Logistical and Transportation Planning Methods
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Massachusetts Institute of Technology
Cambridge, Massachusetts

Quiz #2
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OPEN BOOK
TWO HOURS
5 PAGES, 3 PROBLEMS

PLEASE SHOW ALL YOUR WORK!

INSPIRATIONAL QUOTE:

If you come to a fork in the road, take it.
Yogi Berra, once a baseball catcher for the New York Yankees

Problem 1 (36 points)

In the tree network shown below, demands are generated uniformly along the edges of the network in a Poisson manner at the rate of λ per unit of time. (Think of the edges as city streets, with houses located along the streets.)

(a) (9 points) Find a solution to the 3-absolute-center problem on this network, i.e., locate three facilities in a way that minimizes the maximum distance between any demand and its nearest facility. What is the maximum distance associated with your solution? (Any answer that finds the correct maximum distance will receive full credit.)

(b) (9 points) Find a solution to the 4-absolute-center problem on this network. What is the maximum distance associated with your solution? (Any answer that finds the correct maximum distance will receive full credit.)

Suppose now that we wish to locate a station for a single ambulance on this network in a way that minimizes expected response time to calls from the network. Assume that the station is located at some point X on the network. Response time then consists of the sum of (1) the round-trip travel time between X and the demand and (2) any waiting time until the ambulance is dispatched from point X. Specifically, assume that the speed of the ambulance during its travel to and from each demand is equal to 1 and that the time it spends on the scene of the demand is exactly 1 unit. Service to demands is provided in a first-come, first-served way.

(c) (9 points) Identify the optimal location for the ambulance station when λ is very small $\lambda \approx 0^+$. Please justify your answer briefly.

(d) (9 points) What is the largest value of λ for which this queueing system will be stable? Where should the station be located for this value of λ? Please provide a clear justification for your answer.
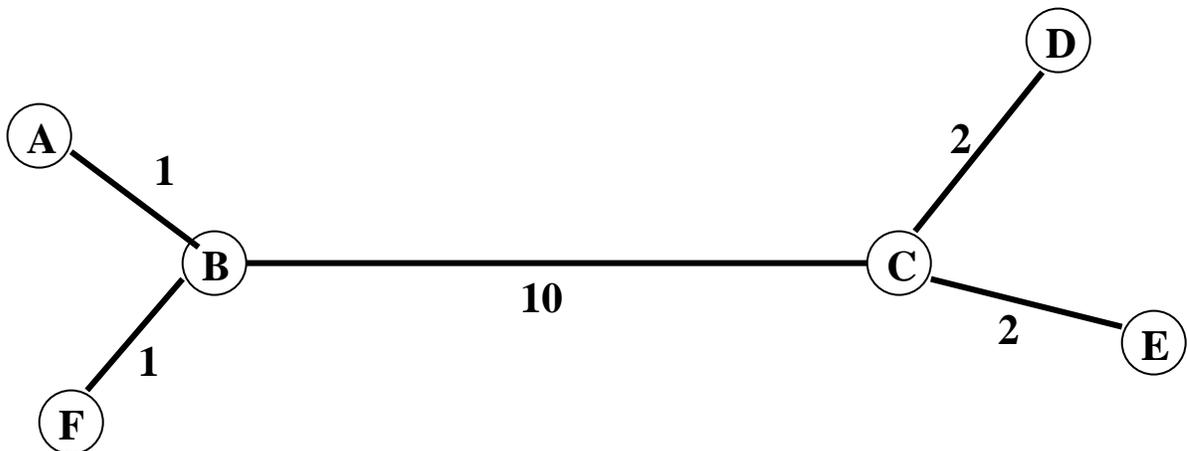
Problem 2 (42 points, with each part worth 7 points)
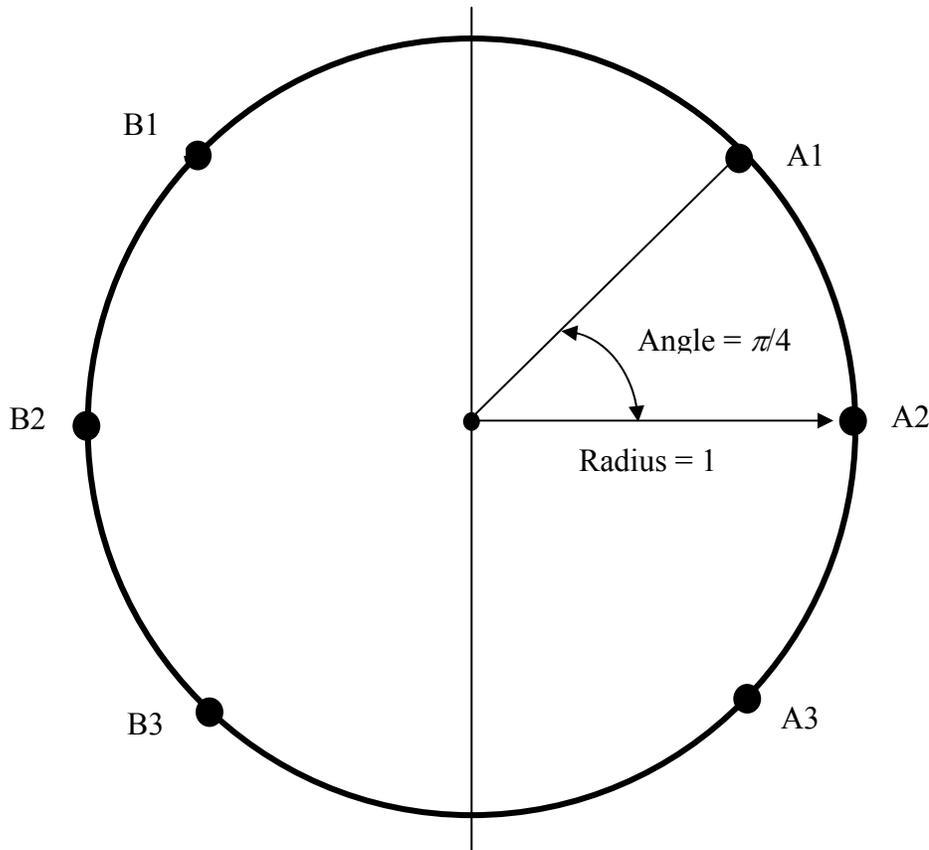
We consider a 2-server spatial queue for ambulances that operates like the traditional Hypercube queueing model. See the symmetric figure above to understand the geometry. Calls for ambulance service are uniformly independently distributed over the **circumference of the circle** of radius 1, as shown in the figure. There are two primary response areas, **A** and **B**. Response area **A** is the half circumference of the circle to the right of the vertical diameter drawn in the figure. Response area **B** is the half circumference to the left of the vertical diameter drawn in the figure. When idle, i.e., not serving customers, ambulance **A** is *equally likely to be stationed at any one of the three parking stations*, **A1, A2** or **A3.** When idle, ambulance **B** is *equally likely to be stationed at any one of its three parking stations*, **B1, B2** or **B3**.

Calls for ambulance service arrive in accordance with a homogeneous Poisson process with rate λ =1.0 call/hour. Travel speed is very fast, so virtually all of the service time is on-scene time. An ambulance always travels the shortest circumferential distance between its location and the call for service to which it is dispatched. Upon completion of service at the scene, the ambulance virtually immediately returns to one of its three parking stations, picking one of the three at random.

We model the service time for each customer as being distributed according to a negative exponential pdf with mean $(1/\mu)$ = 1.0 hour. The ambulance dispatcher always dispatches ambulance **A** to any customer in response area **A**, if ambulance **A** is available.

Otherwise a customer in response area **A** is serviced by ambulance **B**, if ambulance **B** is available. Symmetrically, the ambulance dispatcher always dispatches ambulance **B** to any customer in response area **B**, if ambulance **B** is available. Otherwise a customer in response area **B** is serviced by ambulance **A**, if ambulance **A** is available. Customers who arrive when both ambulances are busy are lost forever, i.e., not served. Thus, we have a zero queue capacity system, a 'loss system.'

(a) Carefully draw the 2-server hypercube state space and transition rate diagram, showing all the upward and downward transition rates.
(b) Find the utilization factor (i.e., the fraction of time busy serving customers) for each ambulance.
(c) Compute the mean response distance traveled by an ambulance to a random customer.

Now assume that the dispatcher has an automatic vehicle location system so he knows the parking station at which each available ambulance is located. The dispatch strategy is now changed so that when both ambulances are free, i.e., not serving customers, the dispatcher will dispatch the closer ambulance to the customer. For instance, if a customer calls in from response area **B**, near the top of the circle and just to the left of the vertical diameter drawn, if ambulance **B** is available and located at **B2** or **B3** and if ambulance **A** is located at **A1**, then ambulance **A** is closer and is dispatched to the customer. The other parts of the dispatch strategy remain unchanged.

(d) Carefully draw the 2-server hypercube state space and transition rate diagram, showing all the upward and downward transition rates.
(e) Find the utilization factor (i.e., the fraction of time busy serving customers) for each ambulance.
(f) Compute the mean response distance traveled by an ambulance to a random customer.

Problem 3 (22 points)

Consider the transportation network shown below. All the link lengths are equal to 1 mile. The two numbers in parentheses next to each node indicate, respectively, the rate of demands per hour generated at each node for a repair service from midnight to noon ("Period 1") and from noon to midnight ("Period 2"). Demands at each node are generated independently and in a Poisson manner. Please note that total demand in each period is equal to 25 and that only two nodes have demand rates different from 1 during each period.

The service's administrator now wishes to locate a facility on the network to minimize the average "cost" of travel time from the facility to a random demand. The cost of travel for period i ($i = 1$ or 2) from point x to node j is equal to $w_i \cdot h_{i,j} \cdot d(x, j)$, where $d(x,j)$ is the distance between x and j, w_i ($0 \le w_i \le 1$) is the "weight" assigned to period i and $h_{i,j}$ is the weight of node j during period i. (For example, $h_{1,5} = 9$.) The administrator will assign weights w_1 and w_2 such that $w_1 + w_2 = 1$.

a) (11 points) Find the optimal location for the facility if $w_1 = w_2 = 1/2$. Please provide a clear justification for your answer.

b) (11 points) Can node 16 be the optimal location for a facility for some combination of w_1 and w_2? If yes, please find one such combination. If no, please explain why.