

CHAPTER 14

MIXED-SIZE SEDIMENTS

INTRODUCTION

1 In a certain sense, this is the most significant chapter in Part 2 of these course notes—inasmuch as virtually all natural sediments comprise a range of particle sizes, not just a single size. Most of what was said in earlier chapters, on threshold, transport mode, and transport rate, involve an implicit assumption that the sediment is effectively of a single size (hence the term “unysize” sediment), in the sense that the effect of the spread of sizes around the mean (that is, the *sorting*) is sufficiently small that it can be ignored, at least for very well sorted sediments. All sedimentationists know, however, that such an assumption cannot be valid even for moderately sorted sediments, to say nothing of poorly sorted sediments, with a wide spread of particle sizes, like the sand–gravel mixtures that are so common in rivers.

2 I will probably be insulting your intelligence when I explain the meaning of “size fraction”. A *size fraction* in a natural sediment or an artificial mixture of sediments is a specified range of sizes within the size distribution of the sediment. Such size fractions are usually perceived or chosen to be very narrow relative to the overall range of sizes in the sediment. The choice of lower and upper size limits of the fraction are basically arbitrary—in practice, usually governed by the subdivisions of the conventional powers-of-two grade scale for sediment size. Keep in mind, however, that the size varies, perhaps non-negligibly, even within a narrowly defined size fraction. A size fraction is *not* a single size.

3 If, for definiteness, we assume a certain definite size-distribution shape for mixed-size sediments, like a log-normal distribution, then the relative size of a given size fraction is specified by three things: the sorting of the distribution, the mean or median size of the distribution, and the position of the given size fraction within the distribution (which is most naturally described by D_i/D_m , where D_m is the mean or median size and D_i is the size of the given fraction). Beyond this, of course, matters become much more complex (hopelessly so?) when we allow the shape of the size distribution to vary, as it does greatly, even to the point of bimodal and trimodal distributions, in natural sediments. (A great many natural sediments, particularly sand–gravel mixtures, are strongly bimodal.) You can see that the task of addressing the problem of threshold and transport of mixed-size sediments is a daunting one.

4 A final note seems in order here. The focus of this chapter is on sediment size. As you saw in Chapter 8, sediments in general have a joint frequency distribution of size, shape, and density. The study of mixed-shape and mixed-density sediment has not progressed as far as study of mixed-size sediment. It

seems fair to say that the effect of mixed shapes is not as significant as the effect of mixed sizes—except, perhaps, for uncommon sediments with extremely nonspherical shapes. The effect of mixed-density sediments is important, for example, in understanding the development of placers. For completeness, these notes should have additional sections on mixed-shape and mixed-density sediments.

A USEFUL THOUGHT EXPERIMENT

5 To get your thinking started, imagine a planar bed of mixed-size sediment, with a wide range of sizes from sand to gravel, over which a uniform flow is arranged to be passed. Assume that the particle-size distribution is unimodal. Suppose that the flow extends uniformly so far upstream and downstream as to be effectively infinite in extent. Clearly this is an idealization of the flow in real streams and rivers—but there is an essential element of reality to it, inasmuch as during a period of strong flow in a river the flow for the most part works on a bed of sediment that was lying there, waiting to be worked on before the event, and the flow picks up and moves what it wants to, without an externally constrained supply of sediment. A sediment-recirculating flume (see Chapter 8) works the same way, and in that sense is a good model for fluvial sediment transport.

6 You could attempt to measure three significant aspects of the transport of the mixed-size sediment in such an experiment. One is the relationship among the load (the sediment in transport at a given time), the bed surface (the sediment that is exposed to the flow at any given time), and the substrate (the bulk sediment from which the flow entrains, transports, and deposits sediment particles of various sizes). A second question has to do with movement thresholds: how do the thresholds for the various size fractions in the sediment mixture differ from one another? A third aspect is the relationship among the rates of transport of the various size fractions (usually called fractional transport rates; see below) of the sediment mixture. These three aspects are considered in some detail in the following sections.

THE BED-SURFACE SIZE DISTRIBUTION

7 If the flow is sufficiently strong, it moves some of the sediment that is resting on the bed surface. The question now arises: after the sediment transport reaches an equilibrium state, *is the size distribution of the sediment on the bed surface the same as that of the sediment in the substrate?* An unsophisticated observer might suppose, beforehand, that it would be the same. In general, however, it is *not*: it is *coarser* than the substrate. In part this develops because the flow selectively entrains the finer fractions in preference to the coarser fractions. In the sediment-transport literature, this has been termed *selective entrainment*. That should seem natural to you in light of what was said in Chapter 9, on movement threshold: it takes a stronger flow to move coarser

sediment than it does to move finer sediment, so at first thought it might seem that in a mixed-size sediment the coarser fractions should be more difficult to move than the finer fractions.

8 There is another effect as well: the flow “develops” the bed surface as it works on it, in such a way that finer particles find their way down beneath coarser particles, leaving a bed-surface layer that is coarser than the underlying layers. Such a coarser surface layer, beneath a flow that is transporting the bed-material sediment in equilibrium, is called *pavement* (Parker and Klingeman, 1982; Parker et al., 1982a; Parker et al., 1982b). Pavement is similar to, but different from, *armor*, which is a coarse surface layer that develops as a flow winnows finer sediment to the point where no more sediment can be entrained by the flow. In other words, armor is a coarse bed-surface layer which, once it is formed, never moves, under ordinary circumstances (it could, of course, be disrupted by a rare, catastrophic event), whereas pavement is a coarse surface layer which, if not in equilibrium with the flow, is moved, at least in part, under ordinary circumstances. (By “ordinary circumstances” here I mean strong flow events that might occur during some number of time periods, small or large, in a typical year.)

9 Another way of thinking about the development of a coarse surface layer during bed-material transport is that, if the coarser fractions are more difficult to transport than the finer fractions, the concentration of those coarser fractions on the bed surface must increase, in order for the flow to transport the sediment it is given to transport. That is true to the extent that the sediment-transporting system is like a sediment-feed flume (See Chapter 8), for which the flow and the bed *must* become adjusted to transport the sediment that is fed, independently, into the flow at the upstream end of the flume.

FRACTIONAL TRANSPORT RATES

10 Suppose, now, that you measured the unit transport rate (that is, transport rate, in mass per unit time, per unit cross-stream width of the flow) of each size fraction in the mixture of transported sediment. These transport rates are called *fractional transport rates*, often denoted by q_{bi} , where q represents the unit transport rate, the subscript i denotes the i th fraction in the mixture, and the subscript b stands for bed load or bed-material load. (In natural flow environments, of course, the size distribution is continuous, so you need to divide the size continuum, arbitrarily, into a large number of narrow fractions.)

11 You might measure the fractional transport rates in the following way—without great difficulty in a laboratory flume, but not without great difficulty, if not impossible, in a real stream or river! Build a slot trap of some kind across the flow at some station and extract all of the passing sediment during some interval of time. That might be called the “transport catch” (an unofficial term). If you divide the mass of the transport catch by the time interval and the width of the trap, you have the total unit transport rate, q_b . Then sieve the transport catch into the various size fractions, to find the proportion p_i of each of the fractions in the

catch, and multiply each of the p_i by the total transport rate q_b to find the fractional transport rates q_{bi} . (That works well for the bed load, but much, if not most, of the suspended bed-material load is likely to pass over the trap. But the problem lies in practice, not in concept.)

GRADATION INDEPENDENCE VERSUS EQUAL MOBILITY

12 For the relationships among the various fractional transport rates, you can think in terms of two end members. At one extreme, you might suppose that *the transport of each size fraction is entirely independent of the presence of all of the other size fractions*. Then the transport rate of each fraction could, in principle at least, be found by appeal to the same considerations that were described in the Chapter 13, on sediment transport rates. Such a situation might be called **gradation independence**. At the other extreme, you might suppose that, if you normalize the fractional transport rates by dividing them by the proportion f_i of the given fraction in the sediment bed *all of the various fractions have the same normalized fractional transport rate*; in other words, the ratio of the fractional transport rate of a given size fraction to the proportion of the given size fraction in the bed sediment is the same for all of the size fractions. Such a condition has been termed **equal mobility**.

13 The condition of equal mobility might strike you as counterintuitive: should it not be more difficult for a flow to transport the coarser fractions than to transport the finer fractions? You might call this the **particle-weight effect**: larger particles are more difficult to move because they are heavier (Figure 14-1). Two important countervailing effects tend to offset the particle-weight effect, though: (1) the **hiding-sheltering effect**, whereby larger particles are more exposed to the flow and thus have exerted on them a greater fluid force, but smaller particles tend to be sheltered from the forces of the flow by the larger particles (Figure 14-2); and (2) the **rollability effect**, whereby larger particles can roll easily over a bed of smaller particles, but smaller particles cannot roll easily over a bed of larger particles (Figure 14-3). The relative importance of the particle-weight effect, on the one hand, and the combination of the hiding-sheltering effect and the rollability effect, on the other hand, is an essential element in mixed-size sediment transport.

14 In the case of a sediment-feed flume (see Chapter 8), in which a bed of sediment is laid down and then a flow is passed over that bed while sediment that is identical to the bed sediment is fed at some rate at the upstream end of the flume, to be caught and discarded at the downstream end, the condition of equal mobility is *forced upon the system*, simply because the flow must transport all of the sediment it is given. Otherwise, the flow and sediment transport could never

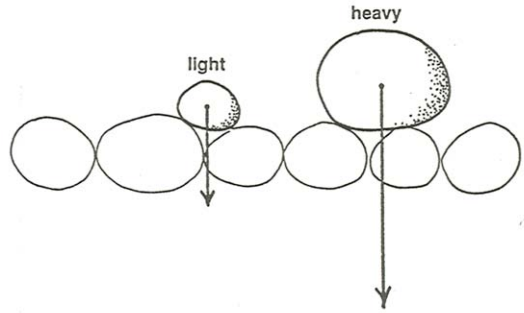


Figure 14-1. The particle-weight effect: larger particles are harder to move because they are heavier.

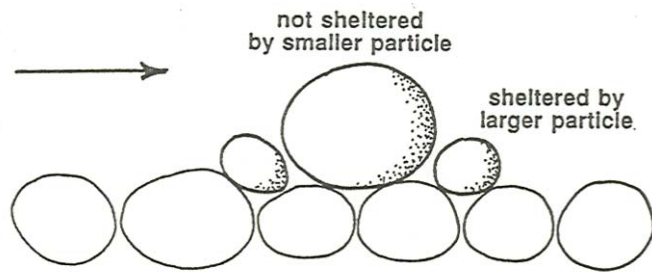


Figure 14-2. The hiding-sheltering effect: larger particles are more exposed to the flow, and smaller particles tend to be sheltered by larger particles.

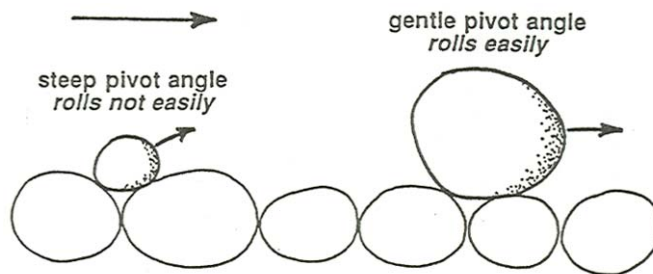


Figure 14-3. The rollability effect: larger particles can roll easily over a bed of smaller particles, but smaller particle cannot easily roll over a bed of larger particles.

attain an equilibrium state. In order to transport the inherently more difficultly transportable fractions—the coarser fractions, presumably—the size distribution of the bed surface must become adjusted in such a way that the proportion of

those difficultly transportable fractions on the bed surface are in greater proportion than they are in the underlying sediment bed. In a sediment-recirculating flume, by contrast, there is no such constraint: the flow is free to adjust its transport of the various size fractions in accordance with their inherent transportability. A fundamental question thus arises: to what extent does transport of mixed-size sediment in a sediment-recirculating flume approach the condition of equal mobility, even though that condition is not forced upon it? The reasons for the importance of that question is that natural rivers and streams, at least over short scales of space and time, seem to behave more like sediment-recirculating flumes than like sediment-feed flumes. The answer to that question will become apparent in a later section of this chapter.

A THOUGHT EXPERIMENT TO DEMONSTRATE THE DIFFERENCE BETWEEN GRADATION INDEPENDENCE AND EQUAL MOBILITY

15 Here is a hypothetical laboratory experiment to reveal more clearly for you the distinction between gradation independence and equal mobility. It would not be dauntingly difficult to do in an appropriately equipped sedimentation laboratory. Obtain or prepare three batches of sediment with nearly perfect sorting: classic “unsize” sediments. Their sizes might range from medium sand to fine gravel. Use each, in turn, for a series of flume runs to measure the unit sediment transport rate q_b (transport rate per unit width of flow), where the subscript b signifies bed-material load, over a wide range of boundary shear stresses τ_0 , from only slightly above the threshold shear stress to a very large boundary shear stress, several times the threshold value. Plot graphs of q_b against τ_0 for each of the three sediments on a single graph (Figure 14-4).

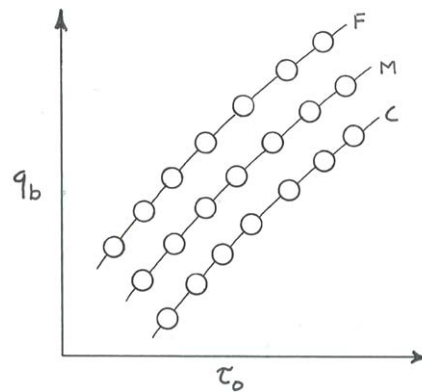


Figure 14-4. Plot of q_b vs. τ for the three unsize sediment batches.

16 You know, beforehand, from the material in Chapter 12, what the graphs would look like, in an approximate way at least: for each sediment, the data points for the runs would fall on an approximately straight line in a log-log plot, with q_b increasing steeply with τ_0 . The curve for the finest sediment would

lie above that for the middle sediment, and the curve for the coarsest sediment would lie below that for the middle sediment—because the flow moves finer sediment more easily than coarser sediment.

17 Another way of viewing the results graphically is to plot the results in a three-dimensional graph, by adding a third axis, the particle size (*D*) (Figure 14-5). Each of the three curves lies in its own plane, corresponding to position on the *D* axis. The earlier graph then is just a projection of the curves in those three separate planes onto the $q_b - \tau_o$ axis plane.

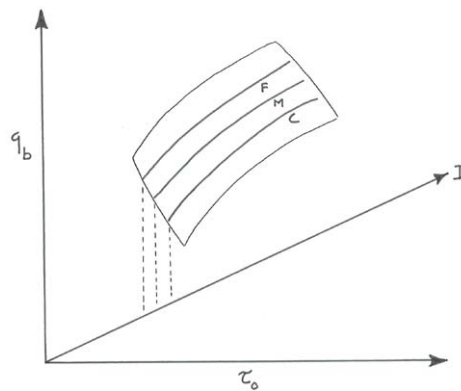


Figure 14-5. Three-dimensional plot of q_b versus τ_o and *D* for the three unisize sediment batches.

18 Now mix the three unisize sediments together, to form a single starkly trimodal sediment mixture. Make a similar series of runs, with similar values of τ_o . For each value of τ_o , you need to compute the fractional transport rate of each of the three size fractions: $q_{bi} = (p_i/f_i)q_b$, where q_b is the total transport rate measured, p_i is the proportion of the transport rate (that is, the “transport catch”; see an earlier section) for the *i*th size fraction (*i* = 1, 2, 3, remember), and f_i is the proportion of the *i*th size fraction in the bulk sediment mixture you placed in the flume. Here we have normalized the q_{bi} by dividing by f_i , to make clearest sense of the results. Again you can plot the results of q_{bi} in a three-dimensional graph, analogous to that in Figure 14-5, of q_{bi} vs. τ_o and D_i (with D_i taking on three values—those of the modes of the trimodal particle-size distribution you created by mixing the three separate unisize batches).

19 Now the question is: what would the graph look like for the end-member cases of complete gradation independence, on the one hand, and perfect equal mobility, on the other hand?

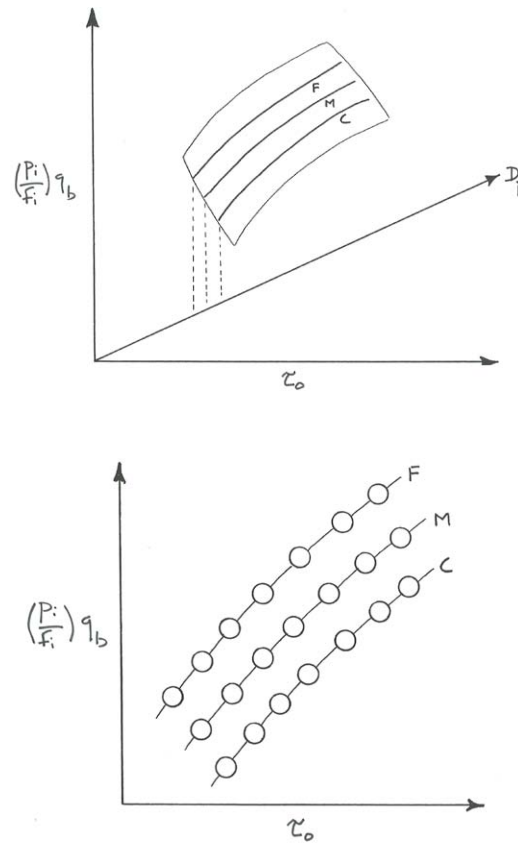


Figure 14-6. **(upper)** Three-dimensional graph of normalized fractional transport rate $(p_i/f_i)q_b$ vs. τ_0 and D_i for the three-size sediment mixture, for the case of perfect gradation independence. **(lower)** The graph in Part A projected onto the q_b - τ_0 plane.

(1) *Gradation independence*: In the case of gradation independence, if the q_{bi}/f_i for the three fractions are what they would be in the absence of the other fractions—that is, each fraction behaves in transport without any interaction with the other size fractions—then the results would plot as three curves in the q_b - τ_0 - D_i graph, one curve in each of three $D_i = \text{constant}$ planes just as with the graph for the separate batches, and the curves would be the same as before, after the change from q_b to $(p_i/f_i)q_b$ is taken into account (Figure 14-6).

(2) *Perfect equal mobility*: In the case of perfect equal mobility, all of the normalized fractional transport rates are the same for a given value of τ_0 : the transport dynamics of the various fractions are so closely interdependent that the transport rates of the various fractions are all the same, when adjusted for their proportions in the sediment mixture. In a graph of normalized fractional transport rate $(p_i/f_i)q_b$ vs. τ_0 and D_i (Figure 14-7), again there are three steeply rising curves,

one for each value of D_i , but now all three curves are the same, and when they are projected onto the $(p_i/f_i)q_b - \tau_o$ plane, they fall on a single curve.

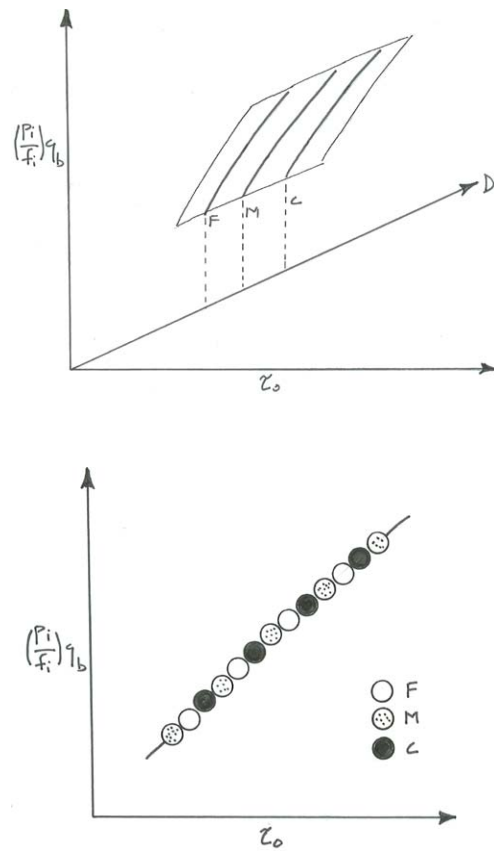


Figure 14-7. **(upper)** Three-dimensional graph of normalized fractional transport rate $(p_i/f_i)q_b$ vs. τ_o and D_i for the three-size sediment mixture, for the case of perfect equal mobility. **(lower)** The graph in Part A projected onto the $q_b - \tau_o$ plane.

20 It is instructive to look also at how the three curves project onto the $(p_i/f_i)q_b$ vs. D_i axis plane of the three-dimensional graph: with each value of τ_o is associated a series of three points, and each of those sets of three points lies on a horizontal line, parallel to the D_i axis (Figure 14-8). If the condition of equal mobility is not fulfilled, however, the curve would not be a horizontal line: if the fractional transport rates decrease with increasing sediment size, the curve would slope downward toward the coarser sizes (Figure 14-9A) and if the fractional transport rates increase with increasing sediment size, the curve would slope upward toward the coarser sizes (Figure 14-9B).

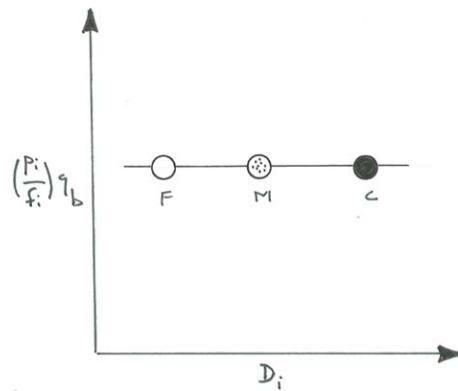


Figure 14-8. Graph of normalized fractional transport rate $(p_i/f_i)q_b$ vs. D_i for the condition of perfect equal mobility of all of the size fractions.

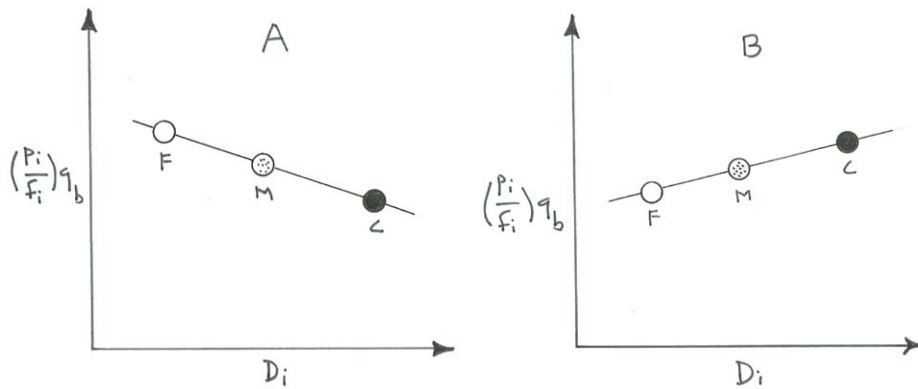


Figure 14-9. **A)** Graph of normalized fractional transport rate $(p_i/f_i)q_b$ vs. D_i if the fractional transport rates *decrease* with increasing sediment size. **B)** Graph of normalized fractional transport rate $(p_i/f_i)q_b$ vs. D_i if the fractional transport rates *increase* with increasing sediment size. The shapes of the curves here are not meant to be significant: they are meant only to show the upward or downward trend of the data.

A LOOK AT SOME REAL DATA ON FRACTIONAL TRANSPORT RATES, FROM THE FLUME AND FROM THE FIELD

21 There has been a long-standing controversy over the reality or importance of equal mobility since the concept was first proposed by Parker et al. (1982b). Some sets of measurements, in flumes and in streams, have shown a close approach to equal mobility, whereas other studies have shown strong deviations from equal mobility.

22 First we look at the results of the most revealing flume studies of fractional transport rates in unimodal sediment made up to now. Wilcock and Southard (1989) made a flume study of fractional transport rates in a sediment-recirculating flume. The sediment was of mixed size, with a mean size of 1.83 mm and a unimodal distribution. In seven runs with increasing bed shear stress, the fractional bed-load transport rates of several size fractions, ranging in size from 0.5 mm to 6 mm, were measured by use of a slot trap that extended across the width of the flume. Once weighed, the samples were returned to the system. Sampling was done at two times during a run: while the bed was still initially planar, and at a later time when the bed and the flow had reached equilibrium. In the runs at lower bed shear stress, the bed remained planar for the entire run, but at higher bed shear stresses, dunes developed on the bed.

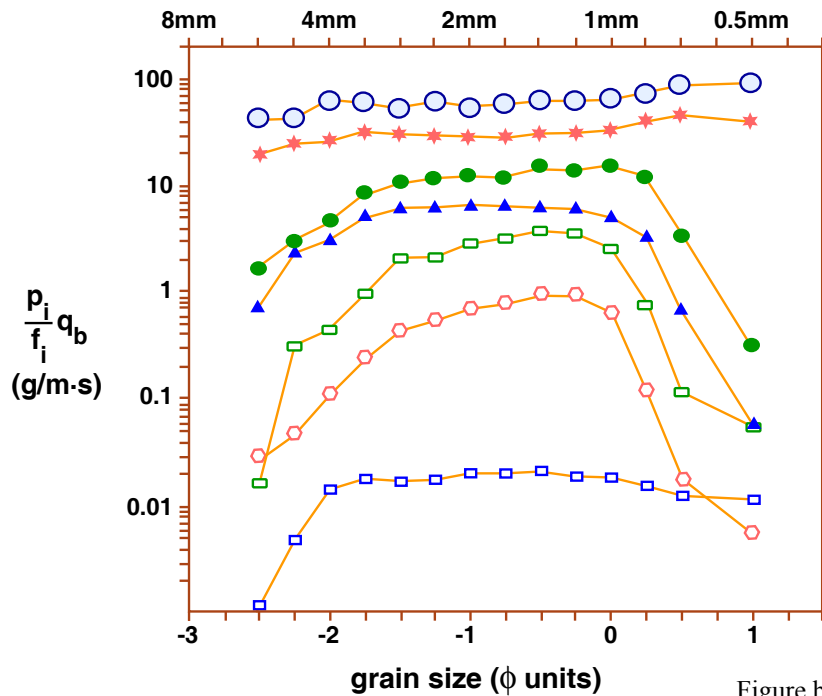


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Figure 14-10. Fractional transport rate $(p_i/f_i)q_b$ vs. particle size for seven runs with increasing bed shear stress. Each curve represents one value of the bed shear stress (not given here). These data were taken at the end of each run, after the flow and the bed had come into equilibrium with the flow. (From Wilcock and Southard, 1988.)

23 You can see from Figure 14-10 (compare this figure with Figures 14-8 and 14-9) that for a wide range of size fractions in the middle part of the size distribution the fractional transport rates are nearly the same: in other words, there is a close approach to the condition of equal mobility for those size fractions. Except at the highest bed shear stresses, however, the curves depart

from the conditions of equal mobility: the fractional transport rates of both the finest fractions and the coarsest fractions are more difficult to transport. You might have guessed that the coarsest fractions would be harder to transport, but it is somewhat surprising that the same is true for the finest fractions.

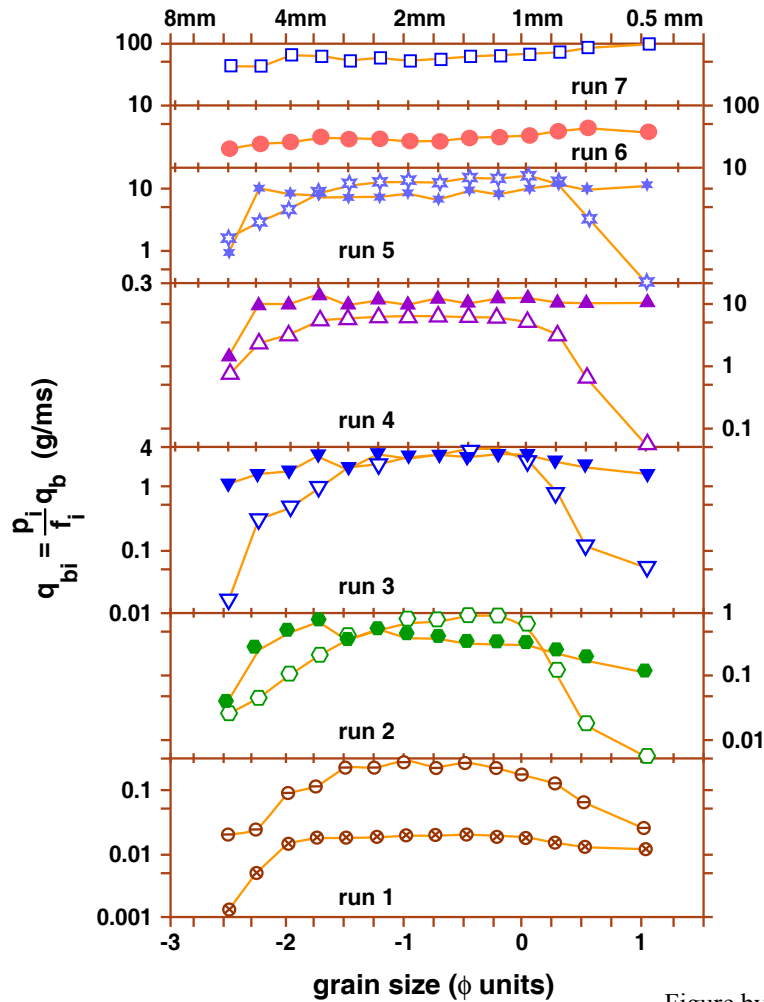


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Figure 14-11. Fractional transport rate $(p_i/f_i)q_b$ vs. particle size for seven runs with increasing bed shear stress. The solid symbols are for initial fractional transport rates, and the open circles are for equilibrium fractional transport rates. (From Wilcock and Southard, 1988.)

24 Figure 14-11, also from Wilcock and Southard (1988), repeats the data in Figure 14-10 but also shows the data for the initial conditions in the runs (except for the two at the highest bed shear stresses). The main difference in the data between the two conditions is that at the initial condition the finest fractions approach the condition of equal mobility more closely than they do at the equilibrium condition. The explanation seems to lie in a combination of two effects: (1) as time goes on, the finer particles find their way downward among the coarser particles to positions below the surface layer; and (2) as a coarse pavements develops on the bed surface, the finer particles are hidden from the flow more effectively.

25 The most widely cited data set on fractional transport rates in natural streams is that of Milhous (1973) from Oak Creek, a gravel-bed stream in Oregon. The Oak Creek data were used by Parker et al. (1982b) in their classic work on the concept of equal mobility.

26 Figure 14-12, a graph of the Oak Creek data on fractional transport rate shows, unsurprisingly, that the fractional transport rates are a steeply increasing function of flow strength. The dimensionless version of the fractional transport rate, called the dimensionless bed-load parameter W_i^* , is equal to $\gamma'q_{bvi}/f_iu_*^3$. (Note: the fractional transport rate, denoted here by q_{bvi} , is by sediment volume, not sediment mass.) The reason for the separation of the curves for the various size fractions is that the dimensionless variable on the horizontal axis, τ_i^* ($= \tau_o/\gamma'D_i$), contains the particle size D_i of the given fraction.

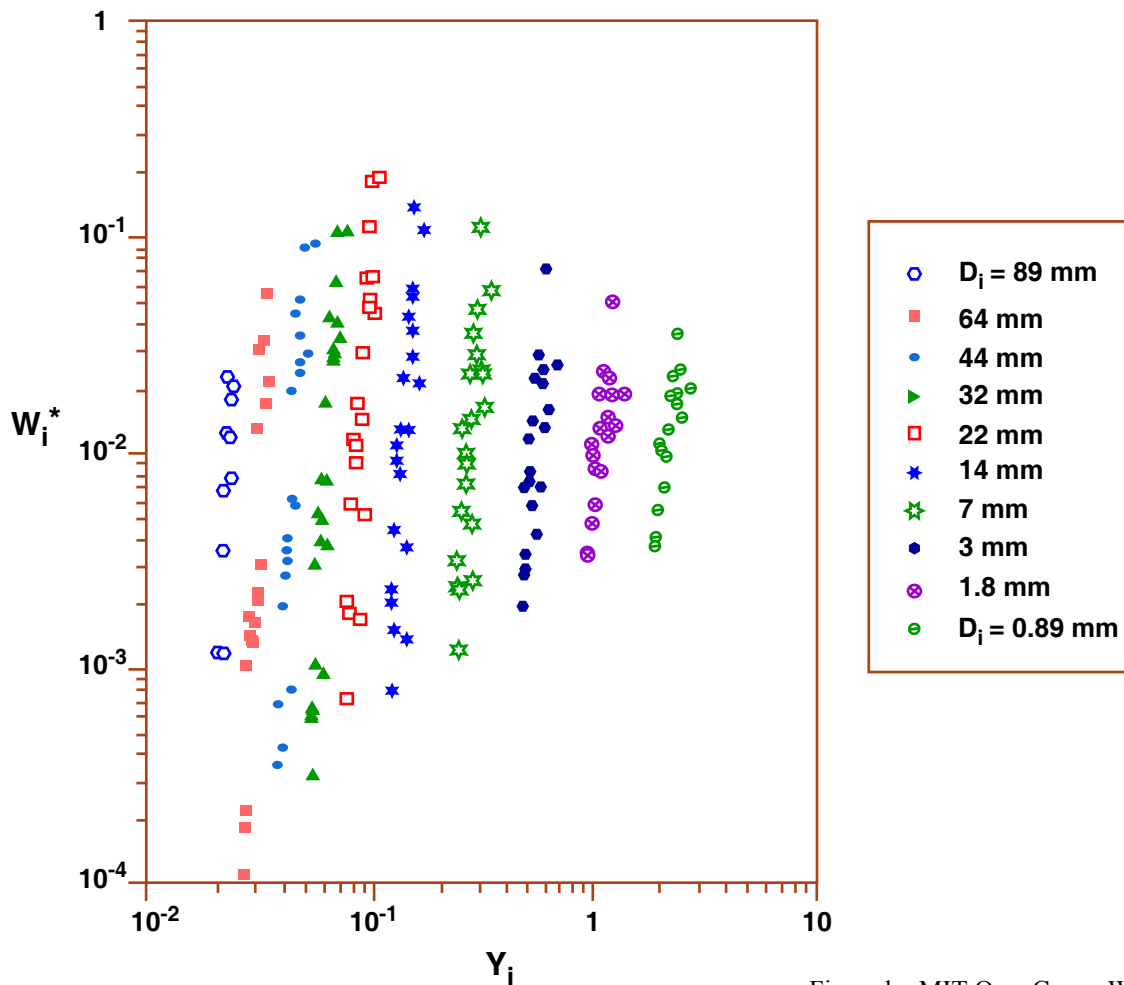
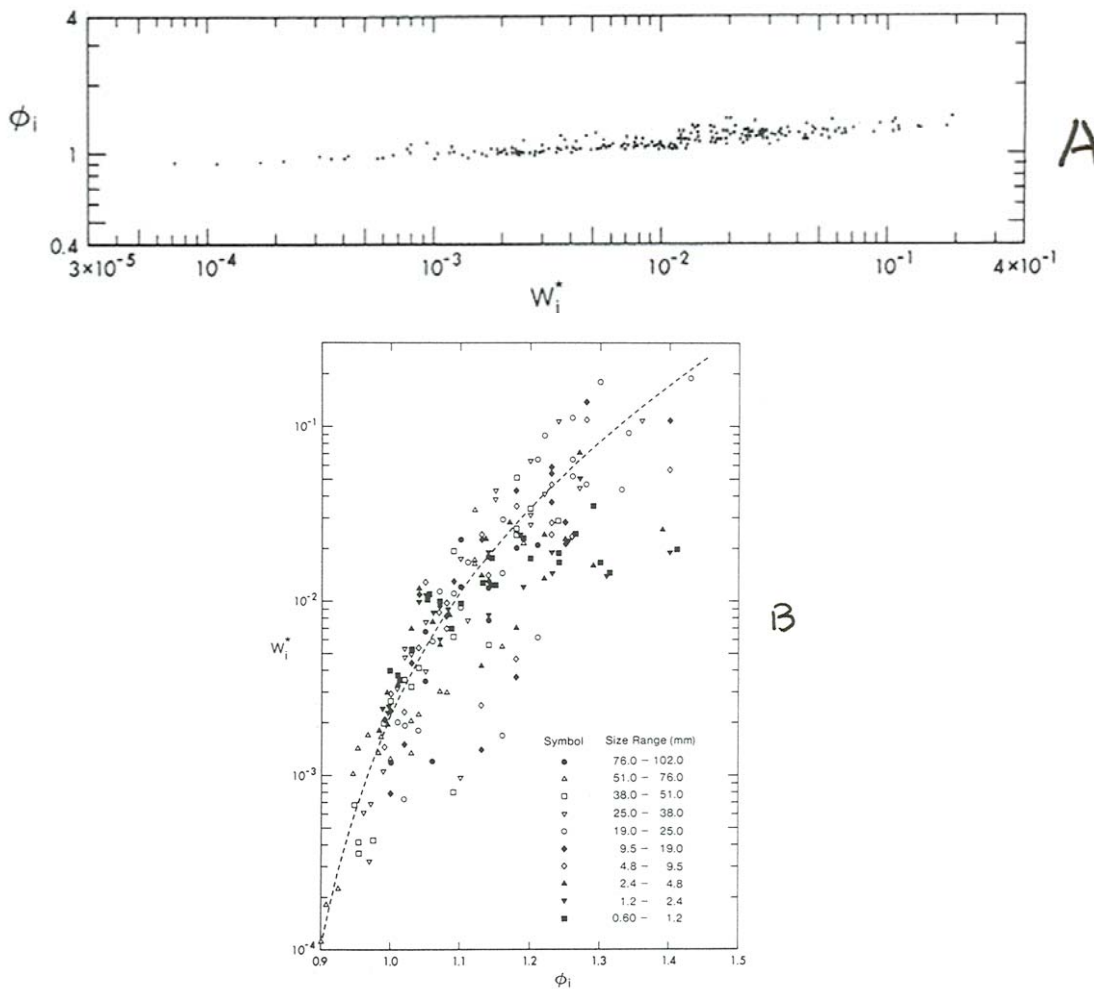


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Figure 14-12. Plot of dimensionless bed-load parameter W_i^* vs. dimensionless bed shear stress τ_i^* for ten size ranges in the Oak Creek data. (From Parker et al., 1982b.)

27 Each curve in Figure 14-12 was extrapolated downward to find the threshold shear stress, defined as the value for which W_i^* was at an arbitrarily chosen reference value of 0.002 (chosen to conform to what would match the commonly accepted condition of movement threshold; see the discussion on the reference-transport rate method of defining the movement threshold, in Chapter 9). Then, in a plot of W_i^* versus τ_r^*/τ_{ri}^* , which Parker et al. denote by ϕ_i , all of the ten curves for fractional transport rate in Figure 14-12 collapse into a single curve—not perfectly, but to a fairly good approximation (Figure 14-13).

28 (Here, the dimensionless variable $\phi_i = \tau_r^* / \tau_{ri}^*$ might need careful attention on your part: it is the reference value of the dimensionless bed shear stress at which the dimensionless total bed-load transport rate equals the reference total bed-load transport rate, divided by the reference value of the dimensionless bed shear stress of the i th size fraction at which the dimensionless bed-load transport rate of the i th fraction equals the reference bed-load transport rate of the i th fraction. (That long sentence requires careful reading.) Basically, it expresses the relative magnitude of the dimensionless bed shear stress at reference threshold condition for the bulk sediment, on the one hand, and the dimensionless bed shear stress at the reference condition for the i th fraction, on the other hand.)



Courtesy of American Society of Civil Engineers. Used with permission.

Figure 14-13. **A)** Plot of W_i^* vs. τ_r^* / τ_{ri}^* for the Oak Creek data. **B)** The same plot, with size ranges given. (From Parker et al., 1982b.)

29 What, then, is the significance of this “collapse” of the individual curves into a single curve? If you go back to the section on the thought experiment and look at Figure 14-7, for the condition of perfect equal mobility, you can see that

Figure 14-13 is of the same nature, because the effect of having the particle size in the denominator of the dimensionless bed shear stress is circumvented by taking the ratio of the two dimensionless bed shear stresses. The conclusion to be drawn is that also in the case of this natural gravel-bed stream the condition of equal mobility is approached, although not met exactly. We must conclude, then, that the effects of hiding–sheltering and rollability combine, in some way, to make the transport of the various size fractions more nearly equal, when normalized by the proportions of the fractions in the mixture, although there still is a tendency for the coarser fractions to be less easily transported.

MOVEMENT THRESHOLD IN MIXED-SIZE SEDIMENTS

Introduction

30 It might seem like “putting the cart before the horse” to deal with movement thresholds after having already considered transport rates, but there is a certain logic to it, inasmuch as the most common way of identifying threshold conditions for mixed-size sediments is to measure transport rates for several values of boundary shear stress and then extrapolate downward to some chosen very low transport rate, called a *reference transport rate*, that corresponds approximately to what seems, by visual observations, to correspond to the boundary shear stress at which movement begins. The discussion in Chapter 9 on how to define the threshold condition in the first place is relevant here.

31 It is abundantly clear, from studies both in flumes and in natural flows, that the threshold shear stress for mixed-size sediments is different from that for unisize sediments. You should not expect to find that the threshold for movement of a certain size fraction in a mixed-size sediment is predictable by reference to the same sediment size in a relationship like the Shields diagram (Chapter 9), which is assumed to hold for unisize or very well-sorted sediment.

32 As with fractional transport rates, it is the outcome of the competition between the particle-weight effect, on the one hand, and the combination of the hiding–sheltering effect and the rollability effect, on the other hand, that is the key to movement threshold in mixed-size sediments. You should expect that to a first approximation the movement thresholds of the size fractions in a mixed-size sediment should be more nearly the same than would be predicted by, say, the Shields diagram for very well-sorted or unisize sediment. The question, again, as with fractional transport rates, is where the true situation lies between the end-member extremes of gradation independence (the threshold for each fraction is the same as for the same sizes of unisize sediment) and equal mobility (all of the size fractions of a mixed-size sediment begin to move at the same value of boundary shear stress).

Setting the Stage

33 Here is a useful framework for thinking about thresholds in mixed-size sediments, in light of what was just said about gradation independence and equal mobility:

Gradation independence: The threshold for each fraction is that same as if it were a unisize sediment, so τ^*_{ci} ($\tau_{oc}/\gamma D_i$, the threshold value of the Shields parameter) is the same for all size fractions:

$$\tau^*_{ci} = \tau^*_{cj} \text{ for all } i, j \quad (14.1)$$

We can massage this by setting, arbitrarily, τ^*_{ci} equal to τ^*_{c50} , the threshold value for the 50th percentile of the sediment mixture. That is,

$$\tau_{ci}/\gamma D_i = \tau_{c50}/\gamma D_{50} \quad (14.2)$$

Dividing both sides by γ and doing a bit of algebra then shows that

$$\tau_{ci}/\tau_{c50} = D_i/D_{50} \quad (14.3)$$

which expresses the condition of gradation independence. This would look like the graph in Figure 14-14. We can carry this a bit further by use of the definition of τ^* : $\tau_{ci} = \gamma D_i \tau^*_{ci}$ and $\tau_{c50} = \gamma D_{50} \tau^*_{c50}$, so the condition $\tau_{ci}/\tau_{c50} = D_i/D_{50}$ can be written

$$\tau^*_{ci}/\tau^*_{c50} = 1 \quad (14.4)$$

Equal mobility: each size fraction has the same movement threshold as all the others, and the particles of all of the size fractions start to move at the same value of τ_0 :

$$\tau_{ci} = \tau_{cj} \text{ for all } i, j \quad (14.5)$$

Again we can massage this by setting τ_{ci} equal to τ_{c50} , for convenience, and then, with some algebra,

$$\tau_{ci}/\tau_{c50} = 1 \quad (14.6)$$

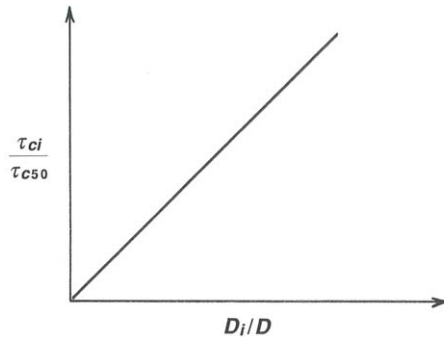


Figure 14-14. Graph of τ_{ci}/τ_{c50} vs. D_i/D_{50} for the condition of gradation independence.

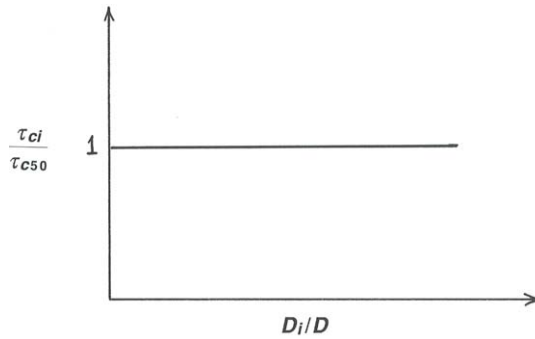


Figure 14-15. Graph of τ_{ci}/τ_{c50} vs. D_i/D_{50} for the condition of equal mobility.

This would look like the graph in Figure 14-15. Again by use of the definition of τ_{ci}^* and τ_{c50}^* , the condition $\tau_{ci} = \tau_{c50}$ can be written

$$\tau_{ci}^*/\tau_{c50}^* = (D_i/D_{50}) - 1 \quad (14.7)$$

Finally, combining the results for both end-member cases, the contrast between gradation independence and equal mobility in graphic form is shown in Figures 14-16 and 14-17.

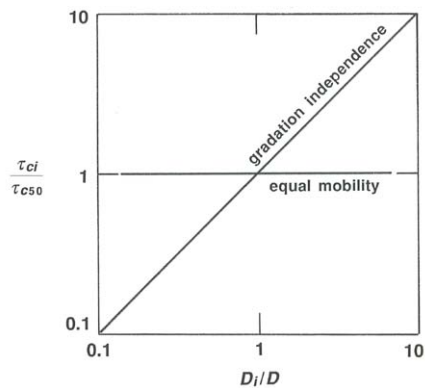


Figure 14-16. Graph of τ_{ci}/τ_{c50} vs. D_i/D_{50} for gradation independence and equal mobility.

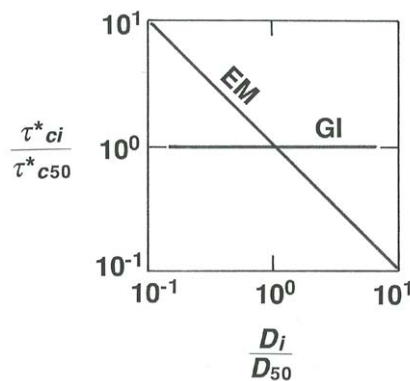


Figure 14-17. Graph of τ^*_{ci}/τ^*_{c50} vs. D_i/D_{50} for the conditions of gradation independence (GI) and equal mobility (EM).

Some Real Data on Thresholds in Mixed-Size Sediments

34 Wilcock and Southard (1988), using the same experimental arrangement described above for fractional transport rates, studied movement thresholds in five batches of mixed-size sediments, made up specifically to represent a range of median size and sorting. The three main batches were chosen to have mean size of about 1.8 mm but with sorting ranging from very well sorted (phi standard deviation 0.20) to moderately poorly sorted (phi standard deviation

0.99). Also used were a well-sorted finer mixture, with mean size 0.66 mm, and a well-sorted coarser mixture, with mean size 5.31 mm. Movement threshold was determined by making several runs over a range of bed shear stress and extrapolating back to a reference transport rate chosen to correspond to a level of weak movement that would generally be agreed to represent threshold conditions.

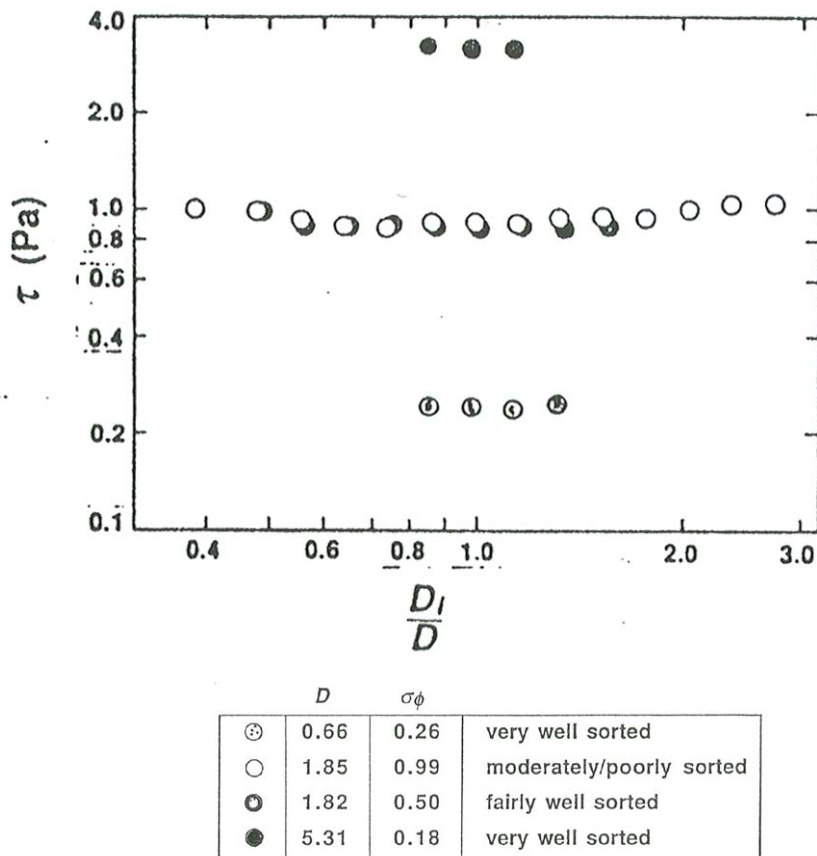


Figure 14-18. Plot of boundary shear stress τ_0 against D_i/D , the ratio of the size of the i th fraction to the mean size of the sediment batch, for four of the sediment batches. (Modified from Wilcock and Southard, 1988.)

35 Figure 14-18 shows results for four of the sediment batches in a plot of τ_0 against D_i/D_{50} , the ratio of the size of the given fraction to the mean size of the sediment batch. Owing to the differences in mean size, the curve for each sediment occupies a different range of τ_0 , but what is interesting is that for each sediment the curves are nearly horizontal, indicating a close approach to equal mobility (see Equation 14-6 and Figure 14-15 above).

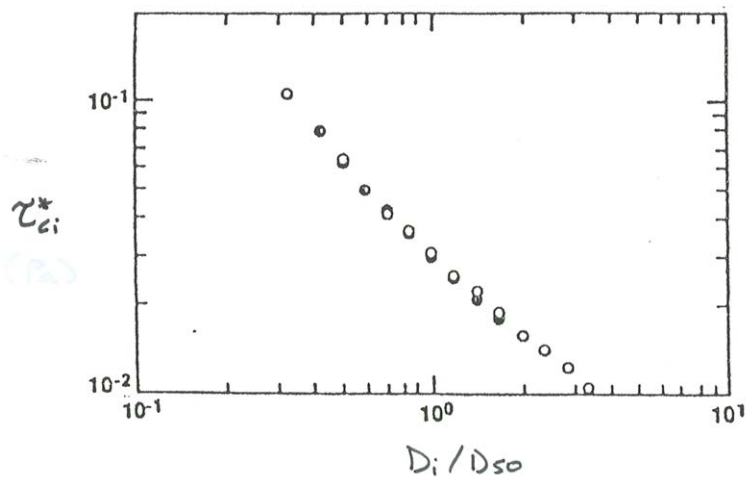


Figure 14-19. Plot of dimensionless threshold bed shear stress τ^*_{ci} against relative size D_i/D_{50} for two sediment batches with almost the same mean size but different sorting. Open circles, phi standard deviation = 0.99; solid circles, phi standard deviation 0.50. (Modified from Wilcock and Southard, 1988.)

36 Figure 14-19 shows results for movement threshold in a plot of τ^*_{ci} , the dimensionless threshold bed shear stress for the i th fraction, against D_i/D , the ratio of the size of the i th fraction to the mean size of the sediment batch. There are two noteworthy things about this graph: (1) The downward trend of the curves shows that the results correspond to a condition close to equal mobility, which is that of a straight line with a slope of -1 (compare with Equation 7 and with the graph in Figure 14-15B). (2) The two curves are almost identical, showing that the sorting of the sediment has little effect on the thresholds of the individual size fractions, once D_{50} and relative size D_i/D_{50} are accounted for.

37 Figure 14-20 is a plot similar to that in Figure 14-19 for sediments from various laboratory and field studies. For these sediments as well, the condition of equal mobility, expressed as slope of -1 in the plot, is approached but not met.

38 An equivalent, but revealing, way of presenting the data in Figure 3-20 is to plot (Figure 14-21) the dimensionless threshold bed shear stress τ^*_{ci} against the dimensionless variable $D^3 \gamma \rho / \mu^2$ (taken to the one-half power here), which is a nondimensionalization of the particle size in a way that does not involve the bed shear stress. The Shield curve for movement threshold is replotted in this graph as the solid curve. The downward slope of each of the curves defined by the data points shows clearly that the finer fractions of the sediment mixtures have

threshold values that lie above the Shields curve, and the coarser fractions have threshold values that lie below the Shields curve.

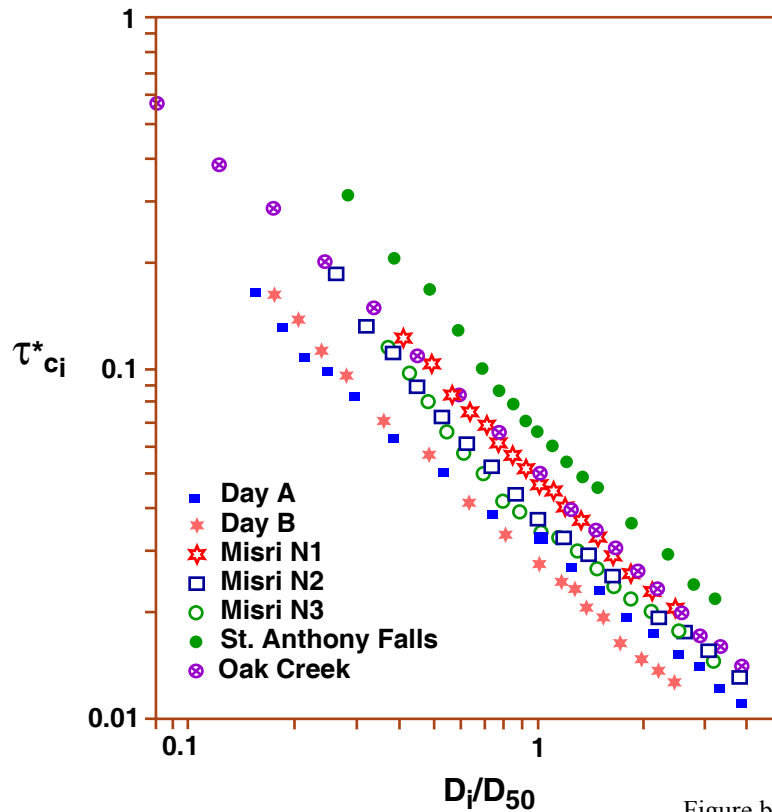


Figure by MIT OpenCourseWare.

Figure 14-20. Plot of dimensionless threshold bed shear stress τ_{ci}^* against relative size D_i/D_{50} for laboratory experiments (Day, 1980; Misri et al., 1984; Dhamotharan et al., 1980; Wilcock, 1987) and field studies (Milhous, 1973).

DEVIATIONS FROM THE CONDITION OF EQUAL MOBILITY

39 Some perspective is needed at this point. You have seen, from discussion of the various data sets presented in the preceding sections, that although the thresholds and transport rates of mixed-size sediments show a much closer approach to the condition of equal mobility than to the condition of gradation independence, there remains a deviation from the condition of equal mobility such that in general the coarser fractions are somewhat more difficult to entrain and transport than the finer fractions; in other words, the combined effects of hiding–sheltering and rollability are insufficient to counteract fully the effect of particle weight.

40 The incomplete approach to the condition of equal mobility leads to two related concepts: selective entrainment and partial transport. The term *selective entrainment* refers to differences in movement thresholds among the various size, shape, and density fractions of a sediment consisting of a mixture of particle sizes,

shapes and densities. The emphasis here is on size-selective entrainment, although density-selective entrainment is one of the keys to the development of placers. Much of the work on selective entrainment is owing to Komar (1987a, 1987b, 1989).

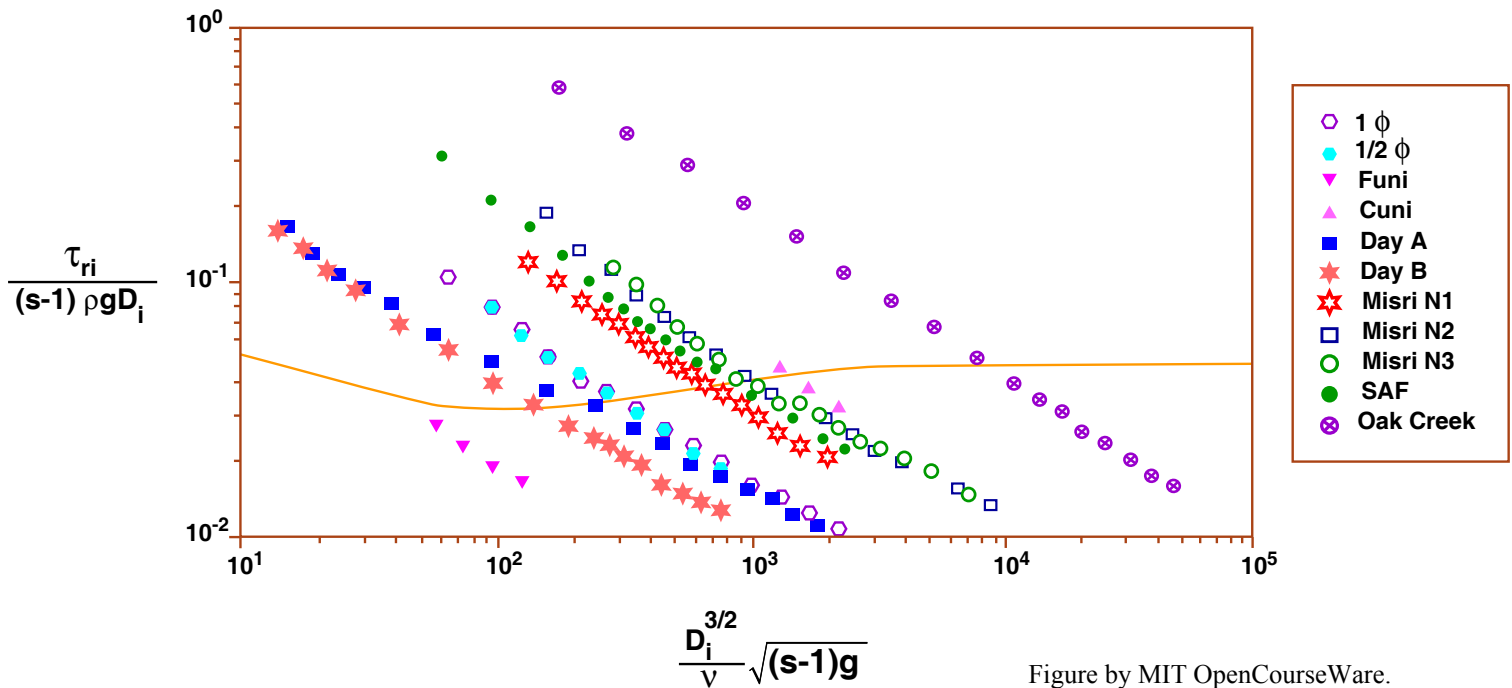


Figure by MIT OpenCourseWare.

Figure 14-21. Plot of τ_{ci}^* against $D^{3/2} \gamma' \rho / \mu^2$ taken to the one-half power (see text) for the same data sets as are shown in Figure 14-19. The solid curve is the Shields curve, transformed into the coordinates of this plot. (From Wilcock and Southard, 1988.)

41 A concept related to that of selective entrainment is that of *partial transport*: for a range of bed shear stresses above the condition of no particle movement, a given size fraction may comprise two populations: (1) particles that are moved, occasionally, by the flow; and (2) particles that are never moved by the flow, and remain motionless on the bed (Wilcock and McArdeU, 1993, 1997). The domain of partial transport lies between the range of bed shear stresses for which there is no motion of any of the particles of the given size fraction, on the one hand, and the range of greater bed shear stress for which all of the particles of the given size fraction are moved by the flow at one time or another. In general, the lower and upper limits of this range differ from size fraction to size fraction. A corollary is that, when all of the size fractions are considered, the domain of partial transport extends from the upper limit of bed shear stress for which no particles of any size fraction are moved by the flow, on the one hand, and the lower limit of bed shear stress for which at least some of the particles of all of the size fractions are moved at one time or another, on the other hand.

42 What is the relationship between partial transport and fractional transport rates? Insight into that question comes from flume experiments on

partial transport by Wilcock and McArdell (1993, 1997). The sediment was unimodal but poorly sorted mixture with the distinctive feature that all of the particles of each of the size fractions was painted a different color, to facilitate observations of particle immobility and particle movement on the sediment bed. Observations of partial transport, as well as fraction transport rates, were made in several runs over a range of bed shear stress that bracketed the domain of partial transport as described above.

43 Figure 14-22, a plot of fractional transport rate against particle size, shows the relationship between partial transport and fractional transport rates for each of five runs. In Figure 14-22 the limiting value of the active proportion of particles in each size fraction, after a long running time, is denoted by Y_i . Each data point in Figure 14-22 represents a given size fraction in a given one of the five runs. The proportion of the particles of the given fraction that are mobile increases from lower right to upper left for each curve.

44 The distinctive feature of the plot in Figure 14-22 is that the curves for fractional transport rate flatten to near horizontality (a condition approaching equal mobility) as flow strength increase. In other words, deviations from equal mobility are large within the domain of partial transport but become small for bed shear stresses above the domain of partial transport.

MORE ON SEDIMENT-DISCHARGE FORMULAS: SUBSURFACE-BASED MODELS VERSUS SURFACE-BASED MODELS

45 All sediment-discharge formulas, including those described briefly in Chapter 13, make use, in one way or another, of the sediment size, usually the median size. Some such approaches have attempted to deal with mixed-size sediments by introducing a “hiding function” that takes account of the hiding–sheltering effect, but even those need to be based on a particular size distribution of the sediment.

46 The question then arises: which size distribution should be used? That of the sediment in the substrate, or that of the bed surface, which the flow actually sees? The latter would seem to be the more natural choice. As you have seen, in mixed-size sediments, especially those with both sand and gravel fractions, the sediment bed surface tends to become paved with sediment that is, on average, coarser than the substrate sediment. The problem is that the surface size distribution is itself a function of the flow. Moreover, only in carefully designed laboratory flume experiments is it possible to observe the surface size distribution, and only a few studies have succeeded in doing so. Up to now, only a few transport models based on the surface size distribution rather than the substrate distribution have been developed (Proffitt and Sutherland, 1983; Parker, 1990; Wilcock and Crowe, 2003).

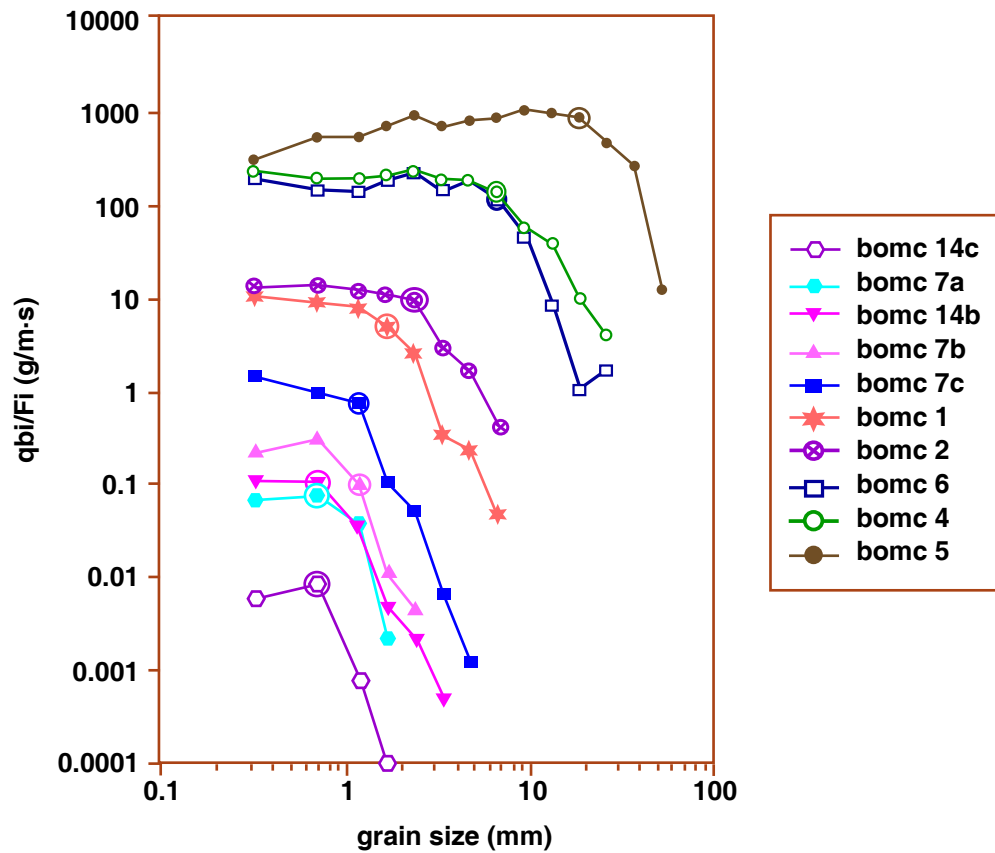


Figure by MIT OpenCourseWare.

Figure 14-22. Plot of fractional transport rate q_{bi}/F_i (where F_i is the proportion of size fraction i on the bed surface) against particle size. Bed shear stress ranges from the lowest values (solid diamonds) to the highest values (solid squares). The large open circles represent the largest fully mobilized particle size in each run. (From Wilcock and McArdell, 1993.)

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