference-transport method, the measurements are usually made after the sediment transport has come into equilibrium with the flow, and there is the possibility that bed forms, especially ripples, have formed on the bed. The sediment transport rate over a rippled bed is in general different from that over a planar bed of the same sediment experiencing the same flow conditions—so the threshold condition is identified in a fundamentally different situation.

## REPRESENTATIONS OF THE MOVEMENT THESHOLD

**42** What has traditionally been done, in studies of incipient movement, is to plot experimental results in a graph with the axis variables arranged in such a way that a unique curve in the graph separates conditions of established movement from conditions of no movement. The earliest such work is that of Shields (1936), who plotted initial-movement data from flume experiments on a graph of boundary shear stress nondimensionalized by dividing by the submerged specific weight and the mean size of the sediment (the resulting dimensionless variable is now called the Shields parameter; see the section on variables in Chapter 8) against the boundary Reynolds number. The result was not the first such attempt, but it became firmly established, especially in the field of hydraulic engineering, by virtue of its rational basis in fluid dynamics.
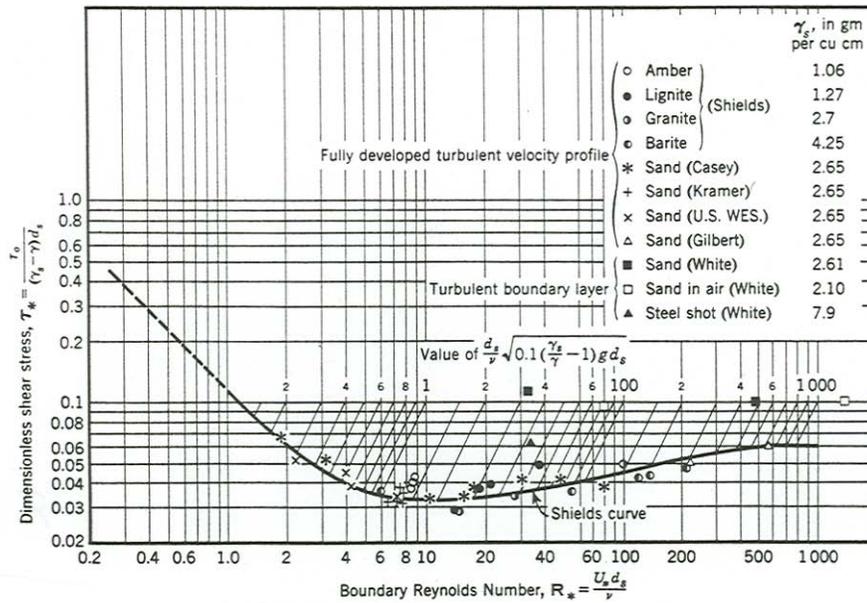
Courtesy of American Society of Civil Engineers.  Used with permission.

Figure 9-7.  The Shields diagram.  (From Vanoni, 1975.)

**43** The experimental results obtained by Shields himself, together with some earlier data
t threshold is called the ***Shields curve***.  There are two striking things about Figure 9-7:

- There is considerable scatter in the points.

- Shields had no data for $Re_*$ less than about 2 or greater than 600.

**44** It helps to evaluate the significance of the Shields diagram if you understand more clearly how the data were obtained.  Shields made his experiments in flumes 0.8 m and 0.4 m wide, with beds composed of granite fragments 0.85 mm to 2.4 mm in diameter, coal ($\rho_S = 1.27$ g/cm$^3$) 1.8 mm to 2.5 mm in diameter, amber ($\rho_S = 1.06$ g/cm$^3$) 1.6 mm in diameter, and barite ($\rho_S = 4.2$ g/cm$^3$) 0.36 mm to 3.4 mm in diameter.  Bed shear stress was determined from the resistance equation, $\tau_0 = \gamma d \sin \phi$ (Chapter 4 in Part I).  The bed was carefully leveled before each run.  Discharge and therefore mean velocity were increased in steps, and the slope was adjusted to maintain uniform flow.  After grains began to move on the bed, bed load was collected in a trap at the end of the flume so that the rate of sediment transport for a given condition could be determined.  For each bed material several observations were made at different discharges and rates of sediment transport, and the beginning of grain movement was determined not so much by direct observation as by plotting the measured rates of transport and extrapolating to the value of $\tau_0$ that corresponded to zero rate of transport.  (Shields did not report exactly how he made the extrapolation.)  Shields observed

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that this corresponded to what other workers had described as "weak movement" of particles.

**45** Shields himself noted that small ripples tend to form on the bed as soon as particles start to move. The presence of these ripples affects the rate of bed-load movement, so there is some question about exactly what was being determined when Shields extrapolated the measured rates to zero: initiation of movement on a plane bed, or initiation of movement on a rippled bed?

**46** It has been observed by other workers that the critical shear stress for initiation of particle movement on a rippled bed is greater than for that on a plane bed, although the mean velocity of flow is less. The explanation is that ripples create form resistance, which contributes most of the measured average bottom shear stress. If the depth does not change, the slope of the flume has to be increased to produce the shear stress needed to balance this form resistance and produce the same velocity. It is observed, however, that the slope and velocity needed to move particles on a rippled bed can be reduced below that necessary to move particles on a plane bed, because the phenomenon of flow separation over the ripples produces locally high and widely fluctuating shear stresses on the bed, which are large enough to move grains even at mean flow velocities lower than those required to move grains on a plane bed.

**47** To make this more explicit, imagine setting up two series of flume experiments, side by side, with exactly the same flow depth and flow velocity, one with a planar sediment bed and one with a rippled bed. Start at a velocity so low that there is no particle movement in either flume. Because of the large form drag on the rippled bed, slope and therefore boundary shear stress is much greater than for the planar bed. Now gradually increase flow velocity in both flumes while keeping flow depth constant but increasing the slope to maintain uniform flow. Boundary shear stress thus increases in both flumes, but at all times it is greater on the rippled bed than on the planar bed. Particle movement starts first on the rippled bed; eventually, at a substantially higher flow velocity, particles begin to be moved on the planar bed also, but boundary shear stress on the planar bed at that point turns out to be lower than that for which particle movement began on the rippled bed.

**48** The original Shields diagram has been used with little modification right up to the present, especially by hydraulic engineers. Miller et al. (1977) updated the Shields diagram, by drawing upon various more recent data to replot the diagram and redraw the curve; see Figure 9-8. The Miller et al. diagram is also in wide use. More recently, Buffington and Montgomery (1997) made an exhaustive and systematic compilation and analysis of studies on movement threshold. They were able to identify a systematic difference in the position of the Shields curve in the Shields diagram between data obtained by the watch-the-bed method and the reference-transport-rate method (Figure 9-9). They found a systematic difference in the location of the Shields curve, such that the curve based on the reference-transport-rate method lies above the curve based on the watch-the-bed method.

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The figure shows axes labeled:

$$\theta_1 = \frac{\tau_o}{(\rho_s - \rho)gD}$$

with values $10^0$, 0.8, 0.6, 0.4, 0.2, $10^{-1}$, 0.08, 0.06, 0.04, 0.02, $10^{-2}$ on the vertical axis and $10^{-2}$, $10^{-1}$, $10^0$, $10^1$, $10^2$, $10^3$, $10^4$ on the horizontal axis.

Labels in plot: gb, y, cs, cs, v, v, v, v, b, b, pvc, pvc, gb, gb, gb

Shields (1936)

calculated by authors

$$Re_* = \frac{U_*D}{\nu} = \frac{\sqrt{\tau_o/\rho}\,D}{\nu}$$
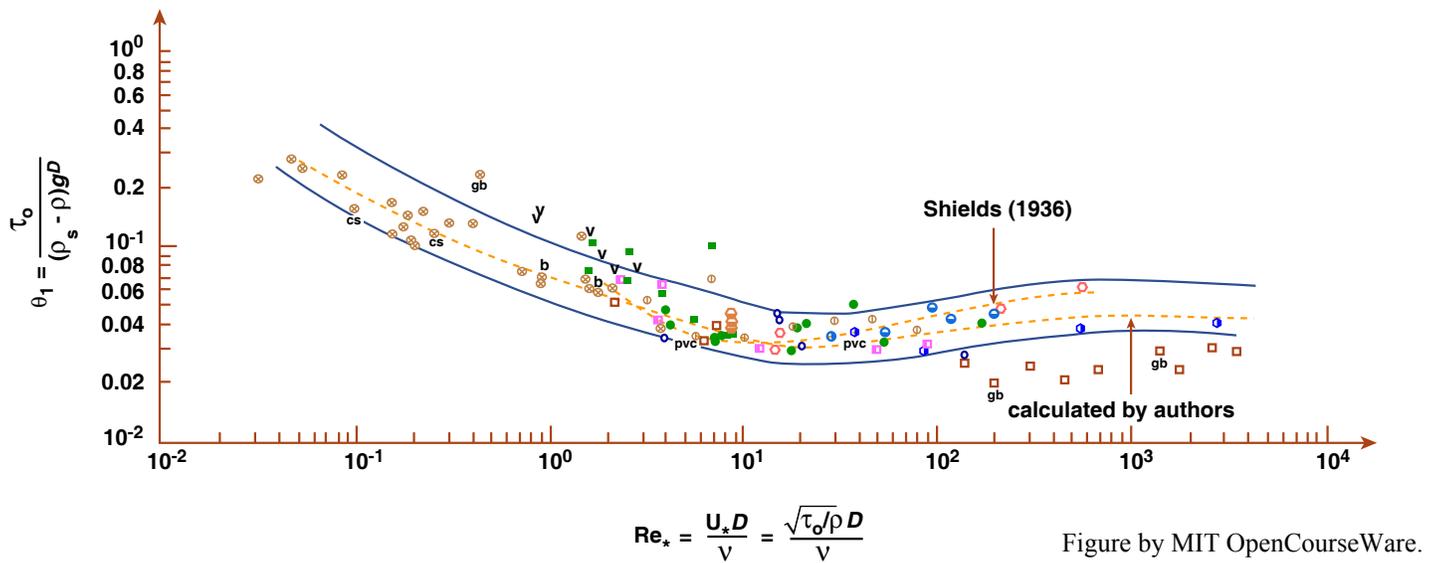
Figure by MIT OpenCourseWare.

Figure 9-8.  A modified and updated version of the Shields diagram.  (From Miller et al., 1977.)

## RECASTING THE SHIELDS DIAGRAM

**49**  The Shields diagram has a satisfying underlying physical significance, but it is awkward to use to find the threshold shear stress that corresponds to a given sediment diameter, or to find the largest sediment diameter that is moved by a given shear stress, because both $\tau_o$ and $D$ appear in both of the axis variables. (This exemplifies the difference between two contrasting goals, both of them laudatory but often in conflict:  expressing results in a form that most directly reveals the underlying processes, or in a form that is most useful in practical work.)  To get around this problem the Shields parameter and the boundary Reynolds number can easily be recast into two equivalent dimensionless variables, one with $D$ but not $\tau_o$ and the other with $\tau_o$ but not $D$.  You can verify for yourself that these are $\tau_o(\rho/\gamma'^2\mu^2)^{1/3}$ and $D(\rho\gamma'/\mu^2)^{1/3}$.  Figure 9-10 shows a

a)

$\tau^*_{c50}$

turbulent flow, $D_{50}/h_c \leq 0.2$

■ $\tau^*_{cr50m}$   ▼ $\tau^*_{cr50s}$

● protruding grains

0.086
0.052

$\text{Re}^*_c$

b)

$\tau^*_{c50}$

turbulent flow, $D_{50}/h_c \leq 0.2$

■ $\tau^*_{cv50m}$

○ platy grains

0.073

0.030

$\text{Re}^*_c$
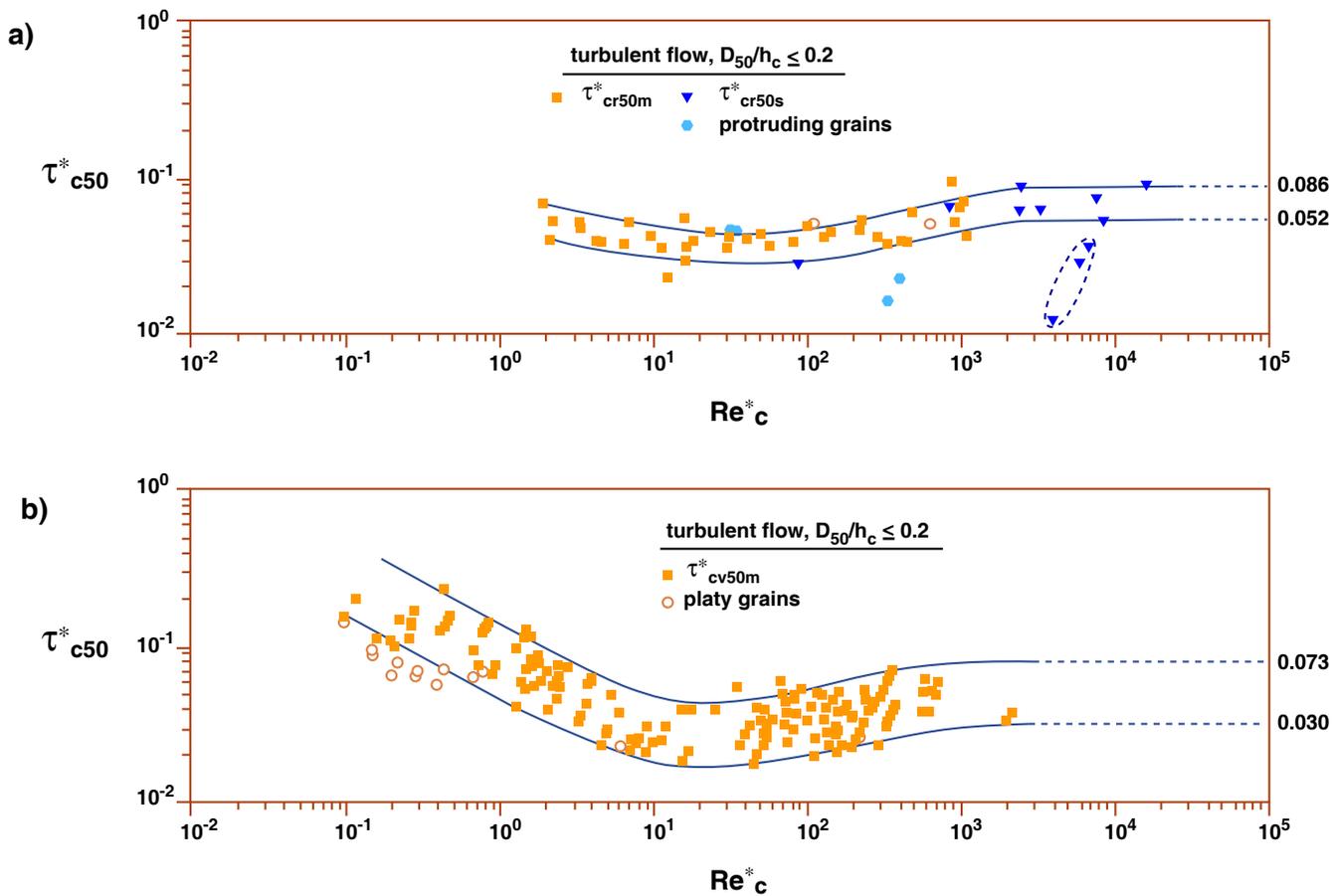
Figure by MIT OpenCourseWare.

Figure 9-9.  Comparison of data on threshold boundary shear stress (expressed as Shields parameter based on median sediment size; $\tau^*_{c50} = \tau_0/\gamma' D_{50}$) for **A)** the reference-transport-rate method and **B)** the watch-the-bed method.  (From Buffington and Montgomery, 1997.)

recast version of the Shields diagram, From Miller et al. (1977).  The axes in Figure 9-10 are labeled with $u_*$ and $D$, so the graph does not look dimensionless, but those values are for a water temperature of 20°C; the compilers just took the dimensionless graph, substituted in the 20° values of $\rho$ and $\mu$ into the axis variables, and ended up with the 20° values of $u_*$ and $D$.  In effect, the graph is still really dimensionless.

**50**  One of the problems about the movement threshold is that there is a wide range of flow conditions for which there's weak but noticeable sediment movement.  That leads to the problem of how to define the condition of incipient movement in the first place.  Quantitative criteria have been proposed, but they have not yet become firmly established.  Nonetheless, the Shields diagram continues to be used, because it gives good ballpark results for both engineering and sedimentological purposes.  Various investigators have tried to establish arbitrary but experimentally reproducible definitions of the beginning of movement.
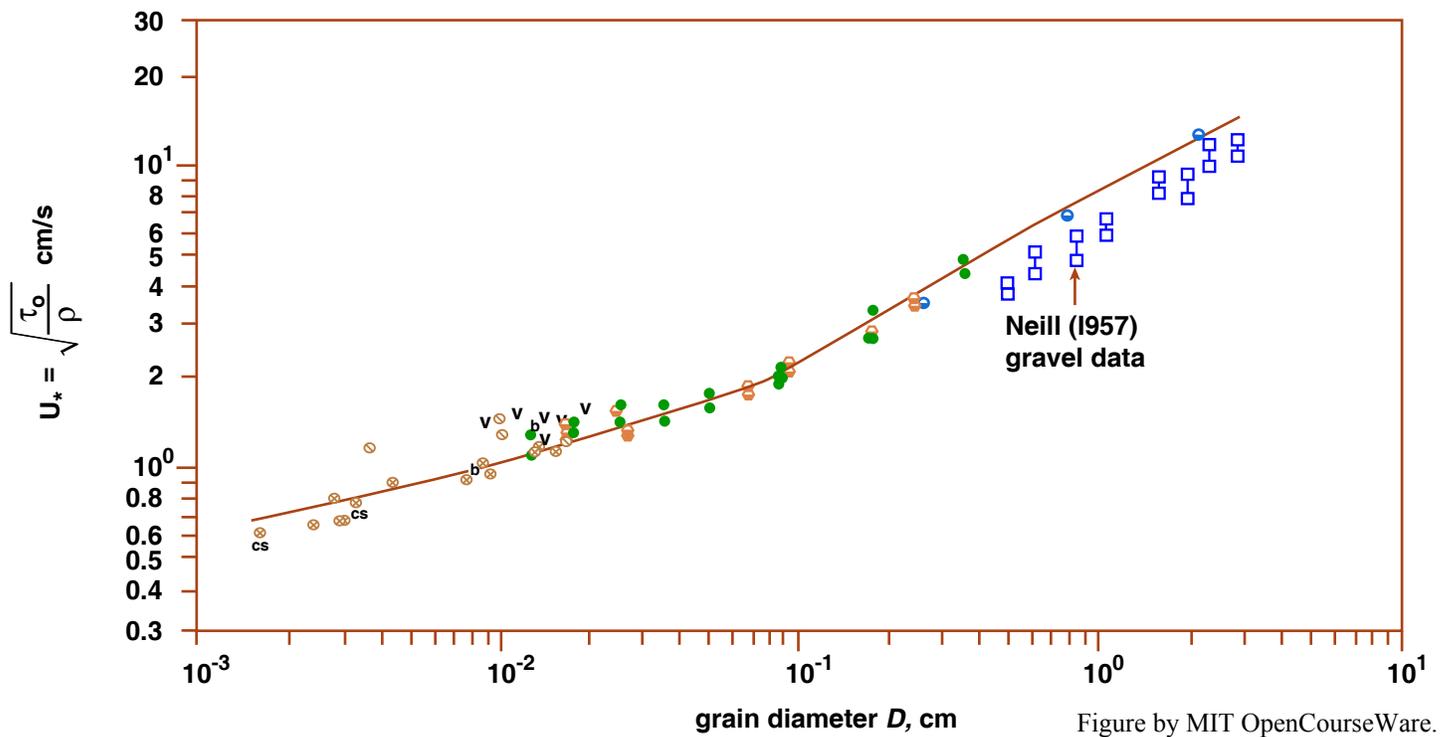
Figure 9-10. A version of the updated Shields diagram, recast in terms of shear velocity $u_*$ and particle diameter $D$, and standardized to temperature 20°C. (From Miller et al., 1977.)
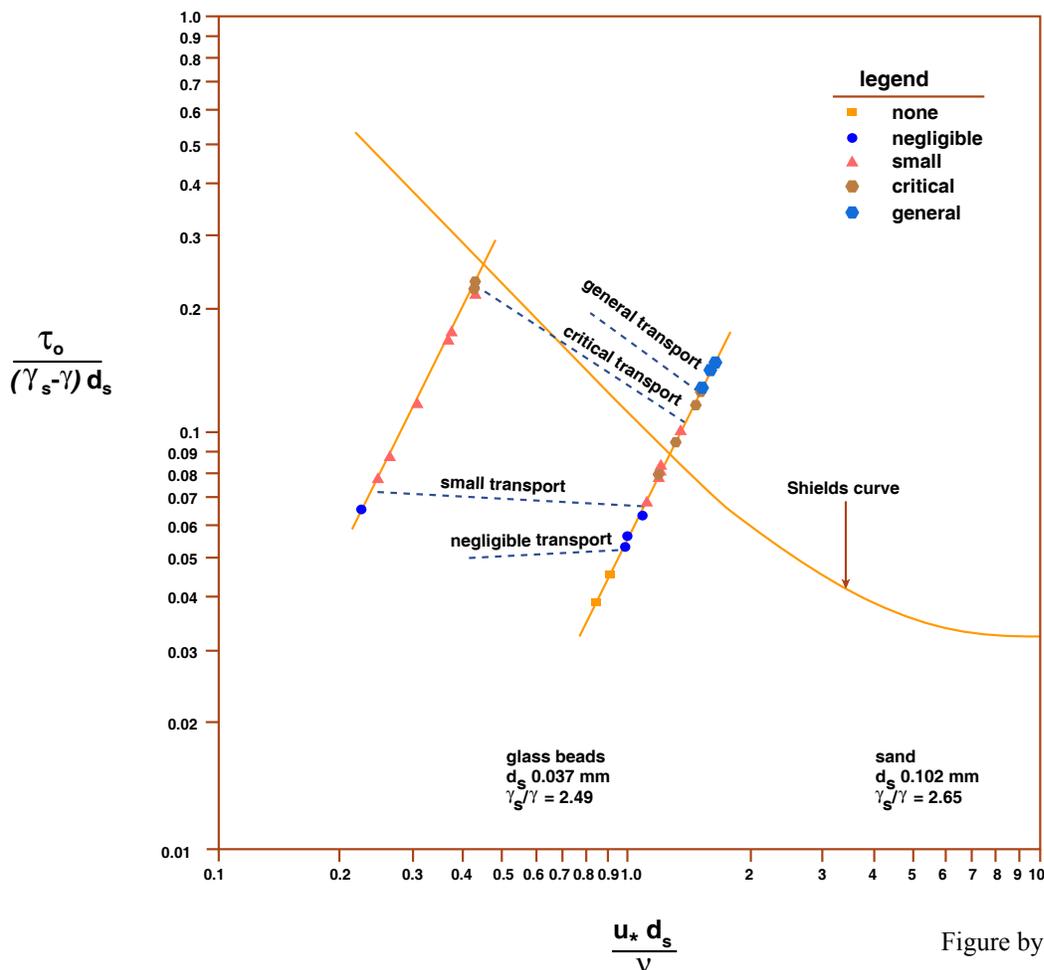


Figure 9-11. Graph of Shields parameter vs. particle Reynolds number for conditions near threshold, for runs with two sediments in a turbulent shear flow. (From Vanoni, 1964.)

**51**  By observation of the bed through a microscope, Vanoni (1964) found that movement was intermittent on any small area of the bed, and, when it did occur, it took place in local gusts with several grains moving at once.  He defined the critical stage of movement in terms of four gust frequencies (Figure 9-11):

> **negligible** ($< 0.1$ s$^{-1}$)
> **small** ($0.1$–$0.33$ s$^{-1}$)
> **critical** ($0.33$–$1$ s$^{-1}$)
> **general** ($> 1$ s$^{-1}$)

**52**  Figure 9-11 makes clear how wide the range of condition is for which an objective observer, unschooled in the intricacies or defining incipient movement, might judge where to locate the threshold condition.  As you have seen, this effect has even led some observers to question whether a criterion for inception of movement can even be formulated at all:  movement tails off so gradually with decreasing flow strength that even in very weak flows, if one waits long enough one might see a particle moved by an instantaneous local fluid force on the bed, that is at the very extreme of the frequency distribution of such forces.

## THE HJULSTRÖM DIAGRAM

**53**  No account of the threshold of sediment movement would be complete without mention of the famous graph proposed long ago by Hjulström (1939) (reproduced without change here as Figure 9-12), which has been used, or at least cited, by generations of sedimentationists.  Hjulström undertook to represent the boundaries among erosion, transportation, and deposition in a graph of flow velocity vs. particle size.  He acknowledged that use of the mean velocity to characterize the flow is inadequate, and viewed it as only "a temporary substitute until more data are available" (Hjulström, 1939, p. 9).  The heavy bands between transportation and erosion, labeled "A", were meant to represent the uncertainty of the data used to define the boundary.  It is clear that Hjulström intended that band to represent the threshold of bed-load transport as the flow velocity is gradually increased.  The word "erosion" in the graph is better read as "traction transport":  the former is best reserved for net removal of sediment from a given area of a sediment bed by the action of the fluid flow.  (There can be transportation without erosion, if the sediment transport rate does not increase in the downstream direction and the local concentration of sediment in transport does not change with time.)  The curve labeled "B" was meant to show cessation of transportation as the flow velocity gradually decreases; Hjulström relied upon some earlier flume observations that indicated that for sediments coarser than about medium sand the flow velocity for cessation of sediment movement about two-thirds that for inception of movement (see comments in the following paragraph). Hjulström showed the part of Curve B for fine sediments as a dashed line presumably because no data were available in that range.  The widening gap between curves A and B is a consequence of the effect of cohesion increasing the velocities needed for initiation of movement.  It seems likely that if the effects of

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cohesion in fine sediments could somehow be eliminated, the gap between curves A and B would be narrow for the entire range of sediment sizes.

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Figure 9-12.  The Hjulström diagram.

Image removed due to copyright restrictions.
Please see: Hjulström, F. "Transportation of Debris by Moving Water." In *Recent Marine Sediments*.
Edited by P. D. Trask, 1939; Tulsa, Oklahoma. "*A Symposium." American Association of Petroleum Geologists*. pp. 5-31.

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**54** The idea that the flow strength for cessation of movement is less than, rather than equal to, the flow strength for initiation of movement may have to do with a kind of hysteresis effect:  as weak transport continues on a granular bed at about threshold conditions, the bed becomes partly armored, in the sense that most of the bed particles have found stable positions, and a small number of particles, not having found such positions and thus being the very most mobile, continue to pick their way across the immobile bed surface at flow strengths slightly lower than that for which inception of movement was judged to have begun.  In any case, the range of flow velocities represented by the gap between Hjulström's curves A and B lies within the rather wide range, noted in an earlier section, for which there is at least some weak transport, so the difference between the two curves seems not to be of great consequence.

**55** The Hjulström diagram was later modified by Sundborg (1956).  Figure 9-13 shows Sundborg's version, taken directly from the original.  Sundborg's graph shows separate curves for the movement threshold corresponding to several water depths, as is necessary if the flow velocity rather than the boundary shear stress (as in the Shields diagram) is used for the flow strength.  The purpose of the

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lightly shaded band that includes these specific curves for the various flow velocities was not explicitly described, but it was probably meant to represent uncertainty in the data. Sundborg made note of Hjulström's lower curve, for cessation of movement, but did not include it in his version.
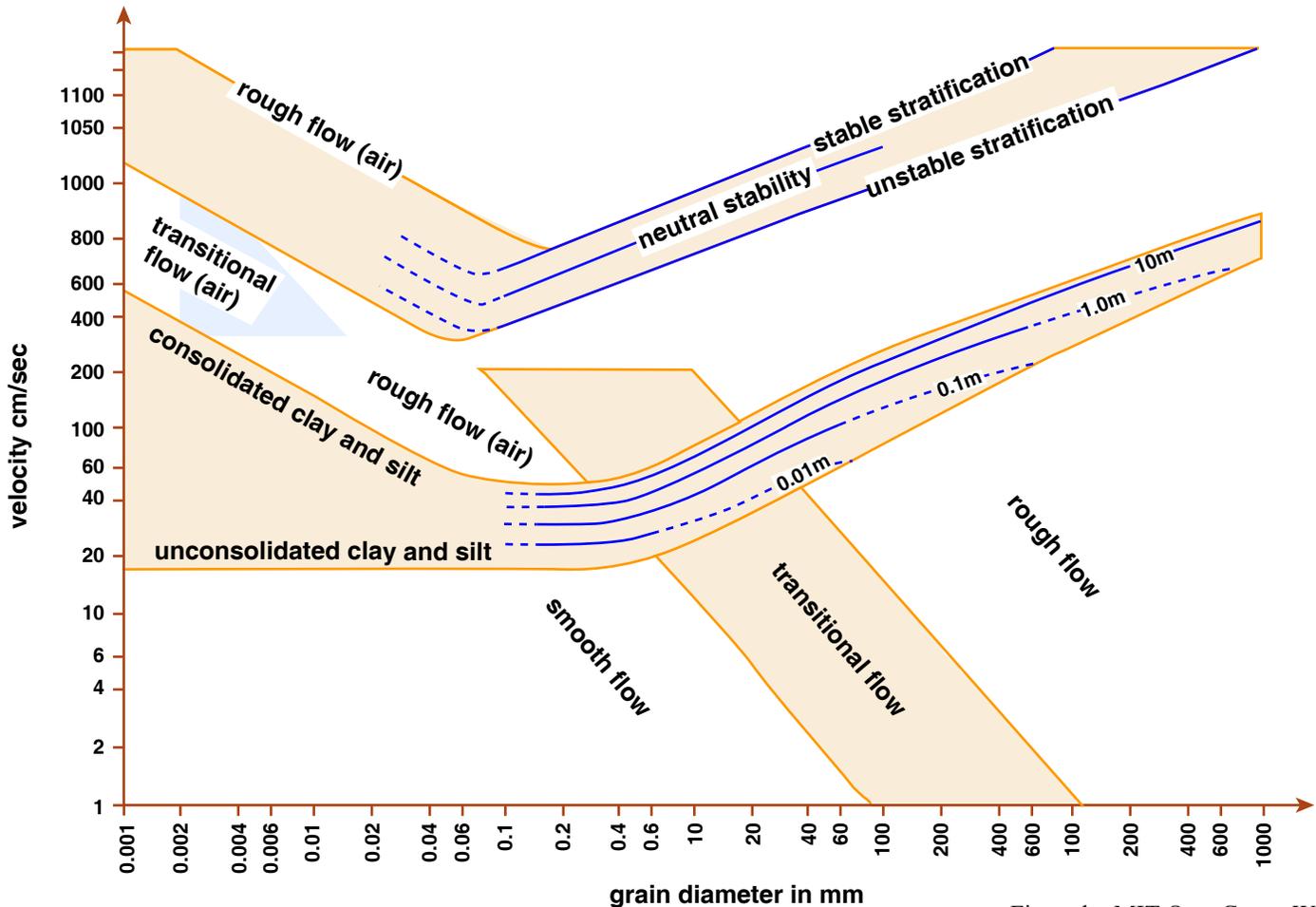


Figure 9-13. Sundborg's modification of the Hjulström diagram. (After Sundborg, 1956.)

**56** Such are the origins of what might be called the Hjulström–Sundborg diagram. A diagram like Sundborg's version, showing a family of threshold curves as a function of flow depth, with the curves based on the same data as the Shields diagram or its successors, has much potential for use by sedimentary geologists, despite the engineers' aversion to using velocity rather than boundary shear stress for this purpose. Various versions of the Hjulström diagram have appeared in a great many textbooks and monographs since Sundborg's contribution, but often in deplorably corrupted form: the curve for cessation of transportation is usually shown far lower than in the Hjulström original, with velocities lower than the curve for incipient movement by a factor of more than five!

**THE EFFECT OF DENSITY RATIO**

**53** It was mentioned in Paragraph 27 above that the density ratio $\rho_s/\rho$ might be important in governing the motion threshold. In that case, the dimensional analysis put forth in the earlier section should be extended in order to include the effect of relative density:

$$\text{threshold} = f(\rho, \rho_s, \mu, \gamma', D, \tau_o) \tag{9.9}$$

and, nondimensionalizing in the same way as before,

$$\frac{\tau_c}{\gamma' D} = f\left(\frac{\rho u_* D}{\mu}, \frac{\rho_s}{\rho}\right) \tag{9.10}$$

where $\tau_c$ is the threshold value of the bed shear stress.

**54** To my knowledge, little consideration has been given to that possibility in the literature on threshold. Shields himself plotted data not just for quartz-density sand in water but for amber (specific gravity 1.06), lignite (specific gravity 1.27), and barite (specific gravity 4.25). The points for amber and lignite lie slightly above the curve, and the points for barite lie slightly below the curve, suggesting that $\rho_s/\rho$ has at least a slight effect. A later study by Ward (1969) showed the same results, although with greater range of difference in threshold with density ratio. I am not aware of any more recent studies of the effect of density ratio on thresholds in water flows. A potential pitfall in interpreting these results, however, is that the experiments were not controlled for possible effects of particle shape, and in the case of plastic beads (styrene, specific gravity 1.18; polyethylene, specific gravity 1.06) in oil, the possibility of subtle particle-to-particle forces were not considered.

**55** The results seem counterintuitive, at least to this observer: should it not be the case that the particles with greater density, and thus greater inertia, would be less likely to be set into motion by suddenly greater fluid force on the particle than a particle with lesser density? A specific set of experiments in which a criterion for threshold is applied uniformly, and in which particle sorting and shape are adjusted to be the same for the various sediment batches with different density, might bring some clarity to the issue.

**56** If the decrease in Shields parameter (the dimensionless threshold shear stress) with increasing density ratio is indeed real, then it should be even greater for the case of quartz particles under the wind, for which the density ratio is far greater than for even the densest solid particles under water flows. Extensive observations of the threshold bed shear stresses for several particle compositions, sizes, and densities under wind (Iversen et al., 1976) gave results of about 0.1 for the threshold Shields parameter over a range of boundary Reynolds numbers from about 5 to about 50, and their results were generally consistent with earlier wind-tunnel studies of threshold (see Chapter 11 for more on eolian thresholds). Thresholds under water flows in this range of $Re_*$ average around 0.6, judging from the modified Shields plots presented by Buffington et al. (1997) (see Figure 9-10 above). The increase in dimensionless threshold with increasing density ratio does therefore seem to be real.

**57**  In a later study by Iversen et al. (1987), in which they synthesized earlier results on threshold over a wide range of density ratios, they purport to show that the dimensionless threshold decreases systematically with increasing density ratio from that characteristic of mineral particles in water to that of mineral particles in the extremely low atmospheric density of Mars; see Figures 9-15 and 9-16.  Be on your guard here, however:  their dimensionless coefficient A uses $\rho_s$ instead of $(\rho_s - \rho)$ in the variable $(\rho_s - \rho)g$, called $\gamma'$ in these notes, in the denominator, so the results for the threshold in water, shown on the left-hand side of Figure 9-14, cannot be compared directly with the conventional Shields curve.  The decrease in their threshold coefficient $A$ from the Venus-wind-tunnel case to the Mars-wind-tunnel case seems to support their position, but even so the values for the Venus wind tunnel, for which the density ratio is not greatly different from that of sand in water, is significantly higher than the generally accepted values shown on the Shields diagram and its later modifications, discussed above.  the matter seems (at least to this writer) not be settled.



| | | $\rho_p/\rho$ | $D_p$ (μm) | |
|---|---|---|---|---|
| ▲ | Mars WT | 2200 to 370000 | 37 to 673 | Greeley et al. (1980) |
| ⬣ | ISU WT | 950 to 9360 | 12 to 1290 | Iversen et al. (1976a) |
| ● | Venus WT | 38 to 414 | 32 to 776 | Greeley et al. (1984) |
| ○ | Water | 1.06 to 4.25 | 36 to 8970 | Graf (1971) |
| △ | Water & oil | 2.65 to 3.08 | 16 to 170 | White (1970) |
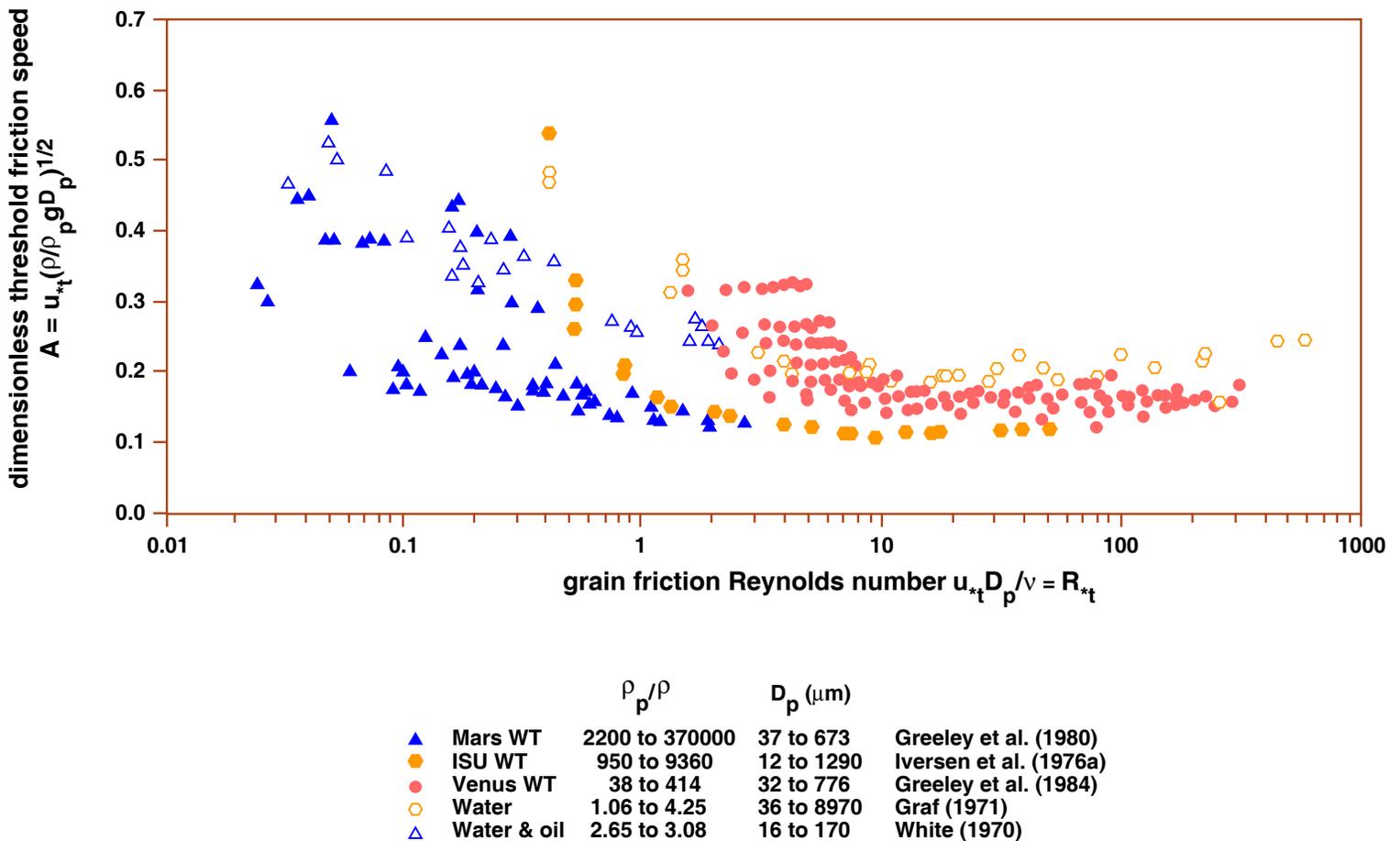
Figure 9-15.  Threshold graph from Iversen et al. (1987).

The figure shows a plot of dimensionless threshold $A = u_{*t}(\rho/\rho_p gD_p)^{1/2}$ versus density ratio $\rho_p/\rho$, with the curve:

$$A = 0.2 \left/ \sqrt{1 + 2.3\left\{1 - \exp\left(-0.0054\left(\frac{\rho_p}{\rho} - 1\right)^{0.86}\right)\right\}}\right.$$

Legend:
- ■ in water (Graf, 1971)
- ● Venus wind tunnel (Greeley et al., 1984)
- ▲ ISU wind tunnel (Iversen et al., 1976a)
  - $D_p > 200\ \mu m$
  - $R_{*t} > 10$
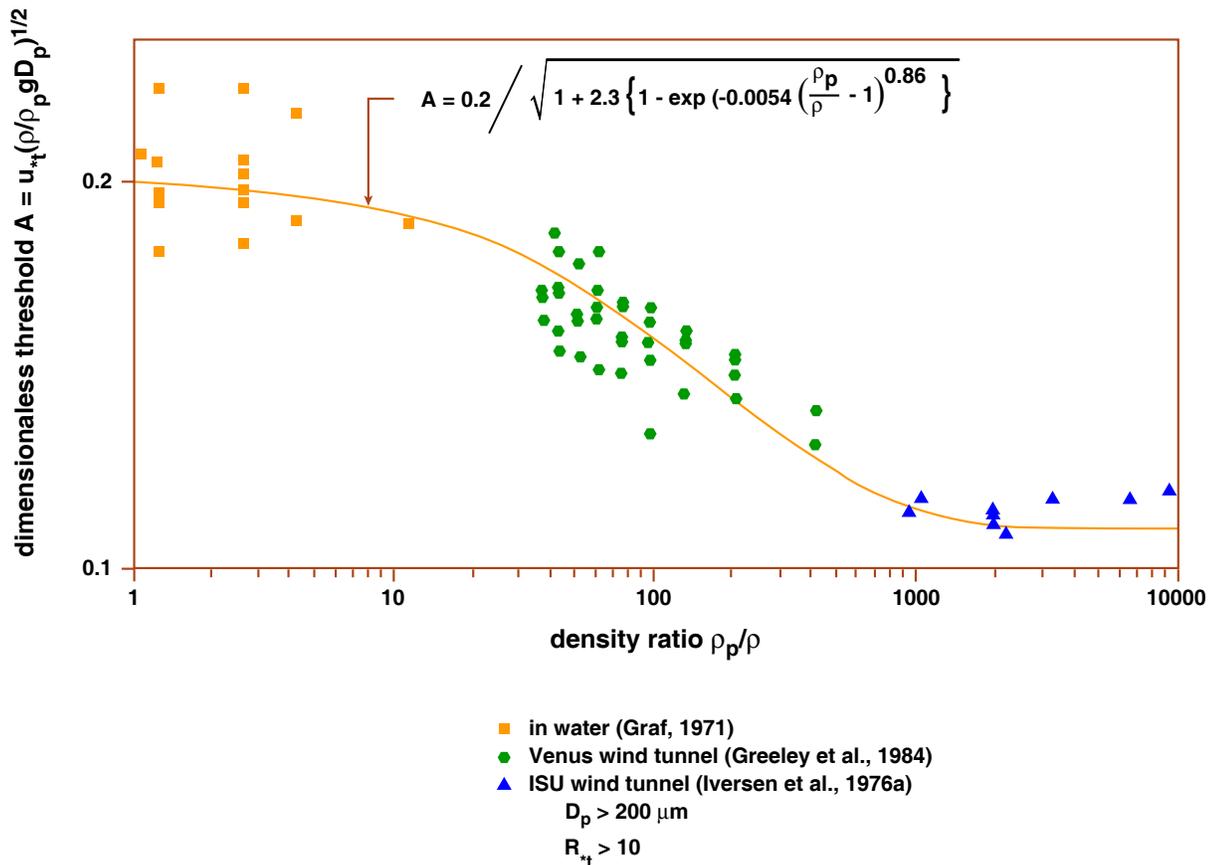
Figure by MIT OpenCourseWare.

Figure 9-16. Effect of density ratio on threshold, for particles larger than 0.2 mm and particle Reynolds numbers greater than 10. (From Iversen et al. 1987.)

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