Homework Set #10

Problem 1

Suppose that hurricanes occur according to a Poisson Point Process with unknown parameter λ. Given that 5 hurricanes occurred during a two-month period, estimate λ by:

(a) The Method of Moments
(b) The Method of Maximum Likelihood, and plot the Likelihood function

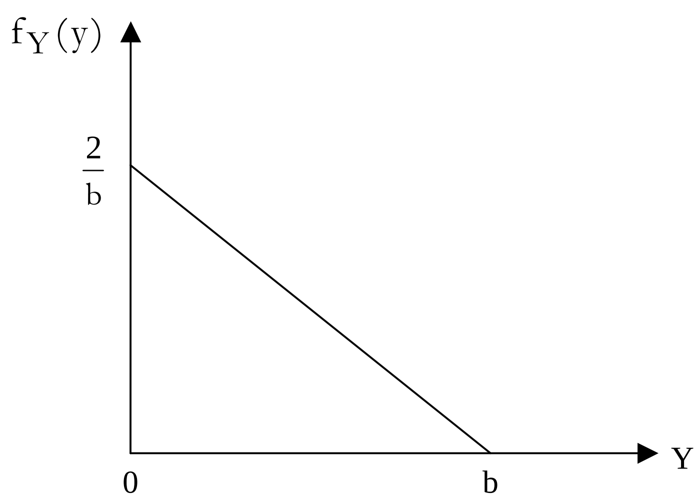
[In this case, you should consider the random variable N = number of hurricanes in a two-month period. Notice that N has Poisson distribution with mean value 2λ, where λ is in units of 1/month]

Problem 2

Consider a random variable Y with probability density function:

$$f_Y(y) = \begin{cases} \frac{2}{b}\left(1 - \frac{y}{b}\right), & 0 \le y \le b \\ 0, & \text{otherwise} \end{cases}$$

where b is an unknown parameter.



The mean value of Y is $m_Y = \frac{1}{3} b$.

Given the following sample $\underline{Y} = \{2, 3, 5\}$ from the distribution of Y:

(a) Estimate b by the method of moments .
(b) Find and plot the likelihood function $\ell(b \mid \underline{Y})$
(c) Find the maximum likelihood estimate of b and compare with the result from (a).

Problem 3

The strength of concrete cylinders, X, is known to have normal distribution with unknown mean value m and known variance $\sigma^2 = (1000\text{psi})^2$.

Suppose that the *prior* distribution of the mean value m is Normal, with mean value $m_m = 7500\text{psi}$, and $\sigma_m^2 = (300\text{psi})^2$. From crushing tests, you collect the following sample of X (in psi):

$\{6500,\ 7000,\ 8000,\ 4000\}$.

Using Bayesian Analysis:

(a) Plot the *prior* distribution of m
(b) Plot the Likelihood function, normalized to have unit area
(c) Plot the *posterior* distribution of m

Problem 4

The compressive strength of concrete cylinders, X, is known to have normal distribution with mean value m and variance σ^2 that depends on the batch considered. In order to estimate m and σ^2 for a specific batch, a laboratory test is performed in which the value of X is measured for n cylinders. Let the resulting statistical sample be $X_1, \ldots, X_n$. You need to estimate m and σ^2 with a certain accuracy, which you set as follows:

$$(1) \quad \sqrt{\mathrm{var}\left(\frac{\overline{X} - m}{m}\right)} < 0.1$$

$$(2) \quad \sqrt{\mathrm{var}\left(\frac{s^2}{\sigma^2}\right)} < 0.1$$

(a) Determine the minimum value of n that satisfies each objective (notice that such minimum value may depend on the actual parameters m and σ^2).
(b) In practice, which of the two conditions do you believe is more restrictive for n?
(c) Based on your response to (b), can you set n without prior knowledge of m and σ^2?