GABRIEL SANCHEZ-MARTINEZ: Let's get started with Lecture 9. Today's lecture is very relevant to the homework. We'll run through some examples of performance models like the ones you have to work on or specify an estimate on your homework.

So we talk about performance, performance models. What is performance? What do we mean by performance?

Typically, this is a key word used to describe output and some sense of how well the output service was, so both in terms of the operator and in terms of the passenger. So things like running times, waiting times, headways, things that you can measure from observations of service being delivered, we call that performance, generally. And the kinds of models that we're going to look at are wait time models, service variation along routes where you'll see how it's not always the same. You might start off at the terminal one way and end up a very different way. We'll look at running time models and dwell time models.

So these are the components of running times. Let's start with waiting time. Well, before that, actually, let's think about-- so here are some kinds of models. Why would we be interested in modeling these things? What applications can you think of for these kinds of models?

That's two ways of putting the same question. Why would we be interested in having a waiting time model or a running time model? Just give some examples. Yeah, over in the back?

AUDIENCE: We want to understand how people are waiting exactly.

GABRIEL SANCHEZ-MARTINEZ: But you can go out and measure it, right? You can go out and do a survey and observe how people are waiting. So why would a model be good? What would a model do that observations don't do?

AUDIENCE: For a new service.

GABRIEL For a new service, OK, we're starting to get some ideas, yeah.

SANCHEZ-MARTINEZ:

AUDIENCE: Or changes in service.

GABRIEL SANCHEZ-MARTINEZ: Changes in service-- OK, so can you give me an example of that?

AUDIENCE: Like [INAUDIBLE] making the Victoria line 20% more frequent.

GABRIEL SANCHEZ-MARTINEZ: OK, so if you increase frequency on the Victoria line, what does that do to demand and to dwell time and to--

AUDIENCE: The Oxford Circuit is now closed for 30 minutes a day because can only interchange, not enter.

GABRIEL SANCHEZ-MARTINEZ: OK, so you can try to use them to predict service changes or changes in performance due to changes in service. What else, any other ideas? [INAUDIBLE]?

AUDIENCE: [INAUDIBLE] service variation, that's [INAUDIBLE] to crowded and to crowded on the bus. It's depending on how the planned headway sort of variance over time will impact how many people end up boarding on a bus. Let's say, a bus that came substantially after the previous bus might be crowded. And I won't be able board. And you might even have-- so denied boardings for--

GABRIEL SANCHEZ-MARTINEZ: [INAUDIBLE] will you summarize that? Maybe I'll try summarizing. You could use one of these models to fill in things about performance that you can't measure directly from data. So you gave an example of measuring headways and then estimating crowding on the bus based on headways. Was that more or less--

AUDIENCE: Yeah.

GABRIEL SANCHEZ-MARTINEZ: --getting at--

AUDIENCE: [INAUDIBLE]

GABRIEL SANCHEZ-MARTINEZ: OK, trying to generalize a little bit-- any other ideas? [INAUDIBLE]?

AUDIENCE: I mean, you could use this as a [INAUDIBLE] model in any other simulation.

GABRIEL SANCHEZ-MARTINEZ: Simulation models, so if you want to do a simulation model of a system that has transit in it, you need all these things to-- right? Because you need to have your [INAUDIBLE] which, it could be a bus, or a passenger waiting, or dwelling, or moving between stops. So you need these things. Any other applications, practical, very practical applications?

So for dwell time and running time models, we all have our prediction of when the bus is coming to the stop. So you need some model to take the bus from where it is right now, which you can measure, to how long will it take me to reach this stop. And so the passenger can look up on the phone, how long will I have to wait. So these are all examples. And there are more.

Let's start with waiting time. So we've already seen this first simple model of waiting time. We say that the expected waiting time is half the headway. And here, we are considering both waiting time and headway to be stochastic quantities. So if we observe many headways over a long period of time, we see a probability distribution of headways. And we're saying, the average headway divided by 2 should equal the average waiting time.

Now, this is a very simple model. And we know there are some problems with it. So we are assuming that passengers arrive independent of vehicle departure times. We are assuming that vehicles are departing at equal intervals deterministically, so they're sort of evenly spaced, and that every passenger can board the first vehicle they see. So nobody is left behind by a bus that is too full.

So obviously, these things don't always hold. And particularly, the part of vehicles departing exactly at equal intervals doesn't hold. It rarely holds.

AUDIENCE: Even if they depart, there are a lot of things along the way that impact the variation.

GABRIEL SANCHEZ-MARTINEZ: Yeah, so but we're saying departing from each stop deterministically, not just the terminal. So obviously, there are some problems with this. And so how do we need to-- well, if we start taking care of some of these things, what will happen to waiting time? Will it decrease or increase?

So if we take this particular one. This is the strongest assumption, right? So that vehicles depart deterministically at equal intervals-- and we say, no, they don't, actually. Some of them depart late. Some of them depart early. There is bunching. So what happens to waiting time when that happens?

AUDIENCE: It goes up.

GABRIEL SANCHEZ-MARTINEZ: It goes up. So how do we adjust this model to account for that? Let's look at that.

AUDIENCE: Doesn't it depend though when the passengers are arriving?

GABRIEL SANCHEZ-MARTINEZ: If you look at an average or a long period of time, and you'd have the same number of vehicles and drivers, so you either-- the best thing you can do if people are arriving randomly is to--

AUDIENCE: OK, they're arriving randomly.

GABRIEL SANCHEZ-MARTINEZ: Yeah, so we're only tackling that assumption. OK, so there are some issues, as we said. There is bulk arrivals. So you have a bus stop right outside of a train station. And the train has just left. And a bunch of people get off the train and they all want to board the bus. That's not being captured by this.

And we can think of the passenger arrival process in steps, from having no information to having a lot of information. So random arrivals is what we typically assume, certainly for high-frequency service, and in some cases, for all kinds of service. And that's more problematic.

If you know how long-headway service, people are going to try to time their arrival at the stop to the schedule. But you will see some models that-- in the literature, that, for simplicity, especially service planning models, for simplicity, might assume this anyway. You have a question.

AUDIENCE: Like random, but Poisson distributed, or just, like, random?

GABRIEL SANCHEZ- Yeah, Poisson distributed-- sometimes it could just be random. Typically the assumption made is that it's a Poisson process. So inter-arrival times are negative exponential distribution.

MARTINEZ:

OK, then some passengers will time their arrivals to minimum waiting. So this could be that you have a phone app or a schedule. And you show up a couple of minutes before.

You're trying to minimize your waiting time. You're trying to arrive shortly before the vehicle departs. And then there is the running to the vehicle before it leaves the stop, and therefore, I have no waiting. That's everyone's favorite, right?

So if we look at a graph of expected headway on the horizontal axis and expected waiting time on the vertical axis, you will see that around 10 minutes-- some people say 15, some people say 10, but somewhere around there, 12-- so you will see that the actual waiting time that you would observe is much lower than what the simple model says it is, which is half the headway for headways longer than 10, 12, 15 minutes. And I think those people will have some strategy to minimize their waiting time. So below that amount, people tend to not time their arrivals at stops. And they tend to arrive randomly, essentially.

So if you try to take the Red line here in Boston, you just show up at whatever time. And typically, most people won't be looking at their phone and trying to time their arrival, although that is happening increasingly. So for those people, the model, the simple model of headway divided by 2 tends to underestimate the actual waiting time. And that's due to headway variability, mostly.

So we have to take care of that assumption. Let's look at a formulation that relaxes that. Let's say that vehicle departures are not regular and deterministic. So let's refine the model to take care of that.

So let's define n as a function of headway to be the number of passengers arriving at some headway and the mean waiting time of headway, h, to be, well, that mean headway of some specific headway that we observe, and then g to be the probability density function of headway. So we observe many of these highways. And we have a histogram of headways.

So if you want to compute the expected waiting time across all passengers, what we want is the expected total passenger waiting time divided by the expected number of passengers over many, many observations of vehicles leaving. And that is expressed mathematically in the equation below. We have integrals, in this case, from 0 to infinity, because headways can't be negative, so we're not going from negative infinity to positive infinity.

And we're saying, yeah, for any given headway, we'll multiply the number of people times how much they wait on average times the probability that I see that headway. And let's integrate over all possible headways and divide that by the number of people on each headway. Does that make sense conceptually?

OK, so now let's make some assumptions. Let's say that people do arrive uniformly with some rate, lambda. And so the number of people arriving in a given headway is going to be some arrival rate, which we've said is constant, times the headway.

So if we say that 10 people arrive per minute, and lambda therefore is 10 passengers per minute, then if the headway is 2 minutes, we have 20 people. Does that make sense? Any questions so far?

OK, and then--

AUDIENCE: Can you repeat just what you said there?

GABRIEL SANCHEZ-MARTINEZ: Yeah, so lambda is the passenger arrival rate. And I gave an example of it being 10 passengers per minute. So is that something you measure. And we're assuming that that amount is constant for a specific time of day at a specific place.

And then we're saying, if you-- once you assume that, then you observe a headway, which could be, say, 2 minutes long. Then you multiply by the arrival rate. Now we say it's 20 passengers for that particular headway. And there might be another headway that is 5 minutes long. That's 50 people-- 1 minute long, 10 people, and so forth. And then for each of those headways, the average waiting time is going to be half of that headway, correct?

AUDIENCE: Correct.

GABRIEL SANCHEZ-MARTINEZ: We're assuming people arrive randomly, so within a particular headway, this is true. There's no problem with this assumption as long as you are comfortable with assuming that people arrive randomly. So now we have this equation for expected waiting time. Let me write on the board how we get to that from the integrals.

Can somebody say why-- so why is it the case that, when we have headway variability, the waiting time goes up? What's the concept that leads to that? Eli?

AUDIENCE: There is unreliability, and so people need to build in more [INAUDIBLE] time instead of--

GABRIEL SANCHEZ-MARTINEZ: There is something else that is sort of more fundamental happening.

AUDIENCE: A bus that comes at a larger headway than it was planned will actually be collecting more people. So more people will be waiting more time.

GABRIEL SANCHEZ-MARTINEZ: So from a passenger's perspective, the probability of arriving during a long headway is greater than arriving during a short headway. What is that called? There is a name for that that's important in transportation.

Random-- random incidence, so this is something you, if you took 200, you should know that. But it's good to know, random incidence. It's called the random incidence paradox, because people without-- if you don't think about it too much, you say it's half the headway.

And then somebody says, no, actually the average waiting time is much longer. Why is that? And you have to sort of solve the paradox. It isn't really a paradox, but here we are.

So the expected waiting time-- we'll start from the equation, the last equation on slide four. And we're just going to plug in what our assumption is. So we have integral from 0 to infinity. And we said we had n of h.

And we said that that's going to be lambda h. So let's just put that in, lambda h. And then we said, for waiting time, we said it was going to be 1/2, so 1/2 h. And then we're going to multiply times the PVF the probability of that headway times dh.

All right, and here we have the same, but without the waiting time. So that's just substitution, shouldn't be anything wrong with that. So now, lambda and 1/2 come out. They're not a function of h, so they are constants. We can take them out.

And we are left with, let's see, lambda over 2 integral from 0 to infinity of h squared-- because we have two h's here-- times dh dh. And here we have-- lambda comes out. And we integrate from 0 to infinity of h dh dh.

OK, now the first observation is that lambda has come out on both sides. So there's just one lambda on the top, one on the bottom, so we can cancel them out. So that's very convenient. What does that mean?

This happens because we are assuming that lambda is constant. That wouldn't happen if it weren't constant. But that's convenient, right? So now we have some quantity that does not depend on the arrival rate. So the same equation is not affected by that.

And let's see if we can recognize what these quantities are. So let's start with the one on the bottom. This is the definition of the expectancy. So this is just the average.

So we're essentially saying, take every-- take the average headway, so take every headway, and multiply times the probability of observing it. Add them all up. That's what the average is.

So we have 2 times the expected headway. And at the top, we have the expectants of h squared. So I think that takes us to where we are on this slide, which is great.

So part of it was just recognizing that that was the definition of expectants, so that's great. And now, somehow, we go from here to this equation. It's not entirely clear, but it's convenient that variance is a function of the expectants of h squared.

So I'll just remind you, if you haven't seen it, then introduce you to the definition-- or not quite the definition, actually. This is you take the definition and you expand it and collect like terms. And then you get that the variance of h is the expectation of h squared minus the expectation of h quantity squared, so means of square minus squares mean. You might have seen that before.

So essentially, if we solve that equation for expectation of h squared and plug that in here, we can collect like terms and get this last equation. So now we have that the expected waiting time is half the headway times some adjustment factor. And that adjustment factor is 1 plus some amount-- that amount is what we call the coefficient of variation-- squared.

So coefficient of variation is standard deviation divided by mean. So we have normalizing the standard deviation. You square that quantity. You add it to 1. And that'll increase your waiting time by some amount.

Let's stop here for a second. Any questions with the derivation and what the meaning of this is? Yes, Henry?

AUDIENCE: Can you go back to that step where you went from the integral to 2 e of h.

GABRIEL Here?

SANCHEZ-MARTINEZ:

AUDIENCE: Yeah.

GABRIEL SANCHEZ-MARTINEZ: So this is a matter of definition. So this here, the integral from 0-- from negative infinity to positive infinity of some amount times the probability of observing that amount over all possibilities of the amount-- that's why we go from negative infinity to positive infinity-- equals the average of that amount, or the expectation of that amount. So that part-- do you understand?

AUDIENCE: [INAUDIBLE]

GABRIEL SANCHEZ-MARTINEZ: Yes? And then here, we're just saying, well, this is the same thing, except of the amount instead of being h is h squared. So we're taking the average of h squared. This comes from here to here from the definition of expectation.

And the reason we're not going from negative infinity to positive infinity is that h can only take non-negative values. It can be 0 or higher. So it's the same thing to integrate from 0 to infinity, in that case. Does that clear up the question? Any other questions?

So now we have this modified model of waiting time, mainly for high-frequency service, where people are arriving randomly but vehicles are not necessarily departing stops deterministically on equal intervals anymore. Now these vehicles can come-- as long as they come independent of the passenger arrival process, it doesn't really matter how they come. This model calculates the expected waiting time across all passengers.

So we have some cases that we can look at. First case is simple. Let's say that the variance is 0. That means we go back to the assumption of, well, if the variance of headway is 0, vehicles are arriving exactly every five minutes, say.

They're deterministically at equally spaced intervals. So we then have that the coefficients of variation is 0. And we end up with the same thing we had before. So that really checks out with our previous model in that special case, but we generalized the model.

So we can try it with different vehicle arrival or departure processes. One that we have here is, let's say that vehicle departures are a Poisson process. So this would mean that they are arriving randomly, essentially, and that the time of arrival of a vehicle at a stop is independent of any other previous arrival.

So that would be a good model for a service that is controlled very loosely. They are hardly controlled. So you essentially sometimes see bunching. Sometimes you don't. Sometimes you have a long headway. Sometimes you don't.

So the definition-- if you go through the Poisson process and look at the definitions of the probability distributions of inter-arrival times, et cetera, you will see that the variance in such a process is the square of the expectation, so the square of the mean. And if we plug that into our equation, we get that the expected waiting time is the expected headway. So people are essentially waiting about as long as your mean headway, which is a bad situation.

You've doubled their waiting time from half the headway to the full headway. If you have a system where your vehicles are all bunched and you're running pairs of bunches, that's more or less what you get, because you have-- if you had vehicles every 5 minutes, now you have two vehicles every 10. Your average headway is five, but your waiting time is now five as well.

OK, what about the third case? The headway sequence is 5, 15, 5, 15, 5, 15-- thank you, keep going like that, so not quite bunched all the way, but there is a lot of variability. So I'm saying that the expected headway is 10. Does that make sense?

It's 5, 15, 5, 15. The average of that process is 10. So we have an average headway of 10. And now what's the expected waiting time? So it helps to draw a timeline and to divide that timeline into pieces, so 5, 15, 5, 15.

So we have vehicles arriving or departing here, here, here, and here. And there is a five-minute headway here, and a 15-minute headway here, a five-minute headway there, and a 15-minute headway here. And people are arriving randomly. People arrive independent of this process.

So what you'll see is that, the first thing is, if you were to arrive on a five-minute headway, the average waiting time for you would be half of that five-minute headway, 2 and 1/2 minutes. And if you arrive on 15-minute headway, then your expected waiting time, given that you arrive on a 15-minute headway is 7 and 1/2 minutes. So that's the 2.5 and the 7.5 on this equation.

And then we have the probabilities of having arrived on any of those headways. Because the 15-minute headway is 3 times longer than the 5-minute headway, you are 3 times more likely to arrive during a 15-minute headway than during a 5-minute headway. So therefore, 25% of passengers will arrive on the 15-minute headways. And you essentially computing a weighted average, so 6 and 1/4 minutes. Questions on this?

All right, another assumption that we were looking at in the first model that we are still having in this model is that people can board the first vehicle that they see arrive. So no vehicle is too full to take passengers. No vehicle is leaving people behind. That's not true.

If we look at any process, and we say here that w0 is the expected waiting time when people can board their first vehicle, so when no vehicle is full, and we then look at all the expected waiting times for passengers including passengers that are left behind, when that capacity is being reached-- so here on the horizontal, we have rho, which is flow over capacity. You may have seen this kind of relationship from queuing theory. So when rho is 0, it means that you will have some supply and nobody is using it.

If rho is 0.5, then your capacity is double of the demand. That's OK usually. If rho is 1, you are-- your capacity equals your demand. And what we see is that the expected waiting time for service will start to approach infinity.

There is a term that we use in, say, queuing theory, for such a system. Does anybody know it? We say that the system blows up when this happens. If you hear that, you'll see that. If you take any sort of queuing theory, you'll hear that.

So as rho equals 1, let's say that you have-- you're providing a bus service with capacity for 100 people per minute. That's pretty high. And then your demand equals 100 people per minute.

So if people are arriving uniformly, you might just provide a capacity for everyone. And nobody is being left behind. But that doesn't happen, because people arrive randomly.

Some people will leave with some remaining capacity. And the next vehicle won't have enough space for that person. And then the vehicles themselves will not arrive perfectly evenly, so now you have a chain reaction of people waiting and not being able to board.

And unless you supply more capacity, unless you increase the capacity, you're going to have more and more people waiting and being left behind. So the actual expected waiting time would increase as your ratio of flow to capacity approaches 1. That's the takeaway here.

And you could think of two different lines, one for low reliability when there is a lot of bunching, and therefore, a lot of the capacity that you provide is wasted on small headways that are not serving as many passengers as you assign them for, and then high reliability, where, yes, you still have some variability that you can't control.

You've done what you can. And this is relatively high reliability, but still you have some variability, and therefore, you can't expect to reach that rho equals 1. It'll still blow up when you-- as you approach 1. But it's closer to 1, so that's good. Questions on this relationship? [INAUDIBLE]?

AUDIENCE: What do most bus services operate at [INAUDIBLE]?

GABRIEL SANCHEZ-MARTINEZ: I don't have a number off the top of my head. It varies a lot. And it varies a lot of by time period. So some services here in Boston operate during the peak leaving people behind every day.

If you look at some of the most crowded stations in the tube in London, certainly they have to meter people going into stations. So they may have approached rho greater than 1, in this case. But it's non-stationary. So that's during the peak of the peak.

And then the demand falls, so eventually, those people are served, of course, because the supply is kept up at some-- at the highest rate that they can keep it. And demand it comes down, so it stabilizes. Any other questions? That's a good question.

All right, so service variation along routes-- so we've been talking about vehicles leaving on time or not on time, and bunching. So here's what happens. This is a space-time diagram. Have we seen these diagrams on this course yet?

AUDIENCE: No.

GABRIEL SANCHEZ-MARTINEZ: No-- OK, well this is a very important diagram in public transportation, certainly.

AUDIENCE: And the homework says to make one. So we should probably--

GABRIEL SANCHEZ-MARTINEZ: Great.

AUDIENCE: --learn it. I was wondering what it was.

GABRIEL SANCHEZ-MARTINEZ: This is a space-time diagram. You have space on the y-axis and time on the x-axis. And what you see are some lines that show how a vehicle is moving across space and time. So there are variations of this. Sometimes people put time on the vertical and space on the horizontal. That's fine. And typically, you see little steps for each stop, because if a vehicle is stopped, time is running and the vehicle is not moving, so you see little steps. So you can see holding. You can see dwell times. You can see speeds. You can see recovery time at a terminal, everything on this diagram. So it's a great diagram for analyzing service.

AUDIENCE: When we make, should we make them like this? Or like--

GABRIEL SANCHEZ-MARTINEZ: The little steps are--

AUDIENCE: Little steps--

GABRIEL SANCHEZ-MARTINEZ: Yeah, I mean, they're helpful. You'll see dwell times, so yeah. So this is a conceptual very simplified diagram, but here is the idea.

So you have some scheduled trajectories in dashed lines. So we're essentially planning to run service every 10 minutes, departing from Stop 1 at 9:00, 9:10, 9:20, et cetera. And if things run according to plan, you will keep a 10-minute headway throughout the route.

But that's not what happens. If we observe-- let's say that the driver that was driving the bus that departed at 9:10 is a slow driver, so that driver drives slowly. So therefore, more time passes covering the same space.

And then let's say that the 9:20 driver is a fast driver. So that driver covers more distance in less time. And they bunch.

They meet somewhere before they reach the end of the route. They've platooned. And they're running together as a bunch. So what does that do to headway?

AUDIENCE: What does what do to headway?

GABRIEL SANCHEZ-MARTINEZ: What does this process of bunching do to headways?

AUDIENCE: It increases.

GABRIEL SANCHEZ-MARTINEZ: Right? Headways are increasing, because now you've-- well, the average headway remains the same, because you have a 0 and a big number. But what people actually see are the long headways, because the chance of arriving and just catching the 30-second headway is much smaller. So people are waiting more because these headways are longer. Yeah?

AUDIENCE: Well, you probably have more people waiting. So there is [INAUDIBLE]--

GABRIEL SANCHEZ-MARTINEZ: Yeah, so there's a vicious cycle here, right? Yeah, we'll get to that on the next slide.

AUDIENCE: It would be better if the fast driver got in front of the slow driver.

GABRIEL SANCHEZ-MARTINEZ: Yeah, we'll talk about that too. We'll talk about that too. OK, so do we understand this process of bunching?

Here is what you were talking about, Ari? So there is an issue when that happens. So now we have the little steps here. So this is Step 2, Step 3. And these are the dwell times.

So nothing happened on bus one. It ran exactly as planned in this hypothetical example. But because this driver was slow, the headway was bigger. And more people arrived during the headway.

So now the dwell times at Stop 2 are going to be longer for that driver that was driving slow. Now that bus has more passengers. And therefore, the probability of stopping at the next stop is higher, because you have more people, so the chance that at least one person on that bus wants to get off at the next one is higher.

And if the bus is really crowded, the dwell time process is going to be slowed down by just friction between passengers. So this bus continues to be delayed, not just by the driver driving slow, but by the dwell time effect. And because that bus is being delayed, fewer people are arriving to see the next bus, the third bus.

The dwell times of that third bus are shorter than planned. Fewer people are waiting for that bus. And therefore, that bus has a lower probability of having to stop. And if it stops, fewer people will board it.

So that bus is going to run fast. Even if the drivers are now driving at the same speed, there is nothing you can do. They will bunch. Well, there is something you can-- you can control them. And that's a later topic in the course.

So they pair or they bunch, that's what we're saying. OK, so if we think about how if you were to survey headways of different points along the route, you would see that, if you start at a terminal-- this is like a probability density function, but they're not normalized. So don't think about scale.

So you have headway on the horizontal. And the probability density on the vertical. And so at the terminal or close to the start of the route, you will see something bell-shaped usually. It'll be around the scheduled departure time, or around the headway.

And there will be some variability due to drivers not being exactly on time when they leave, and supervision issues, or whatever it is. So maybe the boarding process at the first stop introduces some perturbations. But it's essentially bell-shaped. And it has less variability.

As you move to the middle of the route, you'll see that the bell is being-- it's getting fatter and fatter tails, so more variability in the headway because of the bunching problem that we just described. Usually by the time you see bunching, your-- which could be at the end, or it could be before the end, you'll have this distribution with two spots, a lot of vehicles having headway of 0 or very close to 0, and a lot of headways longer than that by some amount with a lot of variability.

So these are pairs that are arriving. So every time you see a pair arriving, one of them is 0. And the other one is some longer headway. And they are affecting these two parts of the probability distribution. Is that understood? Any questions? [INAUDIBLE]

AUDIENCE: So the big point you're talking about is the lower--

GABRIEL SANCHEZ- So essentially--

MARTINEZ:

AUDIENCE: --the lower curve?

GABRIEL SANCHEZ-MARTINEZ: --it's just, yes, this one right here. So essentially, it's just higher variability of headways. So variability of headways tends to be minimized at the start of a run. And then that degrades.

It increases as you run the route until you have bunching. And when you have bunching, you reach this point of having some vehicles that are close to 0 and others that are anywhere else. So that means the waiting time is a function of headway as scheduled, also of headway reliability.

So if you're running a service that is well controlled and you have good headways, then you may never reach this point. And you may actually have everything running somewhat bell-shaped, not too much variance. That's your ideal situation.

And then it's also a function of where you are along the route. So if you're closer to a controlled point, which could be the terminal, or it could be anywhere along the route, if there is control en route, then you will have less variability of headways. As you run downstream of the last control point, then you will see greater variability.

Great, so what factors affect the headway deterioration? Length of route is one. So if he no longer route, then it takes an hour and a half to cover the whole one-direction run, the half cycle, then this deterioration has-- that process has more time to act on the route. So you will see more headway unreliability, more bunching.

The marginal dwell time per passengers is another factor. If you think of-- well, we just saw that the bunching process is largely affected by dwell time, because if you have a longer headway, more people are boarding. So if you think of an extreme case where everybody boards and alights instantly, then you don't have that effect anymore.

So to the extent that people can board and alight fast relative to the time it takes to run the service, then this effect diminishes to the extent that dwell time is a larger percentage of the total runtime. Then it increases. You have more bunching.

The stopping probability-- again, this has to do with how many people are on the bus and the stop spacing. If stops are spaced very close to each other, then you have a higher variance of where you stop. if you have a long distance between stations, if you think of a commuter rail line, then you're going to stop at every stop. And therefore, you decrease the variability of this effect of stopping. So you have the urban bus route in one hand and the commuter rail with long distances between stops, and therefore stopping at every stop, on the other hand.

The schedule headway has an impact. If you schedule the headways every 30 minutes, it's unlikely you'll see bunching. But if you schedule the headways every two minutes, then it's very easy for you to see bunching. You have to control this very well to avoid bunching in that case.

And driver behavior-- if your vehicles are driven by very good drivers or computers on the extreme right, driven with driverless trains, then you can really sort of try to make up for all of these things in driving and try to keep things even. If they are drivers that are not particularly well trained and have not received feedback, for example, on their driving speeds and other behavior, and not just that, but also being on time at the terminal and leaving exactly on time, things like that, would all affect this.

So here is a simple model of how headway will deteriorate. So there is also a mistake on the integration. That should be p i, not p i minus 1, sorry about that. So we have that the headway deviation at some stop-- here, we're thinking about a scheduled headway and the actual headway we see.

So if service is running exactly on time, ei is 0. If that bus is delayed, then ei is positive. And if it's running early, ei is negative. So you start out-- you go from one stop to the next. And the deviation of headway you see at some stop is going to be a function of how-- what the deviation was at the previous stop.

So if you're already delayed, your chances of being delayed at the next stop are higher. That should make sense. Then you add the effect of running from that previous stop to this stop.

If you are particularly slow doing that, then your delay will increase. If you try to drive fast to make up for that, then ti will be negative. And it will decrease the impact on headway deterioration. Yeah?

AUDIENCE: Do they ever just-- so this seems to imply that, like, maybe I could fix my problem by just dropping a bus from the schedule. Like, let's say I was really behind, I could just delay all the other buses and, like, take up the schedule of the next--

GABRIEL SANCHEZ-MARTINEZ: Yeah, but you will have-- you still have a headway, a longer headway, right?

AUDIENCE: Yeah, [INAUDIBLE]--

GABRIEL SANCHEZ-MARTINEZ: So you could make up your schedule that way. But you're essentially doing an accounting trick by doing that. You can make your vehicles look like they're all running on time by doing that. And you dropped one trip. But you still have a longer headway here and shorter headways there, and passengers boarding mostly that delayed vehicle.

And so from a passenger's perspective, that doesn't do much. But then you have to consider the effect of, well, you might have-- you might want to do that if your drivers need to check out at some time in the day. And your options are either, you do this now, or at some time when they check out, you have to drop trips.

And that could be during the peak. So that would be really bad, right? So then you would have to consider that strategy.

AUDIENCE: And do buses typically post, like, a scheduled time that they're supposed to arrive at each stop? Because normally--

GABRIEL SANCHEZ-MARTINEZ: Yes.

AUDIENCE: --I see, like--

GABRIEL SANCHEZ-MARTINEZ: --especially now, so typically--

AUDIENCE: Like up-to-date minutes.

GABRIEL SANCHEZ-MARTINEZ: So London will say that. So if it's high frequency-- and London, the threshold is 12 minutes. So anything below, they'll say, AM peak between-- expect service every eight minutes, for example. And then as soon as the headway increases above that, they'll say the times. But if you look at service here, for example, here in Boston, every bus route here, if you look at the

DTFS file, has specific times at each stop, even though DTFS has the option of giving a headway for high-frequency service, they don't do that here.

AUDIENCE: So only service [INAUDIBLE] time appear?

GABRIEL SANCHEZ-MARTINEZ: Yeah, but those DTFS file has times at every stop. So yeah, and that's another thing, that if you look at printed schedules that are posted somewhere and on paper, typically they'll have the terminals and some time points in between and not every single stop.

AUDIENCE: And the time points [INAUDIBLE] approximately [INAUDIBLE]?

GABRIEL SANCHEZ-MARTINEZ: Well, that depends on the control policy of that agency. So we'll talk more about control in a later lecture. So typically, you don't control at every stop. You control at, sometimes, just the terminal, and sometimes at the terminal and some key points in between. And those are timing points for control.

OK, so back to this model, the headway deviation at some stop is, you start out from what it was in the previous stop. You add the effect of running time to that stop from the previous one. And then you add the effect of dwell time, essentially.

So this pi, not pi minus 1, as I said earlier-- and so this is the arrival rate at the stop right now and multiplied by the boarding time per passenger. So this whole quantity is multiplied by this amount, which is the time it takes-- it's the deviation of the time it takes to arrive. So it's a deviation in headway, essentially.

So if you're headway is now a minute longer than it used to be, then you will have a minute times the boarding rate per passenger times the arrival rate of passengers, extra people boarding, or extra time, extra time of dwell time affecting that bus, and therefore slowing it down further. Does that make sense? Or should I break it down a little bit?

AUDIENCE: Did not answer that question, but just to ask a question?

AUDIENCE: Yeah, [INAUDIBLE] interrupt anyone else.

AUDIENCE: So is this accounting for the fact that, if there is no one getting on or off this stop, the first person to pull into a stop, decelerate, and open the door takes a given amount of time. And the marginal time to add an extra person is quite small, or relatively small. Is that accounted for here?

GABRIEL SANCHEZ-MARTINEZ: No, this model is saying that it's a linear effect. You're not saying that the first person takes--

AUDIENCE: So two would take twice as long as one person.

GABRIEL SANCHEZ-MARTINEZ: Exactly, yeah, so that is a simplification here. And it's a point that we will address towards the end of this lecture.

AUDIENCE: OK, [INAUDIBLE] asking questions.

GABRIEL SANCHEZ-MARTINEZ: Good segues, that's, like, the second one today.

AUDIENCE: This is only extra [INAUDIBLE] right?

GABRIEL SANCHEZ-MARTINEZ: So yeah, exactly, because this is a model of headway deviation. So you have this passenger arrival rate and the boarding time, but because you're only multiplying it-- you're not multiplying it times the whole headway. You're multiplying times the headway deviation, which is the deviation at this stop when you arrive and this stop.

It's the deviation when you were arriving at the previous stop plus the time it took you to reach this stop from that time. So now you have adjusted-- now you have the deviation arriving at this stop. If your headway is a minute longer, than you need to add however much extra dwell time you have to pick up that extra minute of passengers. An that's what that last case is doing.

AUDIENCE: So at the beginning, the e will be 0 at the first stop?

GABRIEL SANCHEZ-MARTINEZ: Hopefully. It depends on your control policy. So if you depart your terminal and your driver didn't show up on time and they left a minute late from the even headway, then you start out with one minute deviation.

So yeah, hopefully that is 0 at the beginning. And then due to effect that we described here, it starts deteriorating. And this is a mathematical model to-- a simple mathematical model to account for that.

So it's still a deterministic one. Here is a probabilistic one. The details for this formulation are on this paper. If you're interested, let me know. I will send it you.

What I want to highlight here are the quantities. So this is a model of headway variance. So we are looking at, now, headway as a stochastic quantity. And we're calculating the variance of headway at some stop.

And again, we see the same pattern. We see that it depends on what it was at the previous stop. And then we add the effect of the running time, so the variance of running times between consecutive stops, to the extent that the drivers are different in driving, then this variance increases.

And then you have these two terms-- really, three terms, that have to do with dwell time. All of these terms have-- the q here is the mean number of passengers per bus served. And c is the marginal dwell time per passenger-- so different notation from the model that we just saw, but same quantities.

So we have something times that probability is-- the bus will skip a stop, et cetera. So all these things together are accounting for the dwell time effect. So the last term here is, again, looking at c, which is the marginal dwell time per passenger.

So this also has to do with dwell time. But this one is also multiplied by the covariance with headway. So this is that effect of the relationship between headway and sort of passenger arrivals, captured here as the covariance between those two quantities.

So the key takeaway from this slide is not exactly why this is the right equation. If you're interested in that, read the paper. I am interested in understanding what goes into this equation in terms of what components lead to a higher variance at some particular stop.

OK, we are ready to move to vehicle running time models if no-- if there are no questions on waiting time headway models. OK, so let's do that. So different levels of detail-- we have some models that are very detailed.

There are microscopic model. It's a simulation. And they look at the vehicle motion, the interaction with other vehicles. So you might have private automobiles interacting with a bus, for example, and traffic blocking the bus. And you have signals. So every little detail is modeled. That's a microscopic model.

On the other hand, you have macroscopic models, which are not looking at all those details. Instead, they're saying, what are the running times that I observe as a function of time of day, and the driver, different components. And macroscopically, we say, this is the running time. So there is something in between called mesoscopic, when you have some parts that are detailed and others that are not-- so different levels of modeling.

So running time, as we know, includes dwell time, the movement time between stops, and any delays. Delays could be because of signals, traffic signals, for example. So dwell time is a function of the number of passengers boarding and arriving as well as technology characteristics. What are some examples of technology characteristics that could affect dwell times?

AUDIENCE: [INAUDIBLE] cards, smart card.

GABRIEL SANCHEZ-MARTINEZ: So what your fare card technology, so if you have smart cards, or coins, paying cash-- very different, right? What else?

AUDIENCE: Off-board fare collection.

GABRIEL SANCHEZ-MARTINEZ: Off-board fare collection-- so if you remove the payment from the vehicle, then that really helps with decreasing the variability. What else?

AUDIENCE: All boarding.

GABRIEL SANCHEZ-MARTINEZ: Sorry?

AUDIENCE: All boarding?

GABRIEL SANCHEZ-MARTINEZ: All-door boarding?

AUDIENCE: All-door boarding.

GABRIEL So boarding through all doors-- of course, if you can board through multiple doors, you have a

SANCHEZ-MARTINEZ: faster dwell time process in this variable. What else?

AUDIENCE: If the station level is the same as the--

GABRIEL SANCHEZ-MARTINEZ: Level boarding-- so if people don't have to climb steps to go from the curb to the vehicle, that also helps. Any other ideas? Eli?

AUDIENCE: The type of the bus stop-- am I pulling into a cutout and then I have to wait for traffic to merge back in.

GABRIEL SANCHEZ-MARTINEZ: Yes.

AUDIENCE: [INAUDIBLE] stay in the lane [INAUDIBLE]--

GABRIEL SANCHEZ-MARTINEZ: And that depends on the definition of dwell time. So if you say dwell time is only the amount of time needed to serve passengers, then that shouldn't really be a factor. But if you think of the dwell time as the whole time it took me to stop and serve that stop, then this would be included. So you're looking at-- let's see if I can get at this, clean a section here.

So one example is a bus bay here. So this is a bus stop right here. And maybe this is an intersection. So the bus comes in here. And it's waiting there.

But if there is traffic and there are cars right here, after that bus is ready, it may have to-- well, if it's right of the signal, it's fine. But if this now merges and there is traffic here, then that bus might be sort of stuck enough to maneuver its way back into the traffic flow. So a [INAUDIBLE] sign or stop sign have an impact. What else? Henry?

AUDIENCE: If you're serving a line where there are a lot of people who are, like, tourists and people, like, ask questions, it could take forever to get on to the bus.

GABRIEL SANCHEZ-MARTINEZ: That's not a technology characteristic, but it is a valid factor that affects dwell times. Or maybe if your technology is very complicated and people have a lot of questions, that could be a way to--

AUDIENCE: Or maybe a lack of technology where, like, you don't have--

GABRIEL SANCHEZ-MARTINEZ: So OK, I think we gave good examples. If we look at the typical bus running time and how we break it down, this is a typical bus in mixed traffic. Somewhere between a 1/2 and 3/4 will be spent moving between stops. Between 10 and a 1/4-- 10% and 1/4 will be spent at stops, serving the stop. And between 10% and 1/4 will be served in traffic or waiting for a signal, so at a red light, essentially. Yeah?

AUDIENCE: [INAUDIBLE] about this a bit, but in movement time, would that include all the time it takes for me to, like, stop the bus?

GABRIEL SANCHEZ-MARTINEZ: Again, it depends on your definition. But yes, I think in this break down, yes. So the slowing down and accelerating is sort of in there, yeah. Any other questions?

OK, so let's look at some dwell time models. Dwell times models are a component of running time models, because dwell times are one of the key pieces of running times. And actually, another point about these, movement time, dwell time, and delay, which of those three do you think the agency has most control over?

AUDIENCE: Dwell time.

GABRIEL SANCHEZ-MARTINEZ: Dwell time, right? Why? Can an agency usually diminish traffic or do something about traffic signals? There are some things you can do, but usually not. It's harder.

And movement time, that has to do with traffic and speed limits, and the vehicle itself, and driver behavior. So that's a little bit harder to change. So dwell times are probably the one thing here that an agency will target.

Some part of it, you can't change, because it has to do with how many people are at the stop. But you could increase frequency. You could change the bus assigned.

We gave a bunch of factors. And a lot of those are in control of the agency, so under the control of the agency. So let's go to the dwell time models for that reason.

There are some examples here, three papers, one on bus dwell time, one on light rail dwell times for the Green line, and one on heavy rail. So we're going to look at the three of them, just a high-level overview.

So let's look at some concepts first. Vehicle dwell times affect system performance. We've discussed why and how. And they affect service quality. We've also discussed why and how.

They are a critical element in vehicle bunching. I think that has also been covered. So they result in high headway variability, high passenger waiting times, and uneven passenger loads. That last point, we have mentioned, but it was indirectly-- the point has been made indirectly, right?

If more people are arriving at some stop, at some bus that has a longer headway, not only did those people wait longer, but they are boarding a more crowded bus. So their experience in the vehicle is also going to be diminished. It's going to be a lower-quality experience. So we've covered that.

Dwell time impact on performance depends on substation spacing, the mean dwell as a proportion of trip time, the mean headway, and operations control procedures. We have touched upon, not necessarily what can be done, but we know that there are some things you can do to diminish the headway variability. Any examples of that, actually? Anybody have suggestions on what can be done? Eli?

AUDIENCE: You tell the bus that is trailing to, like, wait longer.

GABRIEL SANCHEZ-MARTINEZ: Yeah, you try to slow down the buses that are running fast. So there's different ways of doing that. You can tell the driver to drive slowly. You can hold buses at stops for control points.

So you tell them, do not depart. You have to wait a minute before you depart, because you're running too fast, and you're going to catch up to the previous bus. So that's called holding. So there has been a lot of research and holding strategies.

AUDIENCE: Wouldn't the more typical thing would be telling them to hold?

GABRIEL SANCHEZ-MARTINEZ: Yes.

AUDIENCE: It seems weird to ask someone to slow down in traffic.

GABRIEL Eh, it's been done.

SANCHEZ-MARTINEZ:

AUDIENCE: Really?

GABRIEL SANCHEZ-MARTINEZ: Yeah.

AUDIENCE: I wouldn't like to be that bus driver.

GABRIEL SANCHEZ-MARTINEZ: You would kind of surreptitiously and only a little bit.

AUDIENCE: [INAUDIBLE]

GABRIEL SANCHEZ-MARTINEZ: And people don't like being held. So driving a little bit slow is less obvious to the passengers.

AUDIENCE: [INAUDIBLE]

AUDIENCE: Yes, you have a bus hold just a few seconds when the light changes. [INAUDIBLE]

GABRIEL SANCHEZ-MARTINEZ: Yeah.

AUDIENCE: I could really get you--

AUDIENCE: So that's sort of the best hold strategy. Oh, we just missed it.

GABRIEL SANCHEZ-MARTINEZ: OK, so there are operations controlled procedures that can be used here. So some examples based on these factors, on the one hand you have commuter rail. We gave that example earlier.

Little impact of dwell time on performance-- that makes sense. Commuter rail has long distance between stops. It stops at every stop. And most of the time is spent in movement. So the percentage of time spent dwelling is small.

On the other hand, you have a very long, high-frequency bus route. So the likelihood of bunching here is really high. And it's hard to control this.

AUDIENCE: Especially, also the commuter rail, also it's like a scheduled dwell.

GABRIEL SANCHEZ-MARTINEZ: Yeah, it's a little longer than it has to be.

AUDIENCE: [INAUDIBLE] hold. Then you don't actually-- you have a few seconds at the end where no one is boarding or arriving.

GABRIEL SANCHEZ-MARTINEZ: Yeah, because you might have printed the schedule it has departure times for every stop.

AUDIENCE: I would argue that commuter rail has sort of the most potential, especially in a transit system like Boston where you have-- well, the line acceleration, if you get the acceleration better through different technology, and when you don't have high-level platforms, the dwell times get really long when you're boarding a lot of people.

GABRIEL SANCHEZ-MARTINEZ: But you still--

AUDIENCE: Plus then you could pull the schedule down if you were [INAUDIBLE] dwell time.

GABRIEL SANCHEZ-MARTINEZ: So you could, if your schedule is off, that could be an option. Even if your schedule is good on a high-frequency long urban bus tour line, it's not going to help. You still have to control it. And it's hard. And you're going to get bunching. So dwell time depends on many factors. Some of them are human. Some of them have to do with modes, as we just saw, operating policies and practices, weather, all these things.

So here is a list of models of dwell times for trains. So if we look at a single door situation, so people have to board and alight from the same door and there is no congestion of passengers, then this is the simplest model one could have. Dwell time is some constant a, which has to do with how long it takes me to come to a full stop and open doors, plus some amount times the number of people getting on plus some amount times the number of people getting off.

Does that make sense? You sort of measure the average number of seconds per passenger alighting, the average number seconds per passenger boarding, and some constant. You run a regression model. That's what you get.

OK, now what if you notice that, when the train is packed, people take a lot longer to get off and on? Then you want to add the effect of interference, or congestion, or friction. Some people call it friction.

So one way of doing that is to multiply-- so we add a term, the passenger friction term. We multiply the number of people that were getting on and off times the number of standees in the vehicle. STD here stands for standees.

So the number of people getting on and off are the people that would have moved faster if there hadn't been any standees. And to the extent that there are many standees, then they encounter friction as they are getting on or off. And that slows the vehicle down. So that's one way of taking care of this.

If that car has m doors, then you could run a model for every door and then pick the door that was slowest. So that's that model. And if you say that, actually, people are more or less evenly distributed inside the vehicle and evenly distributed on the platform, so if we are willing to assume that, then we can just take the number of doors and divide by number of doors, essentially. So we're back to the previous model, but now we're multiplying-- or dividing by m, by the number of doors, because we have maybe 1,000 passengers or many hundreds of passengers, but we have to divide by the number of doors to bring them down to passengers per door.

If that train has several cars, then you have a model for each car in the train. And you take the maximum, so similar concept. We were looking before at doors in one car. Now we're looking at cars in one train.

And if that is our balance flows, again, we can now divide by number of doors per car and number of cars per train. So we normalize the demand by the size of the train, the number of doors. And we include friction.

So here is some examples, some ideas for your problem set, if you want. Some of them may apply. Others may not. So this is the kind of thing you could think of.

So now we want to look at this study by Milkovits published in 2008. At the time, there had been some studies with manually collected data. So there was very limited data on infrequent events. There was very limited data on crowding.

They were still using, I think, the token system. So the previous study was based on the token system, so it wasn't updated on new fare technology. So there was another study that did have automatically collected data, a little more recent, but they hadn't taken into account the effect of payment type.

So the AFC system does tell you, this is a pass, or this is a ticket, but they hadn't-- they had ignored that variable from the model. And the fit of the model was poor. And then we have a transit capacity and quality of service manual, which says, assume half-second penalty per passenger for crowding, so rule of thumb. So there was interest in developing something more sophisticated.

And they looked at buses and CTA for that. So they looked at these factors, so boarding, alighting passengers, counting them, onboard passengers, so load, the fare media type, the alighting door selection, so whether people boarded-- alighted from the front door or from the back door, which has an impact, and the bus type. There were several bus designs. And some of them were-- had wider doors and fare boxes placed more optimally. So all of that was taken into account.

And yeah, so data from the CTA in Chicago-- they didn't consider timing points, because at timing points, buses can be held. So that could be erroneously included as dwell time, so that was thrown out. Only far-side stops-- so what's the difference between a near-side stop and a far-side stop?

AUDIENCE: Far-side stop is after the intersection.

GABRIEL SANCHEZ-MARTINEZ: Right, after signals-- so far-side stops are after signals. They did not look at near-side stops, which could be affected by the red light, the example that Ari just gave of, as the bus is ready to leave, a light turns red. Now you have a longer dwell time. The bus could leave its doors open just in case. So that was thrown out.

OK, they looked--

AUDIENCE: What do you mean by known stops?

GABRIEL SANCHEZ-MARTINEZ: So if there were any stops that were not properly coded or-- they were thrown out. They had APC, so the Automatic Passenger Counting, all doors. And they threw out data that was of bad APCs, so buses that required zero boardings or zero alightings were thrown out. And they looked at each AFC transaction and matched it to what-- how they paid. Was it a ticket? Was it a smart card, et cetera?

So they looked at a whole month, November 2006. So here's how the model works. First, they realized that it was easier to have one model for when the front door controls the process and a separate model for when the rear door controls the process. So the first step was to predict which of the two processes was going to dominate and control the dwell time, and then from that, select that one model, or the, one or the other. And then they looked at including bus type and traveling as a friction factor in each of these models.

So let's look at the high-level results. This is the front door model. So this is for when the front door dominates the process. They had a pretty good adjusted r squared, 0.733.

And we see some dummy variables. NABI is a type of bus here. And NOVA and New Flyer are also types of buses. So NABI, for some reason, tended to be a half second longer dwell time overall.

This variable is the number of people getting on in the front. They are excluding the first few passengers for the reason that Ari brought up earlier in the lecture, that the first few passengers take a little longer, but once you have a [INAUDIBLE], a stream of passengers going in, then there's a more uniform rate. So this is front on extra.

And we have 3.7, about. But it was a little longer for NOVA buses and a little shorter for NABI buses. So you have to adjust that amount for the different buses. So there was an interaction between that variable and the dummies for bus type.

Then the effect of people getting off in the front was accounted for. So here you have-- if you have three or more people, so people in excess of three getting-- people in excess of two, actually, three or more people getting off in the front, that had an impact as well, a positive impact, of course. So these parts that I described were included in the model for non-crowded and in the model for crowded as well.

So they have separate parts of the model that were for crowded and non-crowded conditions. When it wasn't crowded, cards were about 2.6 second effect for boarding per passenger. Tickets were about 4.8, so almost 2 seconds slower than smart cards.

And New Flyer tickets were not quite as longer as tickets everywhere else. So this is the bus that had pretty wide doors and the fare box was placed in a better way so that people could tap in as they went in. You have a small advantage there.

Here you have the effect of people getting off in the front for the first two passengers, not the ones that are three and up, so 2.8 seconds per passenger extra. And here is a dummy for if the sensor in the front was blocked, which could have different effects. But essentially, that could indicate crowding or something, so that was included there.

What happens when it was crowded? So when the load on the bus was high, it didn't really matter if it was a card or a ticket. So now we're just regressing on the number of AFC transactions. And we're getting an average of 4.3 per passenger.

So it's slower. It's much closer to the ticket quantity. And therefore, the impact, the benefit that you had of dwell time savings by smart card is lost. So that savings, you have when it's not crowded, but you lose when it's crowded. Sonya?

AUDIENCE: Is that the same thing with off-board fare collection?

GABRIEL SANCHEZ-MARTINEZ: No, because when you have off-board fare collection, you can open all doors. And so it becomes more like a train, where everybody gets off first and then everybody gets on. So this model is for more people boarding by the front and alighting from the back. But sometimes, some people get off in the front.

And here we have the friction factor, so the number of passengers-- the number of standees squared times passengers. So this is going to-- it's a small factor, a small coefficient, but it's multiplied by some quantity squared, so that kind of makes up the difference. And because this is squared, this is kind of a polynomial effect.

So the more people that are standing-- it's not a linear increase. If there many more people on the bus that are standing, then it's a much slower dwell. So this is a correction factor for a crowded bus.

OK, here is the rear door model. So this is for when the rear door was predicted to control the dwell time, so more towards the end of the route where more people are getting off than people are getting on. We have, again, some impacts. The design of vehicle was significant. And the number of people getting off was, of course, a variable, so about 1.7 seconds per passenger-- more on NOVA buses, less on NABI buses.

And the friction factor, again-- so you have the sort of general friction factor was 0.005. It was 0.009 for normal buses and 0.002 for NABI buses. So you have to sort of add these up to get the effective friction factor on a particular bus.

All right, so the sort of key takeaways are that the smart media loses the benefit in crowded conditions. We saw that. The crowding impact increases exponentially. These people tried linear standees. And standees squared was a much better predictor.

So bus attributes impact dwell time. The dummies for the bus design were significant. And they had an impact. So some of that had to do with the location of the ticket reader. And some of that had to do with wide doors that allowed people to enter more comfortably and faster. So that's good.

Let's move on to the Green line model, just quickly, at a high level. We're almost out of time, so I'm going to skip this slide. You're, I think, all aware of what the Green line is, so I don't have to cover this.

They looked at one-car train and two-car train models separately. So sometimes the Green line will run single vehicles, single car, or sometimes two cars paired together. And the dwell time here was some constant times the number of people who are getting on times the number-- plus the number people getting off plus some friction factor. And same here for two-car trains, but the coefficients were different. So that's the overall concept that I want to communicate.

Here is a table of the results from that model for the number of people getting on. So this is using the model to forecast. These are not observations. So if you feed 0 people getting on, you just get the constants. If you feed the model 10 people getting on, it depends on the load on the model.

If the load is, say, less than 53, then you don't have the friction factor really controlling anything, so you have 20.3 or 20.2, so a very small effect of friction, and therefore about the same time for both models. If it's very crowded, then you do have a more significant benefit for having a two-car train. That makes sense, right? You have more capacity.

And the same thing happens as you move to 20 passengers getting on and 30 passengers getting on, with those differences increasing when you have a crowded train, and the differences between one-car and two-car trains not being that significant when you don't have much [INAUDIBLE]. So this should, more or less, make sense.

So all this is to show you that you have some ideas about what controls dwell time. You test your hypothesis on the data. And you can try different things. These are just examples.

So the findings from that research, dwell times were quite sensitive to flows and loads. The crowding effect might be non-linear. They looked at non-linear effects . Just like before. The dwell times for multi-car trains, for two-car trains were different than those for one-car trains. The dwell time functions suggest high sensitivity of performance to perturbations. So we saw what those preparations were earlier in this lecture. And this model is sensitive to some of them.

Because of that, effect of real-time operations control should be essential to operating the Green line with even headways. So this is more of a recommendation. And another recommendation is that simulation models of this kind of service should include sophisticated dwell time models like the ones that were estimated here to account for all those effects, because otherwise, you would have a simple model that is not very faithful to reality.

Running a mixed fleet, some with one-car trains, some two-car trains is dangerous. So what do they mean by dangerous? So it wouldn't make much difference if it's not crowded. But according to the model, on crowded conditions, the two-car train will be faster. So then you will have a bunching effect happening, two-car trains catching up to one-car trains, and therefore deterioration of service quality.

Here is a marginal boarding time on heavy rail from the third paper. I think this was looking at the Red line. So this is the marginal boarding time when everybody can sit down.

When you look at the number of through passengers-- that is the number of passengers who are on the train when the train arrives on the platform and don't get off, so passengers riding through the station-- when the number is 0, then this is how much boarding time you have per passenger, more or less. And then as you look at the effect of more and more people are standing, than that kind of slows people down as they board. So this was a way of capturing that effect, again, the fiction factor.

So you see a theme here, that this seems to be relevant. And there have been research studies looking at ways of including that effect in models. These models could be used for improving the accuracy of your waiting time estimate that your smartphone gives you, or improving how faithful a simulation model is, et cetera.

Here is the equation, very similar, same structure we had before. We have some constant. We have the number of people boarding per door, the number of people alighting per door.

In this case, we add them up, because people alight first and then board. And then we have some friction factor. How that friction factor is calculated has been different on every model, but they all have a friction factor.

So these are all services that, at least at times, are quite crowded. So that was important and significant. OK, if there are no questions, you may leave. And if there are, I'll take them. Sorry, don't feel that you have to wait, because it's already 5:30.