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12.003 Atmosphere, Ocean and Climate Dynamics
Fall 2008

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Problem set 3: Hydrostatic balance and lapse rate

Due date: October 1st, 2008

1. A spherical spaceship with radius 10 meters is pressurized to an interior pressure of 1000 hPa. The exterior pressure can be regarded as zero. What is the net force (in Newtons) exerted on the walls of the spaceship?
2. Somewhere (in a galaxy far, far away) there is a planet whose atmosphere is just like that of the Earth in all respects but one—it contains no moisture. The planet's troposphere is maintained by convection to be neutrally stable to vertical displacements. Its stratosphere is in radiative equilibrium, at a uniform temperature -80°C , and temperature is continuous across the tropopause. If the surface pressure is 1000 hPa, and the equatorial surface temperature is 32°C , what is the pressure at the equatorial tropopause?
3. For a perfect gas undergoing changes dT in temperature and dV in specific volume (the inverse of density), the change ds in specific entropy, s , is given by

$$Tds = c_v dT + pdV. \quad (1)$$

- (a) Hence, for unsaturated air, show that potential temperature θ

$$\theta = T \left(\frac{p_0}{p} \right)^k \quad (2)$$

(notation defined in class) is a measure of specific entropy; specifically, that

$$s = c_p \ln \theta + \text{constant}. \quad (3)$$

where c_v and c_p are specific heats at constant volume and constant pressure, respectively.

- (b) Show that if the environmental lapse rate is dry adiabatic, then it has constant potential temperature.
4. In this exercise you need the measurements of pressure you collected at various floors in the Green Building with a barometer. You want to use these pressure measurements to estimate the height of the Green Building and the height of the 9th floor. Be careful with your units.
 - (a) First derive the relationship between the height of the Green Building and the pressure drop between the ground and the top of the building. Then use this relationship to compute the height of the Green Building. Assume that the air is at a constant temperature of $T = 20^\circ\text{C}$.

- (b) Compute the height of the 9th floor using the pressure measurements at the ground and at the 9th floor. Assume that the atmosphere is at a constant temperature of $T = 20^\circ\text{C}$.
- (c) Now assume that the air of the atmosphere drops with height z as $T = 20^\circ\text{C} + \Gamma_a z$, with $\Gamma_a = -6.5^\circ\text{C}/\text{km}$. Derive the relationship between the height of the Green Building and the pressure drop between the ground and the top of the building in this case. Use this relationship to compute the height of the Green Building. Does your estimate differ from what you reported in the answer to 4a?
5. Criticize the statement: *Since the Earth's atmosphere is warmest near the ground, it must necessarily be statically unstable.*
6. A jetliner flying at 40,000 feet altitude has a cabin pressure equivalent to an altitude of 10,000 feet. The plane pumps in ambient air, which must be heated or cooled to the same temperature as the air at the ground level. Does the plane need an air conditioner or a heater?