Optimizing Shipping Lanes:
1.224J Problem Set 3
Solutions

Due: 11/21/01

In this problem set you will be working for a company with operations in the Northeast. They are preparing to issue an RFP for bids on their shipment operations within the region. You have been asked to help prepare the RFP and then to select the best providers after the bids have been received. You should be able to complete this problem set in Excel, taking advantage of the provided spreadsheet. If you prefer, though, you may use another program, such as XPressMP. (If you do use XPressMP, you may save yourself some time if you investigate its ability to read from an Excel spreadsheet.)

Problem 1

The first part of the combinatorial lane optimization is the enumeration of all the possible shipping lanes. While this is simple in a small network, the task can quickly become overwhelming. Your employer has facilities in Boston (BOS), Buffalo, NY (BUF), New York City (NYC) and Cleveland, OH (CLE) and maintains shipping lanes as shown in Figure 1.
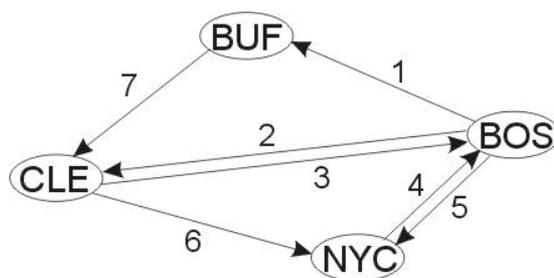

Figure 1: The network of shipping lanes connecting the shipper's facilities with lane numbers shown.

There are a total of 26 possible lane-combinations (we ignore the starting point of "round-trip" routes). In order to ensure some uniformity in the bidding you have been asked to provide a list of all the possible lane combinations. In addition, if the carriers bid on all the combinations, you will have a greater search-space over which to look for an optimal solution, hopefully giving you a lower total cost.[1] In this case, your co-worker was working on the enumeration, but called in sick

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[1]Of course, if the network were much larger, the number of such pieces would be extremely high and neither you nor the bidders would want to deal with an exhaustive enumeration.

today, telling you that he hadn't finished writing down all the pieces. Please find the two missing pieces, add them to the distributed spreadsheet and clearly mark the two missing entries.

Solution: The piece from CLE→BOS→NYC (lanes 3 and 5) was omitted. The piece from BUF→ CLE→NYC→BOS (lanes 7, 6 and 4) was also omitted.

Problem 2

Following the RFP, the company received bids from three carriers. These bids are provided in the attached spreadsheet. Note that the bids are given for the two columns you needed to find in the previous part and are listed as "Missing 2-piece-trip" and "Missing 3-piece-trip". (In other words, you should have found one 2-piece and one 3-piece trip in the previous problem.) With no constraint other than the requirement that all lanes be covered exactly once, please find the optimal lane-carrier matching.

A solution tip: Because the Excel solver is not very powerful or smart, you may have to adjust the options you give it as to the nature of the solution (under the "Options" button). A good first check is to make sure you're giving it enough time. If that's not working, think about what other properties you know about the solution and check the appropriate boxes. If you do have to make any of these assumptions please note this on your solution and state why the assumption is valid. (As a final note, the computer should take no more than a minute or two to find the solution on even the slowest computers – if it's taking you longer you should rethink your assumptions.)

Solution note: In this problem set all the problem formulations should be linear with non-negative solutions. You should, therefore, have been able to check "Assume Linear Model" and "Assume Non-Negative" in the Excel Solver Options.

Solution: We formulate the problem as follows. Let L be the set of lanes, P be the set of packages and C be the set of carriers. Let x_{ij} be a binary decision variable; it will be 1 if we choose award package j to carrier i and 0 otherwise. We introduce an indicator δ_{ij} which is 1 if package j contains lane i. Finally, we denote carrier i's bid on package j by c_{ij}. We can formulate the problem as follows:

$$\text{Minimize}: \qquad \sum_{i \in C} \sum_{j \in P} c_{ij} x_{ij}$$
$$\text{Subject to}: \qquad \sum_{j \in C} \delta_{ij} \sum_{k \in P} x_{jk} = 1 \qquad \forall\, i \in L$$
$$x_{ij} \in \{0, 1\} \,\forall\, i \in C,\ j \in P$$

The cost of the optimal solution is \$3000. Carrier A is assigned package 7-6; Carrier B is assigned package 1 and 2-3; and Carrier C is assigned package 4-5. The constraints used in the Excel solution are shown in Figure 2.

Problem 3

In order to ensure a degree of profitability out of contract with a shipper, a carrier may require that it be awarded some minimum level of business. At the same time, a supplier may want to limit its administrative overhead and deal only with one carrier. You are asked to examine the effects of each such choice. For each part please also include a formulation of the IP as you solved it.

Figure 2: A view of the multiple constraints imposed on the solver in Problem 2.
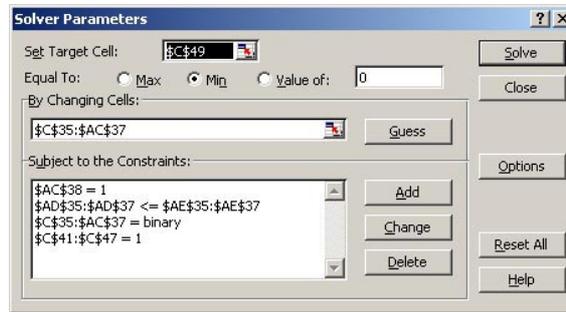
Figure 3: A view of the multiple constraints imposed on the solver in Problem 3a.

(a) Solve the problem with the constraint that no carrier be awarded fewer than two lanes.

Solution: Our formulation for this problem is identical to that of the last, only we must introduce an additional constraint to ensure that each carrier is awarded at least two packages. We do this as follows:

$$\sum_{j \in P} x_{ij} \geq 2 \qquad \forall\, i \in C$$

The cost of the optimal solution under this constraint is $3400. In this solution Carrier A is assigned packages 1 and 4; Carrier B is assigned packages 7 and 2-3; and Carrier C is assigned 5 and 6. The constraints used in the Excel solution are shown in Figure 3. As many a few of you noted, there are actually several optimal solutions to the problem and they are summarized in Table 1;

(b) Solve the problem with the constraint that only one carrier is awarded all the lanes. (Note

Carrier	Sol'n 1	Sol'n 2	Sol'n 3	Sol'n 4	Sol'n 5	Sol'n 6
A	1, 4	1, 5	5, 2-3	4, 2-3	5, 7	4, 7
B	7, 2-3	7, 2-3	1, 7	1, 7	1, 2-3	1, 2-3
C	5, 6	4, 6	4, 6	5, 6	4, 6	5, 6

Table 1: Summary of solutions to Problem 3a.

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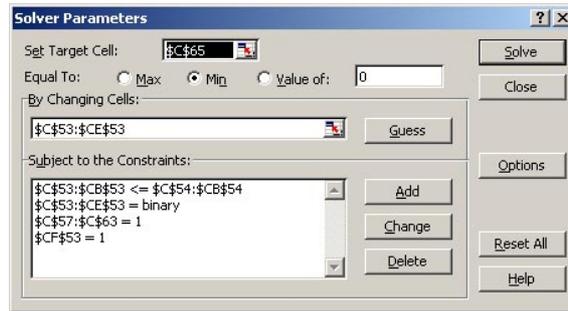
Figure 4: A view of the multiple constraints imposed on the solver in Problem 3b.

that you need to be careful how you formulate the problem, lay it out in Excel and ask the solver to solve it. If it tells you that there is no solution, think: does this make sense? How might you reformulate the problem so that it can find a solution.)

Solution: As many of you realized, this problem is more difficult than it first appears on the surface. We can model it by introducing a decision variable y_i which is 1 if we choose to use a carrier and 0 otherwise. By forcing the sum of the package decision variables to be less than some sufficiently large multiple of the y_i term, we guarantee that we assign carrier i 0 packages if $y_i = 0$ and the optimal number otherwise. In this case we choose the multiple to be 26 since that is the total number of packages; we could have chosen any number as small as 7 as that is the size of the largest set which completely covers all the lanes. We then append our original formulation in Problem 2 with the following constraints:

$$\sum_{j \in P} x_{ij} \leq 26 \cdot y_i \qquad \forall\, i \in C$$

$$\sum_{i \in C} y_i = 1$$

$$y_i \in \{0, 1\} \qquad \forall\, i \in C$$

The cost of the optimal solution is $3075. In this problem, as indicated in the problem set, you have to be careful how you formulate the problem. First, you need to ensure your formulation is linear. Even if it is linear, Excel tends to get rather picky and actually gave the *wrong solution* in one formulation. We kept this problem in because it should serve as a warning to never rely too heavily on the tools you use to reach your solution. The correct solution is to use Carrier A. In this case it is assigned packages 1, 7-6, 2-3 and 4-5. The constraints are shown in Figure 4.

(c) How does your answer to each of the previous two parts compare with the original solution? How do they compare with each other? Do the differences make sense?

Solution: We see that as we further constrain the problem we get further from the optimal solution. We saw that because the second constraint was so restrictive there was a considerable difference between it and the optimal solution. It was nearly the case of assigning each carrier single lanes rather than packages and as such reaped little of the savings. As
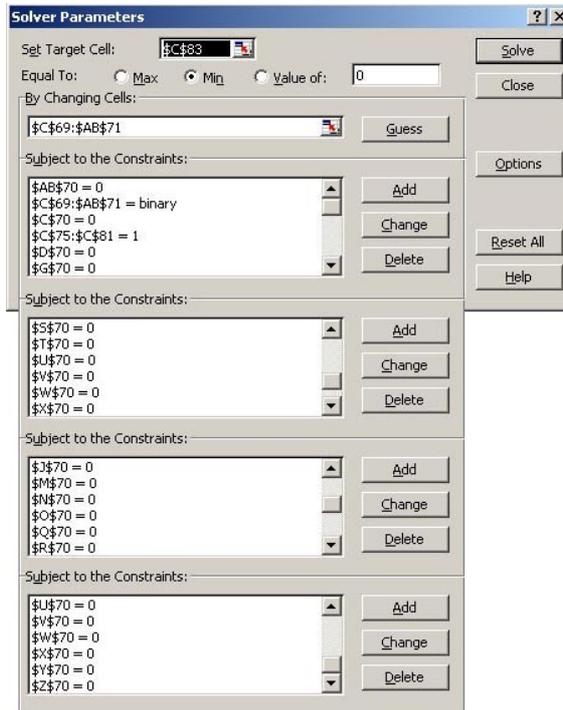
Figure 5: A view of the multiple constraints imposed on the solver in Problem 4. In this case the figure is long because we had to prevent it from choosing a number of routes due to the loss of license.

many of you pointed out, a more logical and more flexible, though more difficult to model, constraint, would be to require a minimum number of lanes (instead of packages).

Problem 4

You have since learned that Carrier B has lost its license to pick up cargo in Boston (they are, however, still allowed to make deliveries). You have been asked to find the lowest-cost alternative, given the above constraint.

Solution: This formulation is identical to that presented initially in Problem 2. The only change is that we force all decision variables which include a lane originating in Boston to be 0. Specifically we added the constraints that x_{Bi}, $i \in P \backslash \{3, 4, 6, 7, 9, 10, 14, 25\}$ In this case the cost only increases slightly from the "optimal" to \$3050. We assign Carrier A packages 1, 7-6 and 2-3; Carrier C is assigned package 4-5 while Carrier B garners none of the business; equivalently Carrier A can be awarded package 2-3 and Carrier C awarded packages 4-5 and 1-7-6. The constraints used in the example solution are shown in Figure 5.

Please submit a hard-copy summary of your results and write-up. You should supplement this by submitting an electronic copy of your spreadsheet to the Command website.