Lecture notes for 12.009, Theoretical Environmental Analysis

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1.1   The big picture

1.1.1   What is theoretical environmental analysis?

Environment: the world around us, living and non-living.

Analysis: an investigation of the component parts of a whole and their relations in making up the whole (http://wordnetweb.princeton.edu).

Theoretical: mathematical, guided by physical reasoning.

1.1.2   What sort of problems do we study?

We focus on problems that

- teach us about the interaction of the physical and biological worlds;

- exhibit interesting principles of dynamics and self-organization; and

- provide opportunities for learning widely useful mathematical tools and physical concepts.

1.1.3   Why focus on *theoretical* analysis?

Observational data is hugely important in environmental science. But these data must be understood.

We focus on problems for which understanding is not merely a matter of measurement.

In this course, we consider that observed data are *understood* when trends in the data can be quantitatively predicted by a theoretical model dependent on no more than a few parameters.

We therefore use mathematics for two reasons:

- to concisely describe complex phenomena; and

- to develop insight (predictions) by deductive reasoning.

In other words: the interplay of theory and observation is a major part of this course.

1.1.4 A broad overview

The course is roughly divided into three parts:

- *Part I* is loosely organized around Earth's carbon cycle, with an emphasis on diffusive transport.

- *Part II* focuses on climate cycles, their relation to celestial mechanics, and the analysis of periodic phenomena.

- *Part III* features ecological organization and dynamics, and nonlinear dynamics more generally.

We proceed to point out some of the highlights of the syllabus.
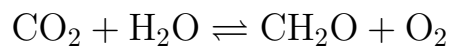
1.2 Earth's carbon cycle

It is conventional to consider two carbon cycles: biological and geochemical.

The *biological* cycle is *fast* (i.e., minutes to 1000s of years).

The *geological* cycle is *slow*, including processes that play out over 10^6–10^8 yr.

1.2.1 The biological cycle

In the simplest statement of the carbon cycle, one has the reaction

$$CO_2 + H_2O \rightleftharpoons CH_2O + O_2$$

Photosynthesis goes to the right, respiration to the left. CH_2O is shorthand for organic carbon, i.e., a carbohydrate "fixed" by photosynthesis from CO_2.

Roughly half this reaction takes place on land, the other half at sea. But nearly all of the CO_2 in the atmosphere and oceans is in the oceans.

This is the essence of the *biological* carbon cycle, which is nearly a closed system: about 99.9% of the carbon fixed by photosynthesis returns back to the oceans and atmosphere via respiration.

1.2.2   The geological cycle

The above description implicitly assumes a kind of single, well-stirred reactor.

The reality contains much spatial structure and relies on passive forms of transport:
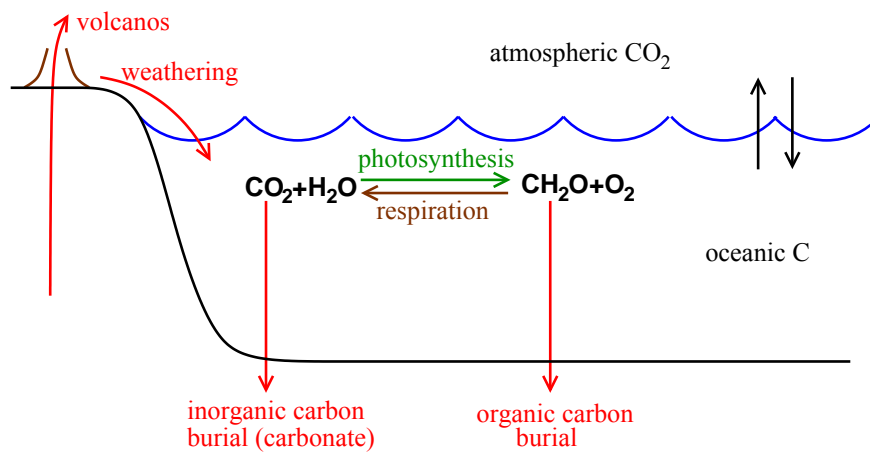
Let us proceed by considering the 0.1% of organic carbon that is *not* recycled "immediately."

This small fraction is *immobilized* or "buried" as rock.

Indeed some inorganic carbon (i.e., carbonate) is also turned into rock.

This process of immobilization amounts to a carbon *sink*.

If CO_2 in the atmosphere and oceans is not to be depleted, there must be a source to counter the burial sink.

1.2.3   Appearance within the syllabus

We begin the term by discussing the mechanism that generates the CO_2 source:

- Plate tectonics: the volcanic source

    - Volcanism as a long-term CO_2 source.
    - Thermal convection within the Earth; the Rayleigh number.
    - Seafloor heat flux, topography, and thermal diffusion.
    - Diffusive scaling.

We then ask a simple question: what is the short-term fate of a pulse of CO_2 injected into the atmosphere?

- Short-term evolution atmospheric CO_2.

    - The Keeling curve.
    - The radiocarbon bomb spike as an impulse response.
    - Microscopic (random-walk) model of molecular diffusion.
   - Diffusive exchange with the oceans.

Next, we consider the structure of the river networks that drain dissolved carbonate (and organic carbon) into the oceans:

- Scaling laws for rivers and runoff.

    - Fluvial transport as a sink for CO_2.
    - The geometry of river basins.
    - Power laws, fractals, allometry, and scale invariance.
    - Random-walk model, null models, and universality.

Note the ubiquitous appearance of diffusive transport, a consequence of the passive nature of these natural processes.

1.3 Climate cycles and the analysis of periodic phenomena

Over the last 400,000 yr, both climate and CO_2 levels have oscillated with a strongly evident 100 Kyr period [1]:
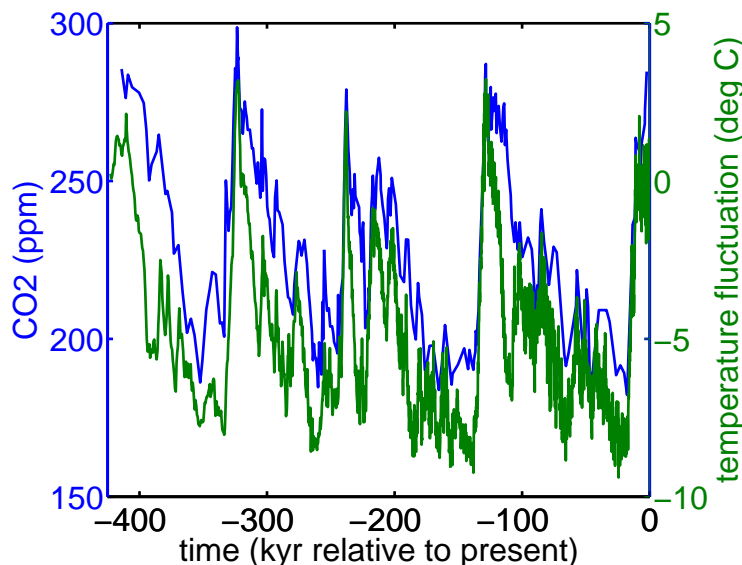
These data represent cycles between *glacial* and *inter-glacial* climates.

We provide a detailed discussion of their foundation in orbital dynamics, and highlight unsolved problems.

- Natural climate change: glacial cycles.
    - Ice-core records of climate change.
    - Milankovitch cycles.
    - Precession, obliquity, eccentricity, and insolation.
    - Enigmatic significance of the eccentricity time scale.
    - Enigmatic correlation of climate and CO_2.

We next discuss the computation and interpretation of power spectra, an important tool for the analysis of periodic phenomena.

- Quantitative analysis of periodic phenomena.

- Discrete Fourier transform.

- Power spectrum and autocorrelation function.

- Power spectra of periodic signals and white noise.

- Identification of spectral peaks.

1.4 Ecological organization and dynamics

Part III focuses on ecology and dynamics.

We begin with a discussion of some fascinating modern ideas rooted in observed *scaling laws* and the geometry of ecological networks.

- Ecological organization.

  - Energetic limits on the length of food chains.

  - Food webs and scale-free networks.

  - Body size, temperature, and metabolic scaling.

  - Ecological equipartition.

Examples of (500 million-year-old) food webs [2]:

Chengjiang Shale

Original Species
$S = 85$, $L = 559$, $C = 0.077$
$TL = 2.99$, Max$TL = 5.15$

Trophic Species
$S = 33$, $L = 99$, $C = 0.091$
$TL = 2.84$, Max$TL = 4.36$

Burgess Shale

Original Species
$S = 142$, $L = 771$, $C = 0.038$
$TL = 2.42$, Max$TL = 3.67$

Trophic Species
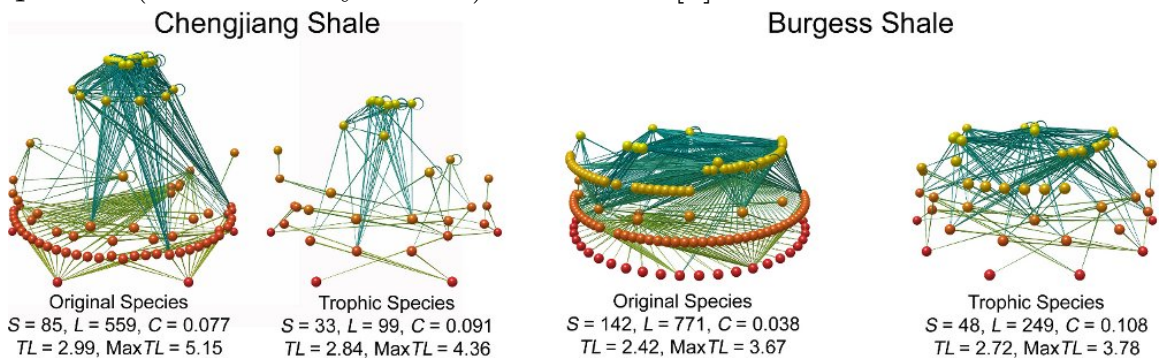$S = 48$, $L = 249$, $C = 0.108$
$TL = 2.72$, Max$TL = 3.78$

Image courtesy of Dunne J. A, Williams R. J, Martinez N. D, Wood R. A, and Erwin D. H. Published by the Public Library of Science.

We next consider what it means for an ecosystem to be stable and consider how stability relates to ecosystem complexity. Considerations of population dynamics lead to an elementary model of chaos.

- Ecosystem stability and chaos.

  – Elements of population dynamics.

  – Linear stability analysis.

  – Predator-prey cycles: the Lotka-Volterra equation.

  – Stability vs. complexity.

  – The fossil record of biodiversity and mass extinctions.

  – Discrete logistic model and period doubling.

  – Chaos.

This fossil record suggests that the global marine animal ecosystem has been anything but stable over geologic time:
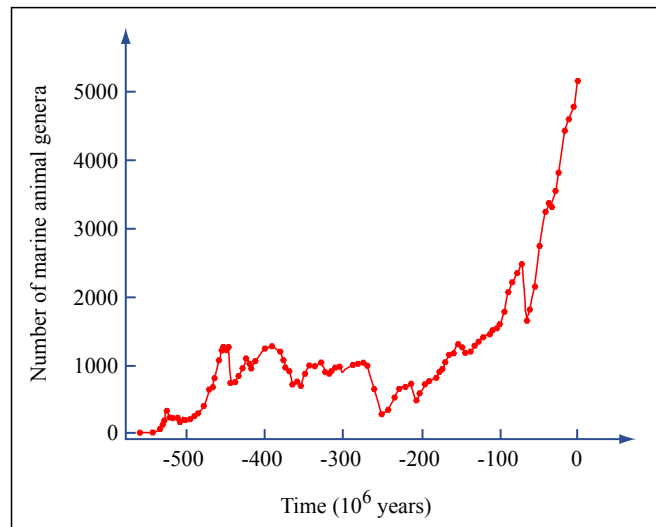

Image by MIT OpenCourseWare.

1.5 A bigger, briefer picture

Note that we begin the term with a discussion of plate tectonics, an idea that revolutionized (in the 1960s) how one views the evolution of the Earth.

We then examine natural cycles of climate change, an observation that greatly changed how we view Earth's climate.

We close the term with a discussion of chaos theory, one of the greatest scientific achievements of the 20th century.

These subjects, and those between, arise naturally in a discussion of *how nature works.*

They are further unified here by their relation to basic physical ideas and methods of analysis.

1.6 Who should take the course?

- Students seeking a quantitative understanding of how the environment works.

- Students who wish to learn physical concepts and mathematical tools useful for understanding how any system works.

- Required for EAPS students, but addressed to all students, without EAPS prereqs.

1.7 Requirements

- Problem sets, roughly once per week. Cooperation ok but acknowledgement of collaborators required.

- Take-home midterm. No collaboration allowed.

- Take-home final exam due the last day of classes. No collaboration allowed.

Grades: Roughly 50% problem sets, 10% midterm, 30% final, and 10% class participation.

References

[1] Petit, J. *et al.* Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica. *Nature* **399**, 429–436 (1999).

[2] Dunne, J. A., Williams, R. J., Martinez, N. D., Wood, R. A. & Erwin, D. H. Compilation and network analyses of Cambrian food webs. *PLoS Biology* **6**, e102 (2008).

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