CHAPTER 7

FLOW IN ROTATING ENVIRONMENTS

INTRODUCTION

1 In everything up to now in these notes, we have assumed that the flow environment is stationary. Just think back to flow around a sphere or flow down a channel. And even if the flow environment is in uniform motion, relative to you as the observer, you have already seen that you can convert the problem to that of a stationary flow environment just by moving with flow environment itself; think back to the problem of relative motion of a sphere and an ambient fluid.

2 But what if the flow environment is accelerating? That turns out to be quite a different matter. This chapter examines some of the effects of steady rotation of the flow environment. (Remember that rotation involves acceleration, of the radial rather than the tangential kind, even with a steady rotation rate.) The effects of rotation on the flow are striking, and I think not intuitive. These effects are of central importance in the study of what are called ***geophysical flows***: the movements of the atmosphere and the oceans on scales of hundreds and thousands of kilometers. The boundaries of these flows are the rotating solid Earth, and we the observers are rotating with that solid Earth. Admittedly, the effects of rotation do not have a very direct bearing on the small-scale dynamics of sediment transport, but the indirect consequences are so far-reaching that I could not resist including this chapter in these notes.

PLAYING ON A ROTATING TABLE

3 In a large unobstructed indoor area (a gymnasium or a warehouse would be best, but a big room in your house would suffice), build a giant flat horizontal turntable—just a disk mounted at its center point on a vertical rotating shaft (Figure 7-1). You can rotate the whole disk at any desired constant rotation rate, described by its angular velocity ω, measured in radians per second. It would be best if you painted the surface of the disk a flat black, the better to observe the motions of the brilliantly white marker spheres you are going to roll around on the surface. To make things really exciting, be sure to coat the white marker spheres with a thick chalky coating of some sort that tracks off evenly onto the surface of the turntable as the spheres roll about.

4 To do things right, you are also going to need an observation perch above the turntable. This perch should be easily movable from place to place above the surface, but (and this is important) stationary relative to the floor of the room while it is in use. One of those mechanized cherry-picker seats, extending horizontally from the margin of the disk, would do nicely, if you can afford it.

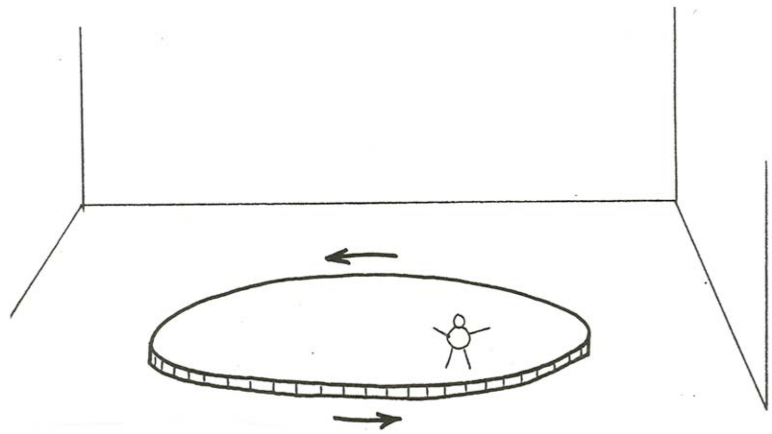

Figure 7-1. Playing on a rotating table.

5 Set the rotation rate, settle into your perch, occupy a point just over the turntable, and roll one of your marked spheres onto the table, just as if you were at a bowling alley. Here is the big question: What would the track of the sphere look like on the turntable? (You are going to have to assume that the turntable exerts no substantial force on the rolling ball. That is not really true, but the effects are small enough that you can safely ignore them for the purposes of this demonstration. If you are uncomfortable with that assumption, you can always imagine a magic air-hockey puck that scoots nearly frictionlessly over the turntable, leaving a powdery white trail behind it.)

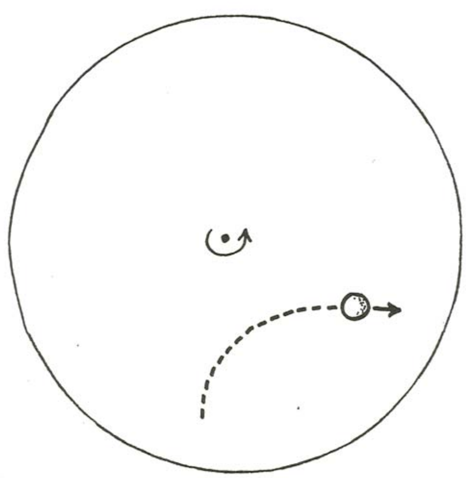

Figure 7-2. The track left by a ball rolling on a rotating table.

6 The big jump that your powers of deduction or imagination have to take here is to see that the track left by the ball on the table would be *curved* (Figure 7-2). And, once you are comfortable with that idea, it might naturally occur to you to think about whether that curved track is a circular arc. The answer turns out to be NO, although the reasons are a bit too intricate to deal with at the moment.

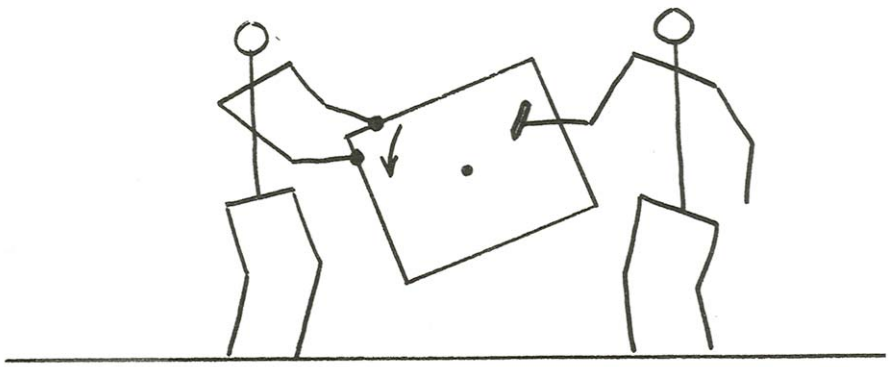

Figure 7-3. How to show that the track of an object moving in a straight line looks curved from the viewpoint of an observer in a rotating frame of reference.

7 If you cannot afford the time and money to build the turntable but you still want to get some results, here is a simpler and much less expensive way of demonstrating the phenomenon (Figure 7-3). Pin a big piece of posterboard to the wall so that it can be rotated about its center point, and have an assistant stand to one side and rotate the posterboard in a hand-over-hand motion as steadily as possible. Stand on one side of the posterboard with a marker pen, and draw a line on the posterboard in such a way that the tip of the pen moves in uniform rectilinear motion *relative to the underlying wall.* That is not easy to do, because you need to try to ignore the surface of the posterboard, and the mark that is being made on it, and instead concentrate on the imaginary path of the pen point on the motionless wall behind. You would find (Figure 7-4) that no matter where you start on the posterboard, and no matter which direction you choose for your line, the mark on the posterboard will be an arc, not a straight line!

8 You do not even need to rig up such a posterboard exercise, really; just think about your rapidly obsolescing phonograph. Pretend that the needle that passively follows the groove in the record is in fact constrained to move from the edge of the record to the center of the record in an almost straight line relative to the underlying stationary phonograph structure. The needle makes a spiral path on the record, with curvature as described in the two foregoing experiments.

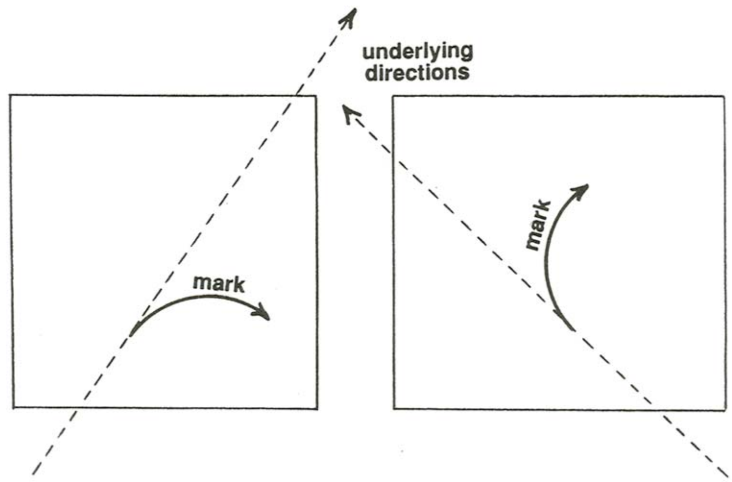

Figure 7-4. The result of the experiment shown in Figure 7-3.

9 And incidentally, this phonograph exercise shows conclusively that the curving arc is not exactly circular: the curvature is tighter along parts of the path located closer to the axis of rotation. That is basically because the velocity of the moving object (the needle) relative to the rotating surface (the record) decreases as the needle makes its way toward the center of the record. Why? Because the speed of the needle relative to the fixed stars is constant but the velocity of points on the record increases from zero at the center to a maximum at the outer edge.

10 But back to the big turntable: have your assistant roll a marker sphere onto the turntable while you are riding on the turntable. Watch the ball as it rolls and leaves its curving track. It will look to you as though some mysterious sideways force is continuously acting on the ball normal to its path to push it off its course. Something seems to be wrong with Newton's first law, which tells you that the ball should be moving in a straight line at constant speed. You know what the problem is, of course: the fictitious side force is an artifact of your observation of the ball from the standpoint of the rotating turntable. If you reoccupied your perch and rolled a clean, bright, chalkless ball onto the dimly lit black surface of the turntable, you would see the ball roll in a nice straight line! The fictitious side force that seems to act on moving bodies in a rotating environment is called the ***Coriolis force***, after the nineteenth-century French mathematician who first analyzed the effect. And the apparent acceleration of the sphere (it is a radial acceleration, not a tangential acceleration, in that only the direction changes, not the speed) is called the ***Coriolis acceleration***. The entire effect is called the ***Coriolis effect***.

11 What is the relevance of this demonstration to the motion of fluids? You could produce all kinds of fluid flows right on the surface of that turntable, by using that surface as your fluid dynamics laboratory: flow in an open channel, a free-convective flow in a big dishpan, or even just a sheet of water flowing freely across the surface of the turntable. In each case, every tiny element of the flowing fluid is subjected to that same Coriolis force. For the right combinations of fluid speeds and rotation rates, the Coriolis effect would have profound consequences for the pattern of fluid movement.

THE CORIOLIS EFFECT ON THE EARTH'S SURFACE

12 Fluid flows you observe on the Earth's surface experience a Coriolis acceleration. That is because the Earth is rotating, and both you and the flowing fluid are rotating with it. The effects you discovered on your turntable show up in those flows as well. The only places this should seem really obvious to you are at the North Pole and the South Pole—where the Earth's surface is perpendicular to the axis of rotation. But the Coriolis acceleration affects fluid motions everywhere else on the Earth's surface also.

13 The complete mathematical development of the Coriolis effect, although straightforward, would take us too far off the path of these notes, so I will give you an abbreviated and incomplete picture, just for the flavor.

14 I mentioned above that the rate of rotation of a rotating body is denoted by ω. But to be specific about such a rotation, you need to describe the orientation and the sense of the rotation as well. The rotation of the Earth is described by its ***angular velocity***—a vector, denoted by $\boldsymbol{\Omega}$, lying within the axis of rotation and with length equal to the rate of rotation ω. By convention, the angular velocity vector points north to express the sense of rotation of the Earth, which is counterclockwise when viewed from above the North Pole (Figure 7-5). The angular velocity $\boldsymbol{\Omega}$ thus specifies the orientation, sense, and rate of rotation of the Earth.

15 Look at a point on the Earth's surface. The speed $\boldsymbol{v}$ of that point, relative to outer space, is equal to the angular velocity ω times the distance from the rotation axis to the point. Expressed in terms of the radius R of the Earth and the latitude angle ϕ, this can be written $R\omega \sin(90°-\phi)$ (Figure 7-6).

16 A more elegant way of looking at the movement of a point on the Earth's surface is to characterize the position of the point by a position vector $\boldsymbol{r}$ that stretches from the center of the Earth to the given point (Figure 7-7) and then express the velocity of the point as $\boldsymbol{v} = \boldsymbol{\Omega} \times \boldsymbol{r}$. The product on the right side is a ***cross product*** of vectors, defined so that the result has a magnitude $r\omega \sin(90°- \phi)$, which is the same as the result in the last paragraph, because $r = R$. Note that the vector product is itself a vector; the vector product is arranged so that its direction and sense correctly describe the velocity of the given point on the Earth's surface. Note also that $\boldsymbol{v}$ is normal to both $\boldsymbol{\Omega}$ and $\boldsymbol{r}$; that is one of the properties of the cross product.

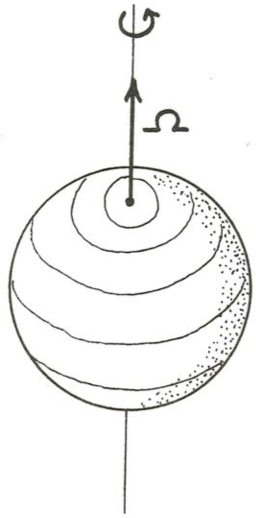

Figure 7-5. The angular velocity vector of the rotating Earth.

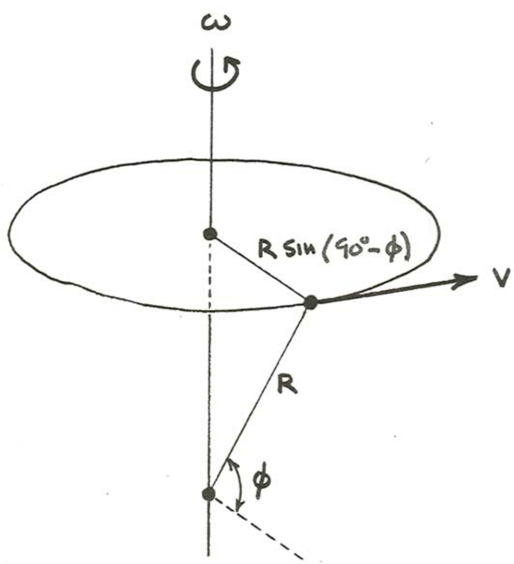

Figure 7-6. Definition sketch for writing the velocity of a point in or on the Earth in terms of the radius of the Earth, the angular velocity of the Earth, and the latitude of the point.

17 Now look at a little marker particle moving along with the air of the atmosphere or the water of the ocean. That particle has its own velocity relative to the solid Earth; call that velocity $\boldsymbol{v_R}$, where the subscript R is meant to suggest that the velocity is *relative* to the rotating Earth. The motion of the particle can also be viewed from outer space; call its velocity relative to that "fixed" frame of reference $\boldsymbol{v_I}$, where the subscript I stands for *inertial*, an adjective that in physics is associated with a frame of reference that is not accelerating. It should make good sense to you that

$$\boldsymbol{v_I} = \boldsymbol{v_R} + \boldsymbol{\Omega} \times \boldsymbol{r} \tag{7.1}$$

Equation 7.1 just tells you that the absolute velocity of the moving particle is the sum of its velocity relative to the rotating Earth plus the rotational velocity of the point, stationary relative to the Earth, past which the particle happens to be moving at a given time.

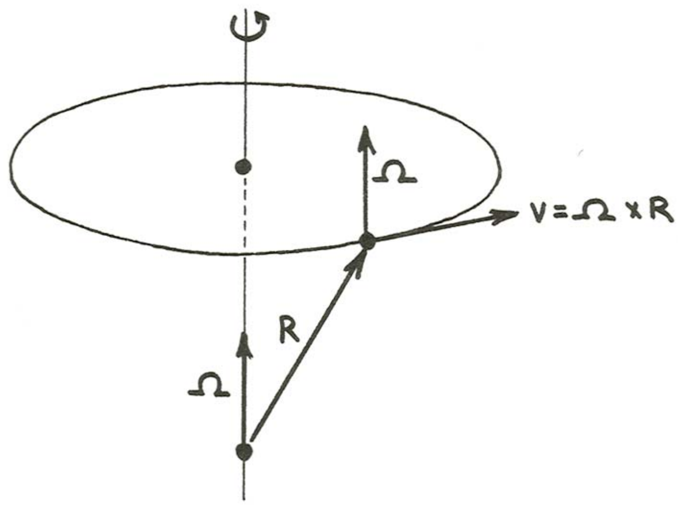

Figure 7-7. Another way of expressing the velocity of a point in or on the Earth.

18 The complications begin when we look at the *acceleration* of the marker particle, not just its velocity. The acceleration of the particle is the time rate of change of its velocity. To find the acceleration you have to differentiate the vector $\boldsymbol{v_I}$ with respect to time, and then, for use of the result in our rotating earthly frame of reference, express the result in terms of quantities like $\boldsymbol{v_R}$ that are observed from within that rotating frame of reference. I will just cite the result for $\boldsymbol{a_I}$, the acceleration of the particle relative to the outer-space reference frame:

$$\boldsymbol{a_I} = \boldsymbol{a_R} + 2\boldsymbol{\Omega} \times \boldsymbol{v_R} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r}) + \frac{d\boldsymbol{\Omega}}{dt} \times \boldsymbol{r} \tag{7.2}$$

where $\boldsymbol{a_R}$ is the acceleration of the particle as observed from the rotating Earth. (For details, see Pedlosky, 1987, Chapter 1.)

19 There are four terms on the right side of Equation 7.2. The first, $\boldsymbol{a_R}$, is easy to understand, but the other three need some explanation. The fourth expresses the effect of *time rate of change of the rotation rate*; it is something we need not worry about for geophysical flows. The third is the *centripetal acceleration*, with which you are all familiar in at least a qualitative way: if you tie a rope around a boulder and swing it around in a big circle, you are producing a centripetal ("center-seeking") acceleration of the boulder, arising from the inward-directed radial force you are exerting on the boulder to constrain it to travel in a circular arc rather than go off straight on its own. The second term is the one we are after in this exercise: it is the *Coriolis acceleration*.

20 Look more closely at the Coriolis acceleration term in Equation 7.2. First of all, it is a vector itself, because it is a cross product of two vectors. Its magnitude is a linear function of the magnitude of the velocity $\boldsymbol{v_R}$: the faster the particle moves, the larger the Coriolis acceleration. Also, its magnitude depends not only on the magnitude of $\boldsymbol{v_R}$ but also on the direction of $\boldsymbol{v_R}$ relative to the Earth's axis. In every case, though, the direction of the Coriolis acceleration is normal to $\boldsymbol{v_R}$ itself (remember the properties of the cross product), and that is consistent with what you learned about the Coriolis acceleration on your big turntable.

21 A good way to study the effects of the Coriolis acceleration on fluids moving at the Earth's surface is to look at a particular point on the Earth's surface at a particular latitude ϕ (Figure 7-8). To a local observer the situation there looks planar, so think about a plane that is tangent to the Earth's surface at the given point. I will call that the ***horizontal plane***, because it is horizontal at the tangent point. The motions of both the atmosphere and the oceans most affected by the Coriolis acceleration are almost entirely horizontal flows: broad-scale vertical motions in the atmosphere and oceans are usually of far smaller velocity than horizontal motions, and locally strong vertical motions, as within cloud convection cells, are on space and time scales for which the Coriolis effect turns out to be unimportant.

22 In the horizontal plane it is natural to set up a coordinate system with one axis vertical, one in the direction of movement of the particle, and the other horizontal and normal to the direction of movement (Figure 7-9). And the Coriolis acceleration itself can be resolved into components in those three directions (Figure 7-10). Think just about the horizontal components of the Coriolis acceleration, because they are what are important for the horizontal motions of fluids; it is easy to show that the vertical component of the Coriolis force in a vertical flow is swamped by the balance between the two large vertically acting forces—pressure and gravity—and is therefore negligible.

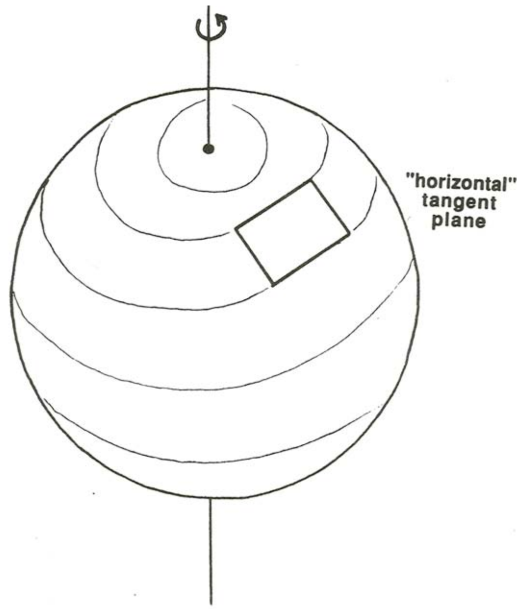

Figure 7-8. The "horizontal" tangent plane.

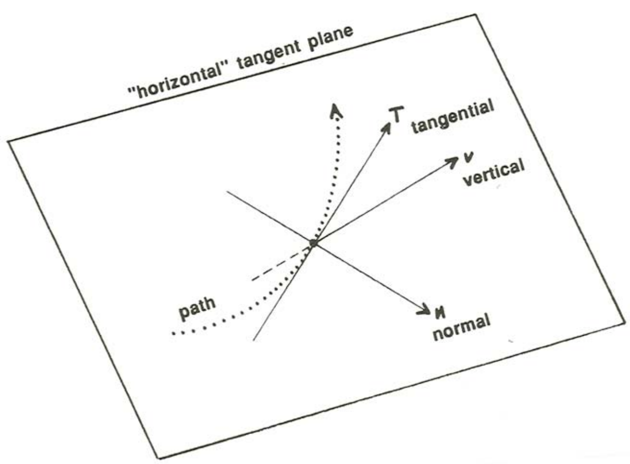

Figure 7-9. A useful coordinate system in the horizontal tangent plane.

23 You have already seen that the component of the Coriolis acceleration in the direction of movement, the *t* axis in Figure 7-10, is always zero. What we need to worry about is the magnitude of the horizontal component of the Coriolis acceleration in the direction normal to the horizontal movement, and how it varies with both latitude and the direction of movement within the horizontal plane. I will do this in a shortcut way by looking at two special directions, north–south and east–west.

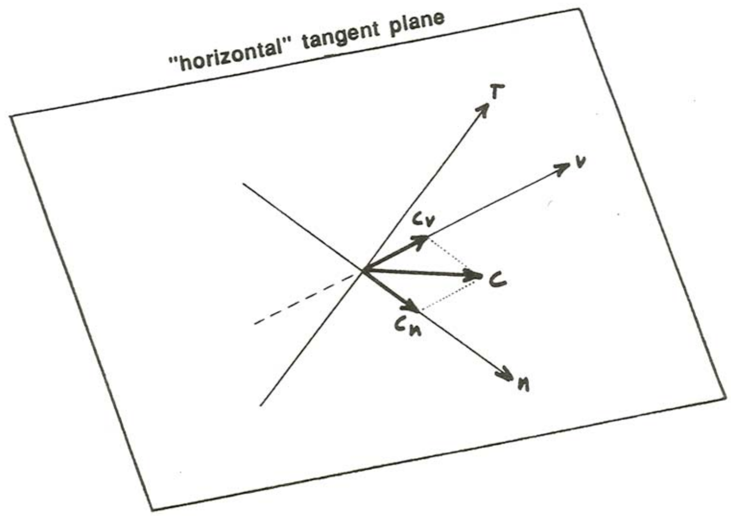

Figure 7-10. Resolving the Coriolis acceleration in the horizontal tangent plane.

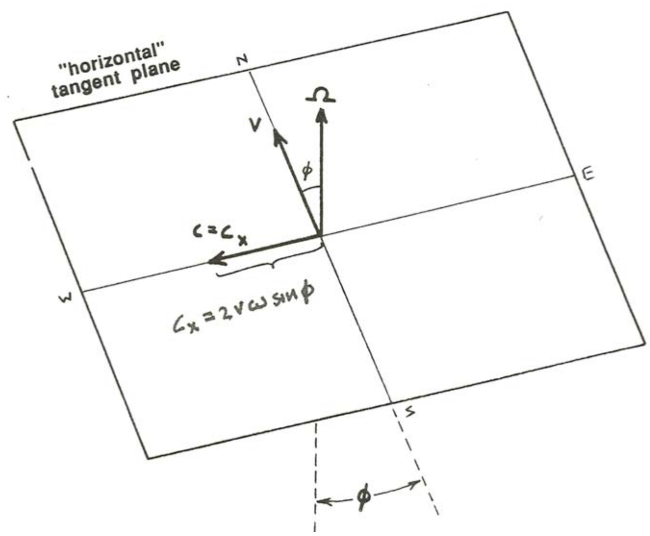

Figure 7-11. The component of the Coriolis acceleration in the east–west orientation in the horizontal tangent plane.

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24 Look at the east–west orientation first. The Coriolis acceleration is directed normal to both ***v*** and ***Ω***, which to an observer on the horizontal plane is a vector that sticks up into the sky toward the south and at an angle to the horizontal that is equal to 90° minus the latitude angle ϕ. By reference to Figure 7-11, if you resolve that vector into the horizontal plane the component has the magnitude $2v\omega\sin\phi$. Now look at the north–south orientation. To an observer on the horizontal plane the Coriolis acceleration vector is horizontal and points due east for a northward movement and due west for a southward movement. By reference to Figure 7-12, the magnitude of the Coriolis acceleration is again $2v\omega\sin\phi$. Although the mathematics is much more intricate, it turns out that the Coriolis acceleration has this same magnitude, $2v\omega\sin\phi$, for any direction of movement in the horizontal plane.

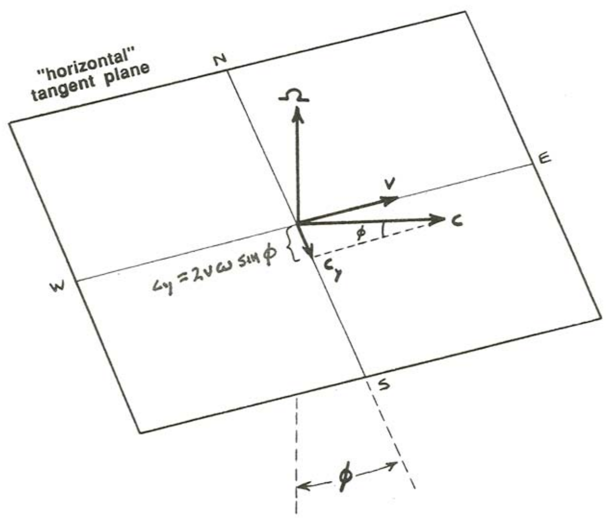

Figure 7-12. The component of the Coriolis acceleration in the north–south orientation in the horizontal tangent plane.

25 So the bottom line on the Coriolis acceleration for horizontal movements on the Earth's surface is this: (1) its magnitude is directly proportional to the speed of the body, (2) its horizontal component is always directed normal to the direction of movement (to the right in the Northern Hemisphere but to the left in the Southern Hemisphere), and (3) its magnitude is always $2v\omega\sin\phi$. So the Coriolis effect is largest at the poles and zero at the Equator. (It works just the opposite way for the *vertical* component of the Coriolis acceleration—largest at the Equator—but, as I said before, the vertical component is unimportant anyway.) What is usually done with this expression for the Coriolis acceleration is to extract the $2\omega\sin\phi$ part and call it the ***Coriolis parameter***, denoted by f.

26 Finally, I want to make sure that you do not confuse the separate effects of the centripetal acceleration and the Coriolis acceleration, both of which affect the motion of bodies on the Earth's surface as viewed by an observer on the Earth. The centripetal acceleration affects all bodies in and on the Earth, whether they are stationary or moving. To reveal its effect, go back to your turntable and place a big shallow pan of water on it. You know what happens when the table is rotated: the water banks up against the outer side of the pan, and the inward-sloping water surface sets up a horizontal component of the gravity force that counterbalances the outward centrifugal force. After the initial adjustment in water-surface slope is made, the centrifugal force has no further direct effect on the movement of the water relative to an observer riding on the turntable. (Lakes and oceans on Earth adjust in this same way—as does the solid Earth itself. It is the reason why the Earth is an oblate spheroid, with equatorial diameter larger than pole-to-pole diameter, rather than a sphere.) Because the velocity of the water relative to the boundaries of the pan is zero, there is no Coriolis acceleration.

27 Now move your hand in the water to produce a gentle current in the interior of the water in the pan. Of course the pattern of flow will be modified by friction and eventually die away, but while the water is moving it experiences the Coriolis acceleration, and the configuration of the current is more or less strongly affected, depending upon the speed of the water and the rate of rotation of the turntable. It is these effects in the atmosphere and oceans that we are going to investigate in the rest of this chapter.

THE ROSSBY NUMBER

28 How can we gain a general idea about whether the motion of a fluid or a solid object on or near the Earth's surface would manifest non-negligibly the Coriolis effect? The answer lies in a dimensionless parameter called the Rossby number. Any such motion, whether it is a flow of a fluid or the flight of a bullet or an artillery shell or a rocket, has some characteristic speed U and moves over some characteristic distance L. Depending upon the latitude, there is some particular value of the Coriolis parameter, ranging from zero at the Equator to a maximum at the poles. The essential idea is this: how long does it take for the material to make its trip, in comparison to how much the Earth rotates under the material while it is making its trip?

29 The only way to combine the three variables U with dimensions of length/time), L (with dimensions of length), and f (with dimensions of 1/length) is U/fL, which is one form of what is called the ***Rossby number***. You can think of the Rossby number as the ratio of two velocities, because the combination fL has the dimensions of a velocity. In the case of the ocean current, moving at a few centimeters per second over a distance of, say, a thousand kilometers, the Rossby number is very small; in the case of a speeding bullet, the velocity is very large and the distance of travel is no more than something like a thousand meters, the Rossby number is very large. In other words, the Earth rotates quite a lot in the time it takes for the ocean current to move from place to place, so the effect of the Earth's rotation on the movement of the fluid is great. By contrast, the Earth rotates very little in the time it takes the bullet to travel from the rifle to the target, so the Coriolis acceleration is negligible.

INERTIA CURRENTS

30 I mentioned earlier that you might generate a current in the interior of the big pan on your turntable and then study the effect of the Coriolis acceleration on that current. If the pan is broad enough and deep enough, the mass of water you set in motion can move for quite some time and distance just by its own inertia before being brought to a stop by friction forces exerted by the adjacent fluid. Currents of this kind are called ***inertia currents***. In nature they might be produced by passage of a sudden squall or a fast-moving windstorm over a large body of water.

31 Consider a small parcel of water anywhere in your inertia current. Your first thought might be that, because no forces are acting on it, it would move in a straight line at constant speed—and that would be true, if your turntable were not rotating. But remember that, from the standpoint of an observer on the turntable, the rotation of the turntable results in a Coriolis acceleration, so you have to deal with a force per unit mass of fluid equal to fv, the Coriolis acceleration (v is the velocity and f is the Coriolis parameter), acting at right angles to the direction of movement. If the turntable is rotating counterclockwise as viewed from above, then the Coriolis force acts to the right of the direction of movement. That sense of rotation is the same as in the Northern Hemisphere on the Earth. If the turntable is rotating clockwise, then the Coriolis force acts to the left of the direction of movement, as in the Southern Hemisphere.

32 The dynamicist's approach to the problem of how the water in the inertia current moves as a function of time is to derive the governing equations of motion, including pressure forces, viscous forces, gravity forces, and Coriolis forces, and then specialize the equations for the particular flow, making any judicious simplifications necessary for good mathematical progress. We will not go through such an exercise here. For inertia currents the equations simplify very nicely, just because we are assuming that the only force we have to deal with is the Coriolis force.

33 When written in an xyz coordinate system with reference to the horizontal plane at some point on the Earth (z being vertical, x being east, and y being north), the two horizontal equations come out to be just

$$\frac{du}{dt} = 2v\omega\sin\phi$$

$$\frac{dv}{dt} = -2u\omega\sin\phi \qquad (7.3)$$

or, using the notation for the Coriolis parameter,

$$\frac{du}{dt} = fv$$

$$\frac{dv}{dt} = -fu \qquad (7.4)$$

where u and v are the x and y components, respectively, of velocity of the fluid relative to the horizontal plane defined above.

34 Do not worry about the signs in front of the Coriolis terms; they come about when the x and y components of the Coriolis acceleration are derived for the horizontal plane. This is a simple set of equations. Its solution, for an initial condition that $u = U$ and $v = 0$ at $t = 0$, is

$$\begin{aligned} u &= U \cos ft \\ v &= -U \sin ft \end{aligned} \tag{7.5}$$

35 If you go back to your storehouse of mathematical knowledge you can easily demonstrate to yourself that this solution represents motion around a circle at constant speed, with the time t as a parameter. For circular motion like this, the radius of the circle is just U/f, and the time it takes a particle to go all the way around the circle is $2\pi/f$. This circle is called an ***inertia circle***, and the period is called the ***inertia period***.

36 That the water in an inertia current in a rotating system moves in circles does not seem intuitively obvious (at least not to me!). Keep in mind here that the situation with the inertia current is not the same as with the rolling ball. The inertia current acts in a body of water that is moving around with the turntable, whereas the rolling ball moved in a straight line relative to the fixed stars and has no connection with the turntable itself.

37 At first thought you might guess that the inertia period is the same as the period of rotation of the turntable itself. But you would be wrong! Call the period of rotation T_r, and the inertia period T_I. I told you above that

$$\begin{aligned} T_I &= 2\pi/f \\ &= 2\pi/2\omega \sin\phi \\ &= \pi/\omega \sin\phi \end{aligned} \tag{7.6}$$

Let me remind you that in any rotatory motion the relationship between the angular velocity and the period is $\omega T_r = 2\pi$, or $T_r = 2\pi/\omega$, or $\omega = 2\pi/T_r$. Substituting this into Equation 7.6,

$$T_I = \frac{T_r}{2 \sin \phi} \tag{7.7}$$

An interesting result, no? The inertia period is *not* the same as the rotation period. For your turntable, which acts like a horizontal plane at the North Pole, where the latitude is 90°, the inertia period is exactly *one-half* the rotation period.

38 Just to give you some feel for how large the inertia circle would be on your turntable and on the real Earth, here are some simple examples. Suppose your turntable is rotating with a period of 100 s—slow enough so that you would not have any trouble staying on board, and you probably would not develop motion sickness either. An inertia current of 1 cm/s in your big pan would have an inertia circle radius of 8 cm, and a current of 10 cm/s would have an inertia circle radius of 80 cm. A current of any speed would have an inertia period of 50 s. So you would actually be able to observe the inertia circles, if the pan is large enough and the current velocity is small enough.

39 At the North Pole the inertia period is 12 hours, and it increases (not linearly!) with decreasing latitude, going to infinity at the Equator. At latitude 45° it is about 17.5 hours, and at latitude 30° it is about 24 hours. At latitude 45° the radius of the inertia circle is about 1 km for an inertial current of 10 cm/s, and about 10 km for an inertial current of 1 m/s.

40 Figure 7-13 shows the track of a tracer in an inertia current that was measured over a period of about a week in the Baltic Sea (Gustafson and Kullenberg, 1933). The periods of the loops match the theoretical inertia period for that latitude very closely. There is net translation of the marker because the inertia current was superimposed on a larger-scale current of some other kind. Note that the size of the inertia circles decreases with time, presumably because the current slowed down because of friction.

THE EKMAN SPIRAL

41 A wind blowing over a water surface exerts a force on the surface, and that force tends to drag or push the water in the direction of the wind. Surface currents of this kind are called ***pure drift currents***. This is in addition to the more readily observable effect of generation of surface waves, discussed in Chapter 6.

42 Your intuition tells you that the wind-driven current is in the direction of the wind, and that its effect decreases downward from the surface. In a nonrotating system both of these suppositions are true, but, from what has been said already above, you should suspect that the Coriolis acceleration complicates matters in a rotating system. This section deals briefly with some of the intricacies of the Coriolis effect on wind-driven currents.

43 What can we deduce about the current, without actually solving the equations of motion? First of all, it should be clear, even without worrying about change in direction with depth, that the speed of the current should decrease downward because of frictional retardation, for just the same reasons as did the fluid contained between a stationary lower plate and a moving upper plate considered way back in Chapter 1.

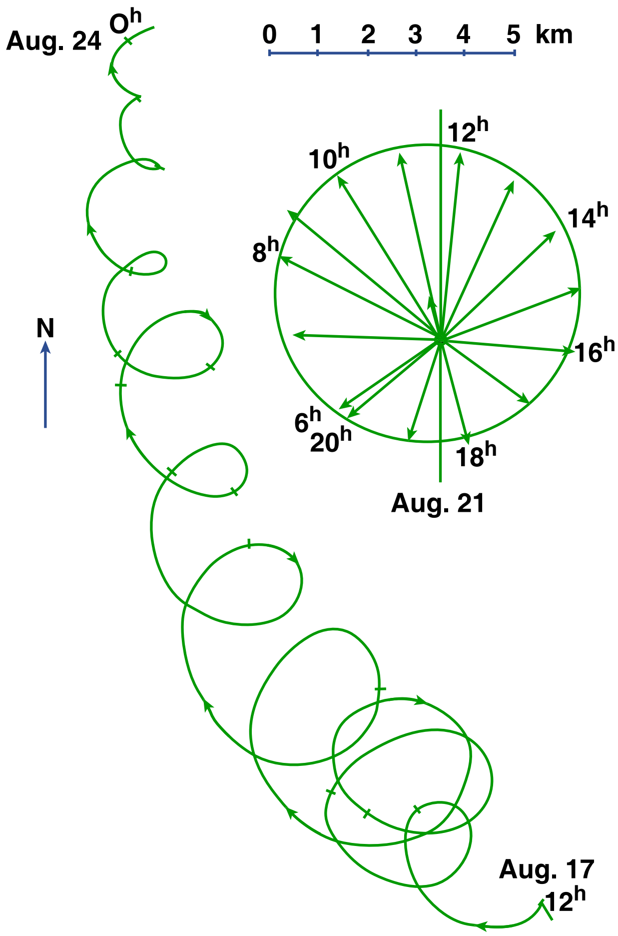

Figure by MIT OpenCourseWare.

Figure 7-13. Inertia circles in the Baltic Sea. (From Neumann and Pierson, 1966.)

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44 But what about the *direction* of the current? Think about the balance of forces on a small parcel of fluid at the water surface (Figure 7-14). The surface water feels not only the wind force, in the direction of the wind, but also the Coriolis force, at right angles to the direction of water motion, and a friction force exerted by the slower water below, in the direction opposite to the surface water motion. Because the parcel of surface water is not accelerating, these three forces have to add up to zero. From Figure 7-14 you can see that the only way you can balance these three forces is for the surface water to move at some angle between 0° and 90° to one side of the wind direction—to the right of the wind in the Northern Hemisphere and to the left of the wind in the Southern Hemisphere.

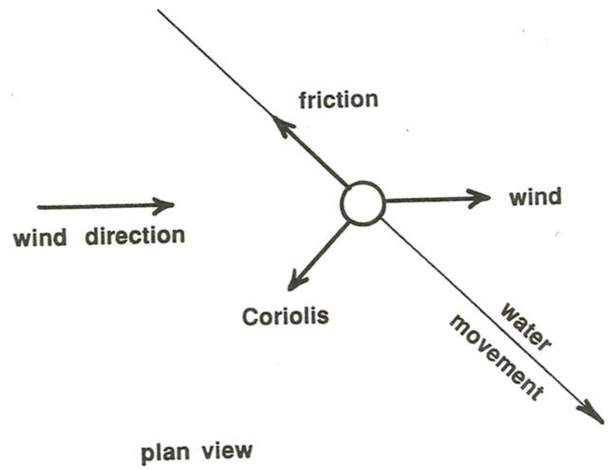

Figure 7-14. Balance of forces on a surface water element under a wind.

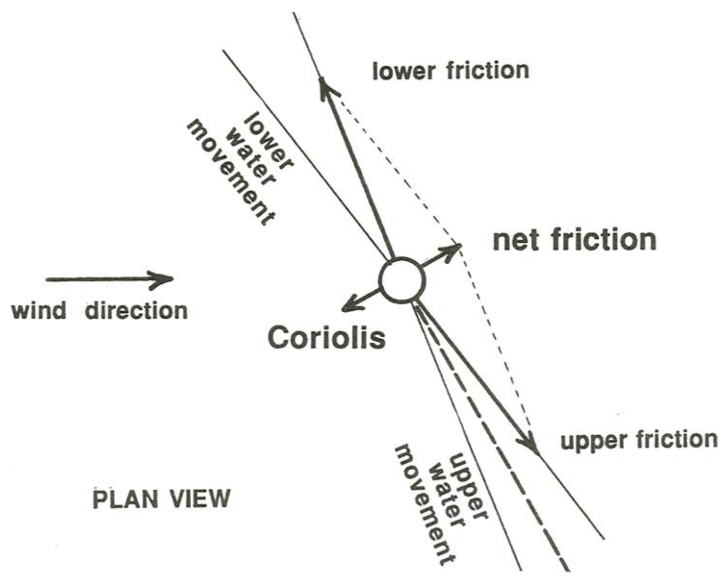

Figure 7-15. Balance of forces on a subsurface water element under a wind.

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45 Now look at a slightly lower layer of water. A frictional force is exerted on its upper surface by the overlying layer, and a frictional force is exerted on its lower surface by the underlying layer. The same line of reasoning we used in Chapter 1 for flow between parallel plates moving relative to each other suggests that these two frictional forces are of the same magnitude but in different directions. But there has to be some left-over net friction force, because that is the only force that is available to balance the Coriolis force. For there to be a friction force directed opposite to the Coriolis force, the velocity vector must be turning continuously away from the wind direction with depth! See Figure 7-15 for how this works. And so on for each successively deeper layer: each lower layer of water moves in a direction farther to the right (or left) of the surface wind direction.

46 The only trouble with the argument in the last two paragraphs is that it is discrete rather than continuous. To solve the problem right, we would have to write the differential equations of motion, subject to the appropriate boundary conditions, and solve them for the vertical distribution of current speed and direction, both of which must of course vary continuously with depth. But the above argument gives you the qualitative physical essence of what is happening.

47 This problem was first solved analytically by Sven Ekman, a Norwegian physical oceanographer, in 1905. The assumptions behind the solution are that the surface wind is uniform in both speed and direction everywhere and that the ocean is of infinite extent horizontally, so that water does not get piled up in the downwind direction to create an extra force in the form of a horizontal pressure gradient and complicate matters, and also that the ocean is infinitely deep. In case you want to play around with the solution, here it is:

$$\begin{aligned} u &= U_0\, e^{-(\pi/D)z} \cos[(\pi/4) - (\pi/D)z] \\ v &= U_0\, e^{-(\pi/D)z} \sin[(\pi/4) - (\pi/D)z] \end{aligned} \tag{7.8}$$

where v is the velocity component at any depth in the wind direction, u is the velocity component to the right of the wind direction, z is depth below the water surface, U_0 is the water speed at the surface, and D is a parameter that contains the water density ρ, the Coriolis parameter f, and a viscosity coefficient A:

$$D = \pi\sqrt{\frac{A}{\rho f}} \tag{7.9}$$

48 What Equations 7.8 tell you is that at the water surface the velocity is directed at exactly 45° to the right of the wind, and with depth the direction of the velocity swings farther and farther to the right while the speed falls off exponentially. Figures 7-16A and 7-16B show two graphical views of the velocity profile with depth. You can see from these figures why the solution is called the ***Ekman spiral***!

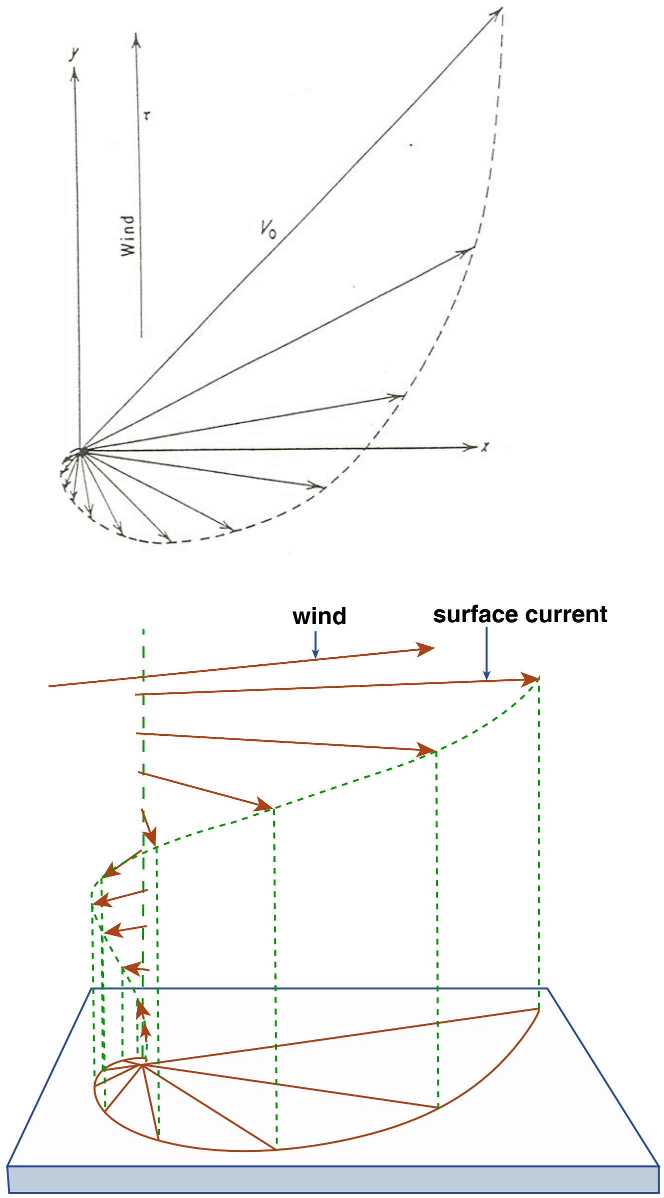

Figure by MIT OpenCourseWare.

Figure 7-16. **A)** The Ekman spiral. (From Neumann and Pierson, 1966.) **B)** A three-dimensional view of the Ekman spiral. (From Gross, 1990.)

49 Here are some interesting properties of Equations 7.8. If you substitute $z = D$ into the equations, you find that at depth D the velocity components are

$$u = U_0 e^{-\pi} \cos(\pi/4 - \pi)$$
$$v = U_0 e^{-\pi} \sin(\pi/4 - \pi) \qquad (7.10)$$

which tell you that at depth D the speed has decreased to $e^{-\pi}$ times the surface velocity (which is less than five percent), and the direction is *opposite* to the surface current! One can also find the total mass transport of water involved in this Ekman-spiral current, by multiplying the velocity components u and v by the water density ρ and integrating Equations 7.8 over the depth from 0 to ∞. The result is spectacular: *the net transport of water is directed at exactly 90° to the surface wind!* I suppose that will seem counterintuitive, but such are the mysteries of the Coriolis effect.

50 It has been difficult for oceanographers to apply or verify Ekman's results, for three reasons. First, in the real ocean it is difficult to find pure drift currents that satisfy all of Ekman's assumptions. Second, there is a great difficulty in figuring out what to use for the eddy viscosity A. It certainly should not be the molecular viscosity, because the flow in the surface layer is hardly laminar, given all the agitation by wave motions; there must be appreciable vertical transport of horizontal fluid momentum by turbulence. Third, there is the practical difficulty of measuring the time-average velocity components at a point below the time-average water surface, both because current meters have a difficult time with wave–current flows and because it is not easy to establish and then maintain the right water depth, with the sea surface moving around so much. But it is clear that pure drift currents in the real ocean do indeed work much as predicted by Ekman, at least qualitatively.

GEOSTROPHIC MOTION

51 So far I have discussed some oceanic consequences of the Coriolis effect. Although important, these are on fairly local scales. But the Coriolis effect also plays a striking and fundamentally important role in the dynamics of the large-scale currents and circulations in both the oceans and the atmosphere.

52 Here is the background you need to know. All of the large-scale motions of the oceans and atmosphere, of the kind you would see on a weather map of North America or a chart of North Atlantic currents, owe their existence to *horizontal pressure gradients*: changes in pressure from place to place when viewed at the same altitude (in the atmosphere) or the same depth (in the oceans).

53 This is not the place to describe in much detail how these horizontal pressure gradients come about; I hope it will suffice to say that in the atmosphere they arise from differential heating and cooling and the resulting expansion and contraction of the atmosphere, and in the oceans they arise from a number of effects, including large-scale differences in temperature and salinity and also the horizontal movement and "piling up" of surface waters in response to winds.

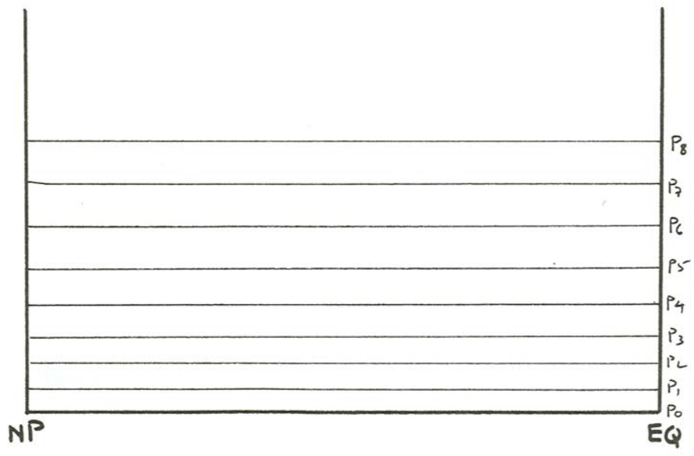

Figure 7-17. North–south slice through a hypothetical atmosphere, before the action. NP, North Pole; EQ, Equator.

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54 Just to give you some feeling for the origin of horizontal pressure differences in the atmosphere, think about a hypothetical convection cell, one that is greatly oversimplified but fundamentally representative of what really happens in the atmosphere on a large scale. Unfortunately this is not something I can expect you to build in your back yard—although essentially the same thing happens in a big tank of differentially heated and cooled water. Figure 7-17 shows a gigantic north–south slice through a hypothetical atmosphere. Suppose that, before convection begins, the atmosphere is at the same temperature everywhere at any given altitude. Because air density is a function of temperature, and air pressure is a function of how the air density varies with altitude above the given altitude level, the pressure is the same everywhere at the given altitude, as shown by the horizontal lines in Figure 7-17, which represent the intersections of horizontal planes with surfaces of equal pressure (called ***isobaric surfaces***).

55 To get the convection cell going, suppose that the atmosphere is heated at low latitudes, near the Equator, and cooled at high latitudes, near the North Pole. At low latitudes the entire column of the atmosphere expands upward, and at high latitudes the entire column of the atmosphere contracts downward, in response to the change in temperature and therefore in density. In response to this expansion and contraction, the isobaric surfaces everywhere above the ground surface take on a poleward slope, as shown in Figure 7-18. The effect is a movement of air from low latitudes to high latitudes, as can be seen by considering how the pressure varies across some arbitrary horizontal surface in the atmosphere, shown by the dashed line in Figure 7-18. Because of the way the isobaric surfaces cut the horizontal surface, there is a south-to-north decrease in pressure along the horizontal surface, and this pressure gradient causes south-to-north air movement.

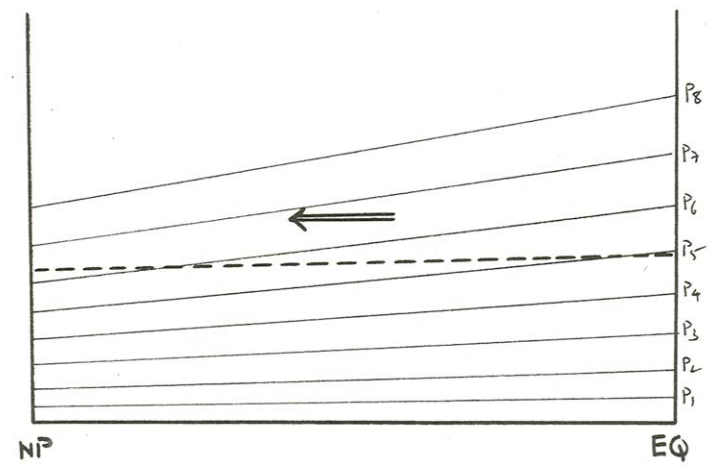

Figure 7-18. North–south slice through the hypothetical atmosphere, after heating and cooling begins.

56 The south-to-north air movement results in a greater total mass of air in an atmospheric column near the North Pole than in an atmospheric column near the Equator. At and near the Earth's surface, therefore, the atmospheric pressure is greater near the North Pole than near the Equator, so along some horizontal surface low in the atmosphere the horizontal pressure gradient is north-to-south rather than south-to-north. The equilibrium pattern of convective motion then shows (Figure 7-19) northward flow in the upper atmosphere, where the horizontal pressure gradient is to the north, and southward flow in the lower atmosphere, where the pressure gradient is to the south. At some intermediate level, the isobaric surfaces are horizontal, and there is no horizontal motion.

57 The foregoing exercise is meant to show how the actual motions in familiar thermal convection cells are qualitatively understandable responses to horizontal pressure gradients set up by differential heating and cooling and consequent expansion and contraction. What I want you to carry away from this example is the idea that there are always going to be horizontal pressure gradients in the atmosphere, which tend to generate winds. (Currents in the deep ocean are generated by such pressure gradients as well.) Now we have to see how the Coriolis acceleration affects these pressure-gradient-generated winds.

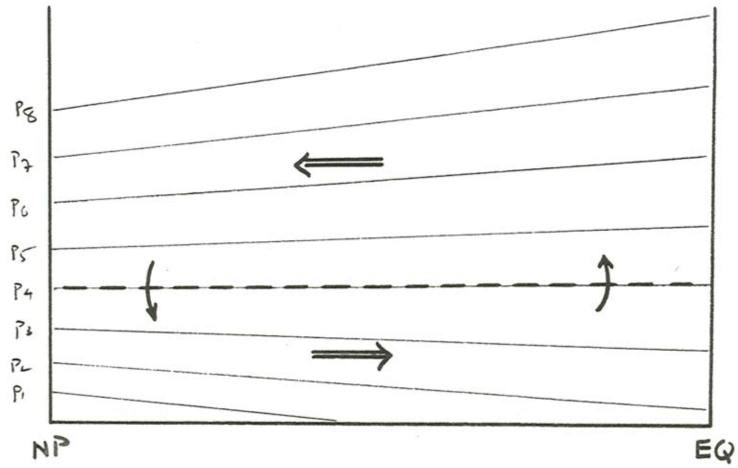

Figure 7-19. Circulation produced in the hypothetical atmosphere by heating and cooling.

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58 I said in an earlier section that the motions of the atmosphere and oceans on the Earth's surface are dominantly horizontal on a large scale (keep in mind that the slopes of the isobaric surfaces in Figure 7-19 are greatly exaggerated) and that only the horizontal component of the Coriolis acceleration has to be considered in these horizontal motions. Think about the fundamental balance of forces that govern these motions. To do a complete job of this we would have to pick apart the governing equations of motion in some detail, but the important effects should make good sense to you without that.

59 First of all, just as with flow in pipes and channels (Chapter 4), we can think about the balance of forces in the plane parallel to the solid boundary, to which the motions are parallel, and be content in the knowledge that in the normal-to-boundary direction the equation of motion boils down to a balance between the downward weight of the fluid and the upward pressure gradient, as a manifestation of what I called the *hydrostatic balance* in Chapter 1.

60 Which horizontal forces need we take into account? The candidates are five: *pressure*, *friction*, *gravity*, *Coriolis*, and (if the wind blows in a horizontally curved path) *centrifugal force*. By what was said in the last paragraph, gravity need not be included. And the centrifugal force is really important only in tightly curving winds, as in tornadoes; even around the eye of a hurricane it is not the dominant effect, although it is not negligible either. That leaves pressure, friction, and Coriolis.

61 I hope it will make sense to you when I claim that, well above the layer of the atmosphere that is in the immediate vicinity of the surface, frictional effects should be very small compared with the other forces. This is indeed the case, as you will see from the outcome of the present line of argument later, but it is not easy to justify. Then the dominant balance of forces is just between the pressure gradient and the Coriolis force. It is this force balance that has such far-reaching consequences for the nature of atmospheric and oceanic motions.

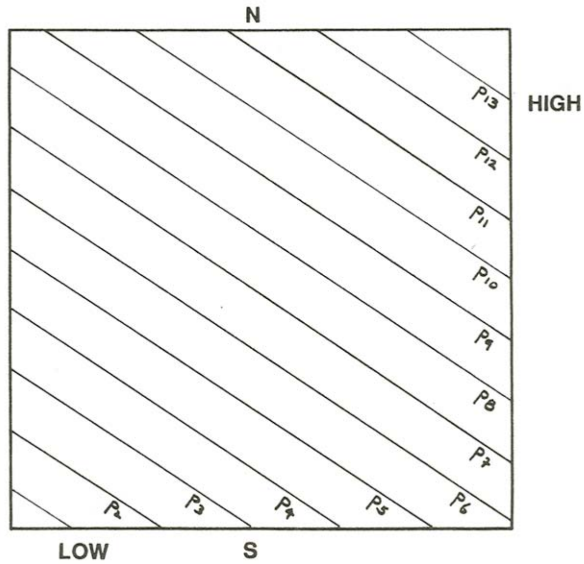

Figure 7-20. Plan view of a large area of the atmosphere in which there is a horizontal pressure gradient.

62 Think about the direction of the wind, relative to the pressure gradient, when the air is moving under the influence of a balance between the pressure-gradient force and the Coriolis force. Use Figure 7-20, which is a plan view of some large area over which a horizontal pressure gradient in the atmosphere acts, as a guide. In Figure 7-20 the light lines represent the intersections of the isobaric surfaces with a horizontal plane at some fixed altitude at which we are considering the atmospheric motion; these lines, called ***isobars***, are exactly the same as the those they used to show on the weather maps in the newspapers and on television.

63 The pressure-gradient force acts at right angles to the isobars, in the direction of decreasing pressure. So (Figure 7-21) in the absence of Coriolis effects the wind should blow straight across the isobars. But on our rotating Earth (Figure 7-22), because the Coriolis force always acts at right angles to the direction of motion, the only way there can be a balance between the pressure gradient and the Coriolis is for the wind to blow *along* the isobars, not *across* them!

64 If the startling conclusion in the last paragraph seems fishy to you, or you do not fully understand what is going on, try drawing the balance of the two forces when a parcel of air is moving horizontally in some arbitrary direction other than parallel to the isobars (Figure 7-23). You would see that the resultant of the two forces always has a component off to one side of the assumed direction of motion, and the direction of that component is such as to swing the trajectory of the air lump closer to being along the isobars. There can be no balance until those two force vectors act in directions exactly opposite to each other, and that direction is parallel to the isobars.

65 Planetary fluid motion of the kind analyzed above, for which the balance between the pressure-gradient force and the Coriolis force dictates a direction of movement along the isobars, is called ***geostrophic motion***, and the balance of forces itself is called the ***geostrophic balance***. It is of fundamental importance in both the atmosphere and the oceans. In fact, it seems safe to say that geostrophic motion is by far the most striking aspect of large-scale planetary fluid motions.

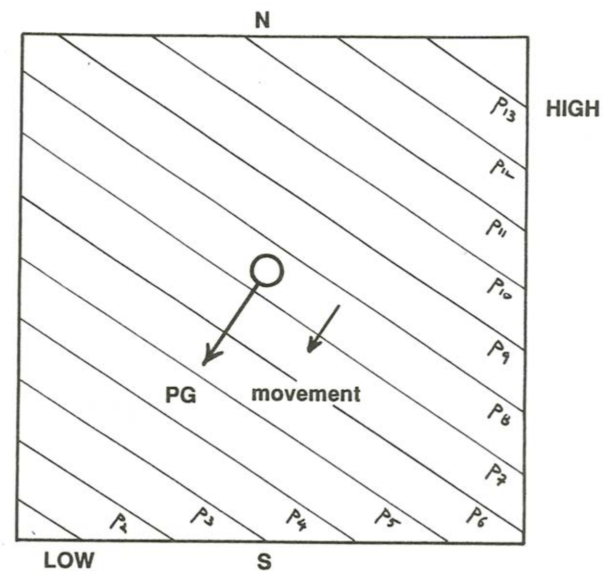

Figure 7-21. In the absence of Coriolis effects, the wind should blow across the isobars toward lower pressure.

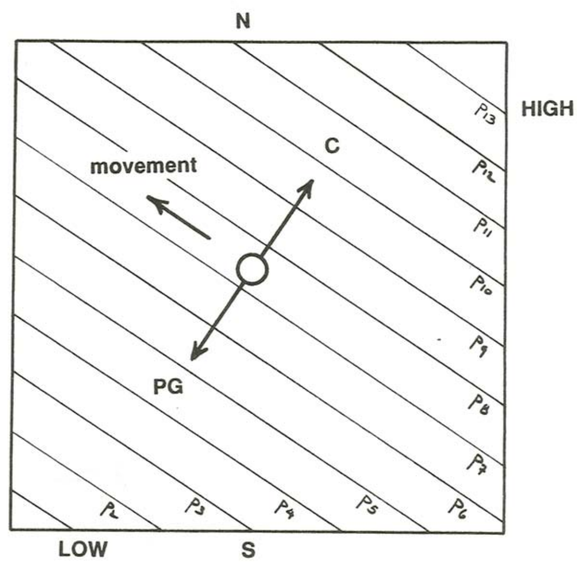

Figure 7-22. In the presence of Coriolis effects, the wind blows parallel to the isobars.

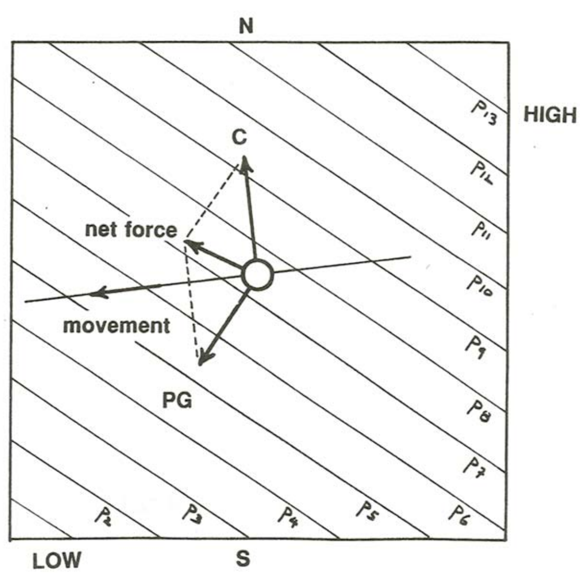

Figure 7-23. The net force on a parcel of air that is not moving parallel to the isobars is such as to cause its movement to become more nearly parallel to the isobars.

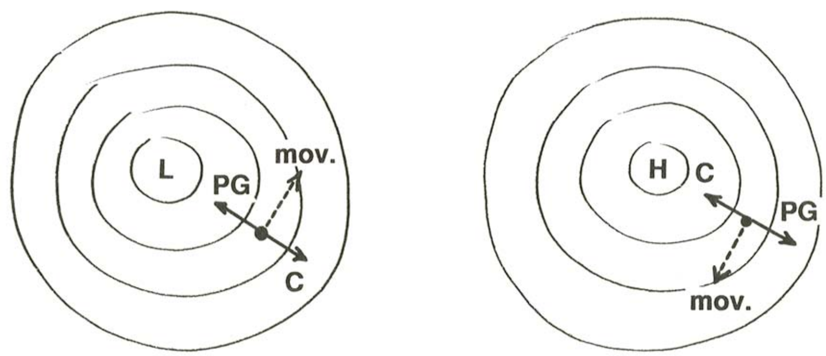

Figure 7-24. In the Northern Hemisphere, winds move counterclockwise around low-pressure area and clockwise around high-pressure areas.

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66 The reality of geostrophic motion is apparent from even a cursory glance at a "real" weather map (one that shows isobars): the winds are almost parallel to the isobars, and in such a sense that in the Northern Hemisphere if you look across the isobars down the pressure gradient, toward the region of low pressure, the wind blows from your left to your right. Another way of saying that is that (in the Northern Hemisphere) the winds move counterclockwise around areas of low pressure and clockwise around regions of high pressure (Figure 7-24).

EKMAN LAYERS

67 It might have occurred to you, the perceptive reader, that there has to be something more to the phenomenon of geostrophic motion than what I have shown above—because the air somehow has to get down the pressure gradient, or else the pressure gradients would keep on increasing. (Think back to the convection-cell example I presented earlier.) Somehow there has to be movement of the air across the isobars, as in the more direct flow that would be set up in the absence of rotation.

68 The way out of this dilemma is to include the effect of friction forces in the lowermost part of the atmosphere. Think about the friction forces on a small test parcel of air moving near the ground. The overlying and underlying layers of air exert friction forces on the top and bottom surfaces of the parcel, and these forces are in different directions: the lower layer exerts a force opposite to the direction of motion, and the upper layer exerts a force in the direction of motion. The key point (Figure 7-25) is that the upper of these shear forces is slightly smaller than the lower, because the shear stress decreases upward from a maximum at the ground, just as in open-channel flow we considered in Chapter 4. So the net friction force should act opposite to the direction of motion.

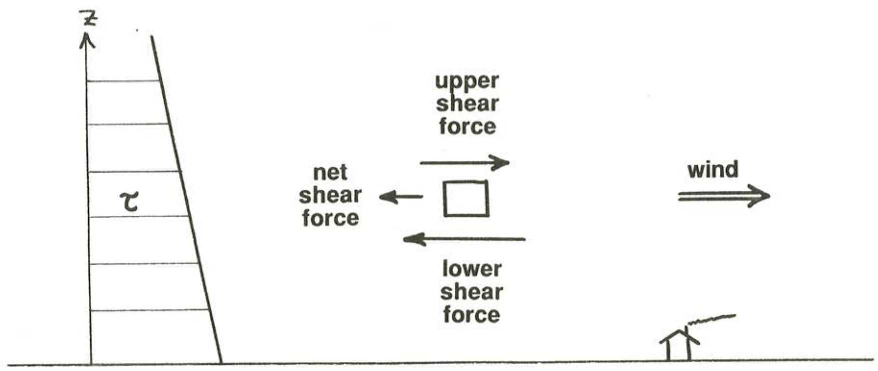

Figure 7-25. The net shear force on a parcel of air parcel moving near the Earth's surface or on a parcel of water moving near the ocean bottom. (The effect of the turning of the flow in the bottom Ekman layer is not taken into account in this approximate sketch; see the section on the bottom Ekman layer in the text for a more realistic account.)

69 Now a qualitative balance among the three forces (pressure gradient, large; Coriolis, large; friction, small but nonnegligible) shows (Figure 7-26) that the wind blows across the isobars at a small angle from high pressure to low pressure. This you can see clearly on any good weather map (of which the map in Figure 7-27 is a cartoon version) drawn for conditions at the surface, where the frictional influence is greatest: the air spirals inward toward the low-pressure area, there to have a small upward component to its motion and then flow outward at high altitudes in compensation for the inflow at low altitudes. The origin of the friction force shown in Figure 7-26 is of the same nature as was shown in Figure 7-15 for the surface Ekman layer: as the flow turns, the directions, as well as the magnitudes, of the friction forces on the top and on the bottom of a fluid element are slightly different, leading to a net friction force at a large angle to the direction of motion of the element.

70 The lowermost layer of the atmosphere or the ocean, adjacent to the land surface in the case of the atmosphere or to the ocean bottom in the case of the ocean, in which the direction of flow turns gradually from the direction of the overlying geostrophic flow, is called the ***Ekman layer*** (or the bottom Ekman layer). The mathematical solution for the structure of the Ekman layer was developed by Ekman in 1905. (See Pedlosky, 1979, Chapter 4, for a lucid derivation.) The Ekman layer is the region of the flow in which the frictional effect of the bottom boundary is significant. Downward through the Ekman layer, the magnitude of the velocity decreases from its geostrophic value to zero, by the no-slip condition, at the bottom and the direction of the velocity turns from the geostrophic direction to its maximum deviation at the bottom, theoretically at 45 degrees to the geostrophic direction—to the left in the Northern Hemisphere and to the right in the Southern Hemisphere. Figures 7-28 and 7-29 show two views of the horizontal velocity vector within the Ekman layer. You can see from Figure 7-28 that the velocity vector executes a spiral, similar to the Ekman spiral at the ocean surface under a wind, but not the same. Figure 7-28, in particular, shows clearly that the magnitude of the velocity reaches a maximum near the top of the Ekman layer before settling into its geostrophic value farther upward.

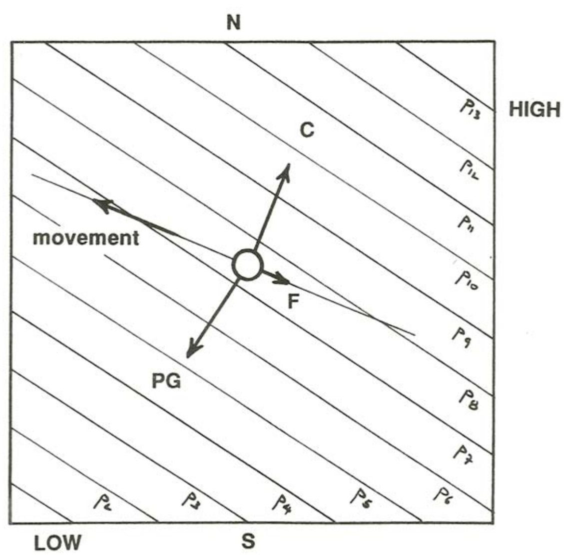

Figure 7-26. The effect of near-surface friction makes the wind blow across the isobars at a small angle toward lower pressure.

71 The thickness of the bottom Ekman layer should depend on the viscosity of the fluid and on the Coriolis parameter. These two factors work against one another: the greater the viscosity, the greater the thickness affected by friction, and the greater the Coriolis effect, as measured by the Coriolis parameter f, the less the thickness. The conventional measure of the Ekman-layer thickness is $[A_V/(f/2)]^{1/2}$, conventionally denoted by δ_E. (Do not be concerned with the reasons for the factor 2 in the denominator.) Upward in the Ekman layer the effect of friction tails off rapidly, but keep in mind that the upper limit is indefinite: the velocity decreases upward as a negative exponential function of height above the bottom boundary. It is best to view δ_E as *representative* of the thickness of the Ekman layer; in the parlance of geophysical fluid dynamics, the Ekman-layer thickness is of order δ_E. Finally, it will probably seem counterintuitive to you that the thickness of the Ekman layer does *not* depend on the geostrophic velocity.

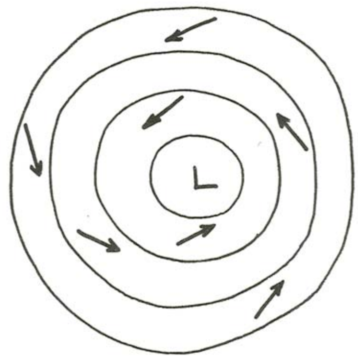

Figure 7-27. Cartoon weather map showing a low-pressure area, with winds blowing counterclockwise around the place of lower pressure but with a component spiraling inward across the isobars.

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72 As is so often the case, the value to use for the kinematic viscosity in the theory for the bottom Ekman layer is a problem. The molecular kinematic viscosity ν could be used, and that gives an exact solution for a laminar Ekman layer. The problem is that the mean-flow Reynolds numbers of large-scale flow in both the ocean and the atmosphere are so large that the Ekman layer must be turbulent rather than laminar. Alternatively, the eddy viscosity A_V can be used, but you have seen earlier that the eddy viscosity is itself a function of the flow, rather than a property of the fluid, and it might be expected to vary with height above the bottom boundary. With what seem to be reasonable values of eddy viscosity, the Ekman-layer thickness is on the order of some tens of meters, which is about what has been observed in the atmosphere and oceans.

PLANETARY BOUNDARY LAYERS: THE EKMAN LAYER, THE LOGARITHMIC LAYER, AND THE MIXED LAYER

Introduction

73 I am now going to subject you to a further set of flow layers, one of which, the Ekman layer, you have already encountered. You can think of this concluding section as a continuation of the material on boundary layers in Chapter 4, in the context of the large-scale bottom boundary layers in the atmosphere and in the oceans—which are called ***planetary boundary layers***. (Note on terminology: in the atmosphere, it is called the atmospheric boundary layer; in flow in the shallow ocean, on continental shelves, for example, it is often called the bottom boundary layer; in the deep ocean, it is commonly called the benthic boundary layer. They are all the same, in essential dynamics)

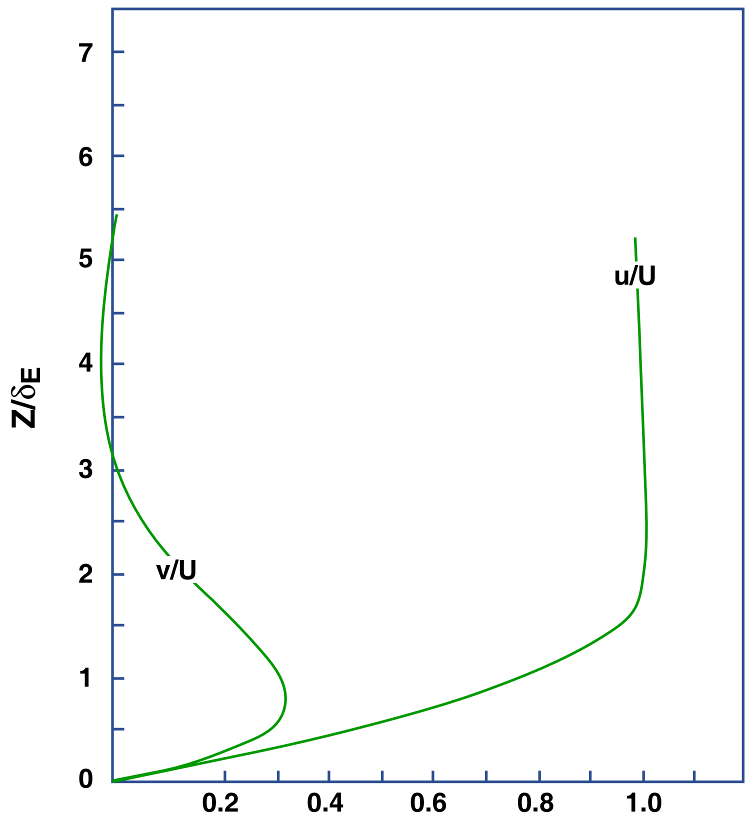

Figure by MIT OpenCourseWare.

Figure 7-28. The velocity in the bottom Ekman layer. The velocity *u* is the current speed in the direction of the overlying geostrophic flow, and the velocity *v* is the current speed in the direction normal to that. Both *u* and *v* are normalized by dividing by the geostrophic current speed *U*. The numbers associated with the velocity vectors are values of z/δ_E, where z is the height normal to the bottom and δ_E is the conventional Ekman-layer thickness. From Pedlosky (1979).

74 Given that the winds and the ocean bottom currents flow over a solid boundary, it is understandable that there must be a bottom boundary layer in those settings. All of what was said about boundary layers in Chapter 4 can be applied to these planetary boundary layers—but with the important additional effect of the Earth's rotation. In the previous section you learned about the Ekman layer, which, as a part of the planetary boundary layer, is an additional boundary-layer element brought about by the Coriolis effect.

75 Stable stratification is ubiquitous in the atmosphere and the oceans. By stable stratification I mean that the vertical profile of fluid density in the medium is such that if you were to capture a small parcel of the fluid and move it bodily upward, without allowing any exchange of thermal energy between the parcel and its surroundings (in thermodynamics that is called an *adiabatic process*), it would arrive with density greater than its surroundings, and if you moved it downward, it would arrive with density less than its surroundings. That means that there is no tendency for convective vertical mixing: the density stratification is such that the parcel would always be pushed back toward the place where it started.

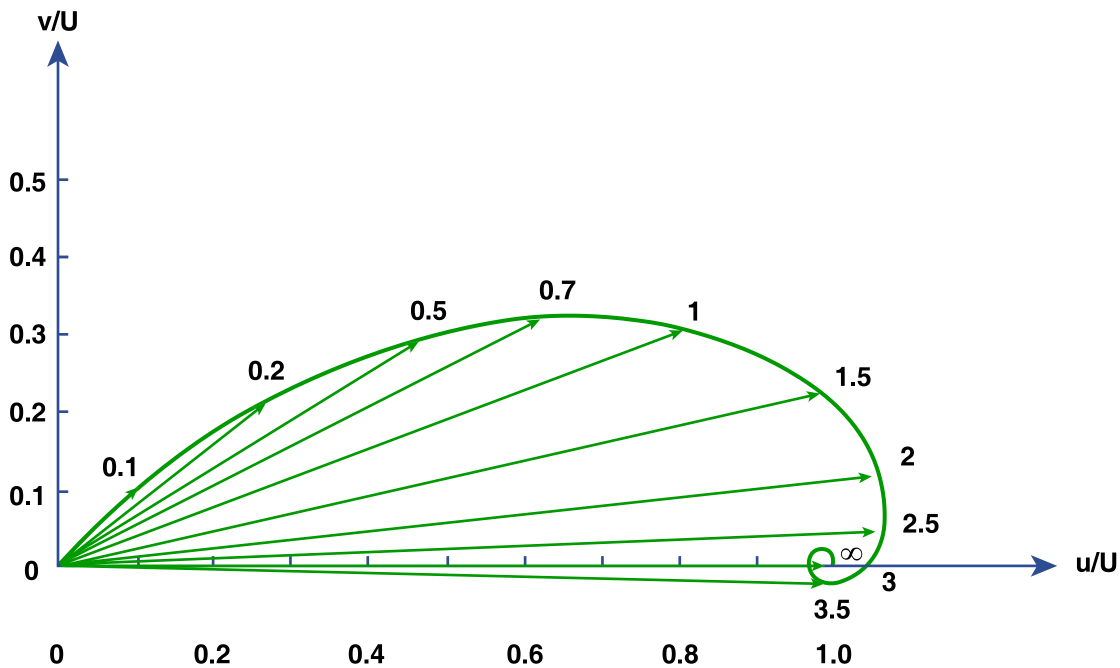

Figure by MIT OpenCourseWare.

Figure 7-29. Vertical profiles of u, the component of Ekman-layer velocity in the direction of the overlying geostrophic flow, and v, the component normal to the direction of the geostrophic flow. Both velocity components are nondimensionalized by dividing by the magnitude of the geostrophic velocity, U. The height above the boundary is nondimensionalized by dividing by the conventional Ekman-layer thickness δ_E. From Pedlosky (1979).

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76 The atmosphere and the oceans are fundamentally different in respect to the origin of stable stratification. The main reason is that the atmosphere is heated at its base: the sun warms the ground surface, and the lowermost layer of the atmosphere is in turn warmed by the ground—during the day, that is, and not at all times and places even then. By contrast, the heating of the lowermost layer of the ocean by heat flow from the substrate is not very important in the dynamics of near-bottom flow.

77 You learned back in Chapters 3 and 4 that shear produces turbulence, and the stronger the shearing, the more likely it is that turbulence is produced. You also learned that, in flow of a viscous fluid past a boundary, the shear is strongest near the boundary and decreases way from the boundary. Here you need to think in terms of a *competition*: one the one hand, stratification tends to damp turbulence, whereas on the other hand, shear tends to produce turbulence. That leads to the concept that under the right conditions a turbulent layer develops adjacent to the boundary, in which turbulence produced by shear causes vertical mixing, as you learned about in Chapter 4. Such a layer is called the ***mixed layer***. So the thickness of the mixed layer depends upon the competition between the intensity of the near-boundary shear and the strength of the stratification. Figure 7-30 shows a striking example of a mixed layer in the ocean.

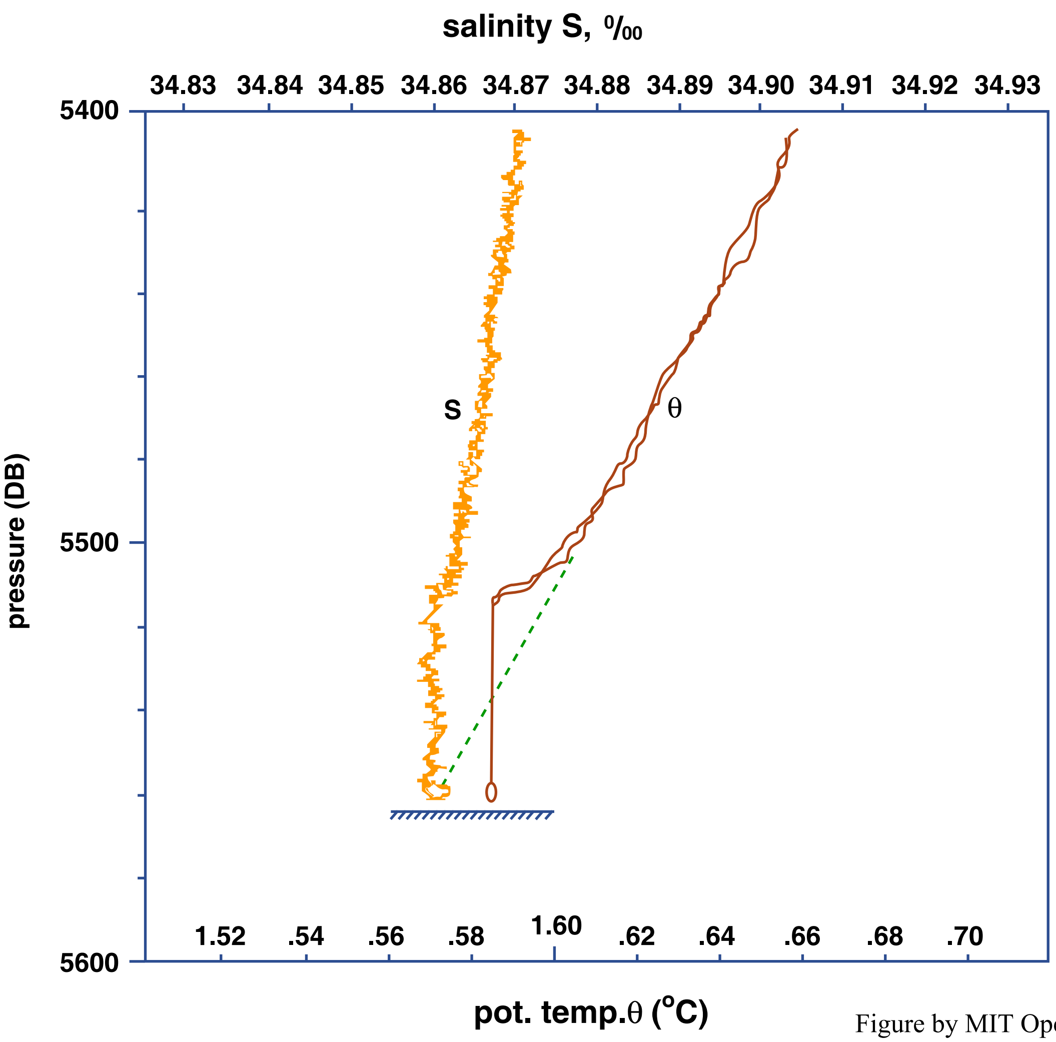

Figure by MIT OpenCourseWare.

Figure 7-30. An example of the mixed layer in the deep ocean. The most natural way of explaining why the potential temperature in the lowermost layer shows no variation with depth is that this layer is thoroughly mixed by shear turbulence. (The potential temperature is what the temperature of an element of the fluid medium would be if the element were transported to some reference level without exchange of thermal energy with its surroundings. A constant potential temperature means neutrally stable stratification: neither stable nor unstable. Mechanical mixing tends to produce such neutrally stable stratification.) (From Armi and Millard, 1976.)

78 Things are different in the atmosphere, because of the tendency for diurnal (day to night) difference in heating of the ground by solar radiation during the day and cooling of the ground by long-wave radiation to space during the night. The tendency for development of a mixed layer by shear turbulence is present in the atmosphere as well as the ocean, but the other effect, strong convection by ground heating, is even more important. The two processes act together to produce a well-defined mixed layer, even when the atmosphere is overall stably stratified (as is true most of the time, except in the vicinity of low-pressure areas, where uprise of air through a deep layer is caused by other effects). At times when the convectively uprising air reaches the local condensation level, puffy cumulus clouds form, and mark the top of the mixed layer nicely. You probably have experienced the structure of the atmospheric mixed layer yourself: as you gain altitude in your jetliner on a nice sunny day, the ride is bumpy until you reach the tops of the cumulus clouds, whereafter you have a much smoother ride.

79 What is the relationship between the mixed layer, on the one hand, and the Ekman layer, which you learned about earlier in this chapter, on the other hand? Commonly, the thickness of the mixed layer in the ocean is the greater part of a hundred meters, and that of the mixed layer in the atmosphere is on the order of a thousand meters. That's several times the thickness of the Ekman layer. So you can think of the Ekman layer as typically being embedded deeply in the mixed layer. The flow is turbulent at heights above the bottom well above the turning of the flow velocity in the Ekman layer.

80 A grand large-scale mental experiment might not be out of place here. Suppose that you could somehow set up a gigantic tank, as large as an ocean basin, on a rotating platform. To a depth of a few thousands of meters, fill the tank with water that is at the same temperature (and salinity) throughout, so that there is no density stratification whatsoever. Somehow, produce a broad current in the medium, on a scale of thousands of kilometers on both the along-flow and cross-flow directions. In the parlance of geophysical fluid dynamics, what we are aiming for is a flow with a very small Rossby number.

81 What would you observe? A vertical profile of velocity that is the same, in essential respects, as the fully developed boundary-layer flow, akin to what is shown in Figure 3-26, back in Chapter 3 on boundary layers. The boundary layer would have grown to occupy the full depth of the flow, and the flow would be turbulent throughout. The Ekman layer, however, would be there, immediately above the bottom: that's the fundamental difference between a deep uniform flow in a nonrotating system and one in a rotating system. The fundamental reason why such a flow is never observed in the deep ocean is the ubiquitous stable stratification, described above: the thickness of the turbulent boundary-layer flow is severely limited by that stratification.

The Logarithmic Layer

82 You might be wondering, at this point, about the relevance of the large body of material on flow resistance and velocity profiles in turbulent shear flows past a solid boundary, treated in much detail in Chapter 4, to planetary boundary-layer flow. The essential point is easy to state: everything developed in Chapter 4 is relevant to the planetary boundary layer. Very near the bottom boundary, deep in the mixed layer (and at the base of the Ekman layer) the same arguments used in Chapter 4 to deal with the inner layer and the outer layer of the flow hold equally well in the case of the planetary boundary layer: there may or may not be a viscous sublayer, depending upon the particular the value of the boundary Reynolds number, but in any case there is a part of the inner layer that is described by the law of the wall. Also, there is an outer layer, where a "velocity defect law" must hold, and there is an overlap layer, which (as mentioned in Chapter 4 but not developed in detail) requires that both the law of the wall and the velocity defect law be logarithmic. In the context of the planetary boundary layer, this lowermost layer, in which the velocity profile is logarithmic, is called the ***logarithmic layer***. In the ocean, the logarithmic layer is typically only few meters thick—but it is what the bottom sediment feels! It is thicker in the atmosphere, but still not nearly as thick as the entire Ekman layer.

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