MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

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1.60/1.995 Fluid Mechanics

In-Class Examination, 6 May, 2005

Problem No. 1 (30%)

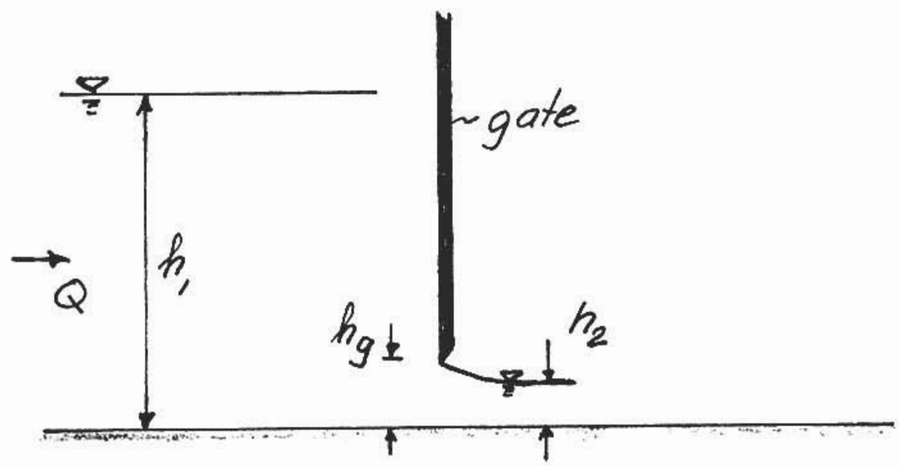
An extremely long rectangular concrete channel ($\varepsilon = 3$mm) carries a discharge $Q = 100$ m^3/s. The width of the channel is b = 20m and it slopes at an angle $\beta = 0.05°$ relative to horizontal.

a) Determine normal depth in this channel (Default value $h_n = 1.8$m)

b) Is normal flow sub- or supercritical

c) For the given discharge and channel geometry, determine the slope, S_{oc}, for which normal flow would be critical.

Problem No: 2 (40%)



A discharge of $Q = 100$ m^3/s passes under a vertical gate located in a rectangular channel of width b = 20m. The depth of flow a short distance after passing under the gate is $h_2 = 0.60$m (see sketch).

a) Estimate the height of the gap, h_g, under the gate.

b) Determine the depth, h_1, a short distance upstream of the gate (see sketch).

c) Determine the force exerted on the gate by the flow.

d) Determine the force on the gate if the pressure is assumed to be hydrostatic on the upstream face of the gate.

e) Explain why the answers to (c) and (d) differ.

Problem No: 3 (30%)

Assuming the vertical gate in Problem No: 2 is located near the mid-portion of the very long channel in Problem No: 1, describe:

a) The nature of the flow upstream of the gate.

b) The nature of the flow downstream of the gate.

Your answers should include the values of depths of flow "far" from the gate as well as a very rough estimate of how "far" far is, and an identification of how the depth varies with distance from the gate upstream and downstream directions, i.e. identify types of gradually varied flow profiles and hydraulic jump specifications (if any).

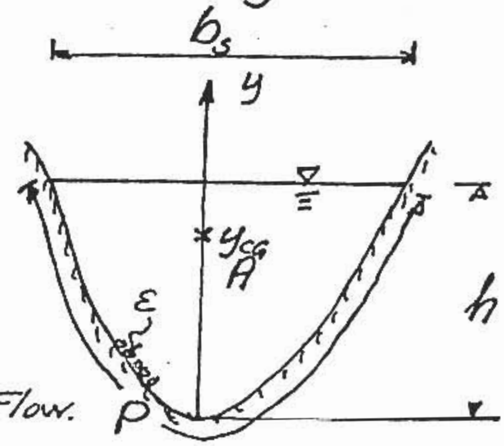
1.060 ENGINEERING MECHANICS II

Cheat Sheet for Test No: 3

UNIFORM FLOW:

$$V = \sqrt{\frac{8g}{f}}\, R_h^{1/2} S_o^{1/2} = \dot{C}\, R_h^{1/2} S_o^{1/2} = \frac{1}{n} R_h^{2/3} S_o^{1/2} = \frac{Q}{A}$$

Darcy-Weisbach, Chezy, Manning



S_o = channel slope

R_h = hydraulic Radius $= \frac{A}{P}$

$$Fr^2 = \frac{Q^2 b_s}{g A^3} = \frac{V^2}{g(A/b_s)} = \frac{V^2}{g h_m}$$

S_f = slope of EGL = S_o ⇒ Uniform Flow.

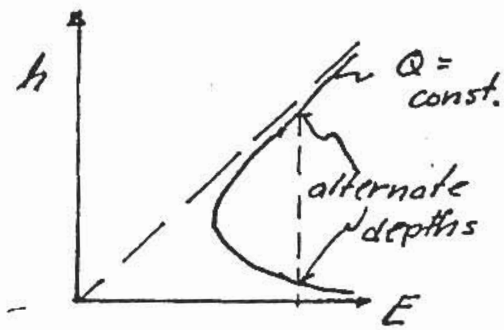
n = Manning's $n = 0.038\, \varepsilon^{1/6}$ (SI-units)

ENERGY PRINCIPLE

$$H = \frac{V^2}{2g} + h + z_o$$

E = Specific Energy $= H - z_o$

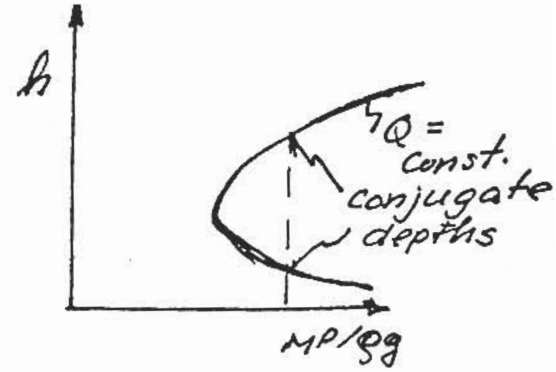
$$E = \frac{Q^2}{2gA^2} + h$$



MOMENTUM PRINCIPLE

$$MP/(\rho g) = \frac{Q^2}{gA} + (h - y_{CG})A$$

y_{CG} = y-value of centroid of A



(OVER)

HYDRAULIC JUMP (UNASSISTED) IN RECTANGULAR CHANNEL

$Fr_1 > 1$; $Fr_2 < 1$

$$\frac{h_2}{h_1} = \frac{1}{2}\left(-1 + \sqrt{1+8Fr_1^2}\right); \quad \frac{h_1}{h_2} = \frac{1}{2}\left(-1 + \sqrt{1+8Fr_2^2}\right)$$

$$\Delta H_{jump} = H_1 - H_2 = E_1 - E_2 = \left(\frac{V_1^2}{2g} + h_1\right) - \left(\frac{V_2^2}{2g} + h_2\right) = \frac{(h_2 - h_1)^3}{4 h_1 h_2}$$

GRADUALLY VARIED FLOW

$$\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

$S_o = S_f$ for uniform flow ⇒ Normal Depth

S_f replaces S_o in formulas for V

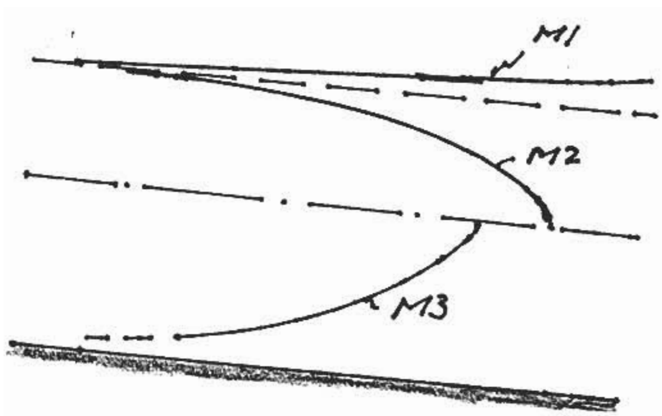
$Fr^2 = 1$ ⇒ Critical Depth

$$S_f = \frac{f}{8g}\frac{Q^2}{A^3/P} = \frac{1}{C^2}\frac{Q^2}{A^3/P} = \frac{n^2 Q^2}{A^{10/3}/P^{4/3}} = \frac{\tau_s}{\rho g A/P}$$

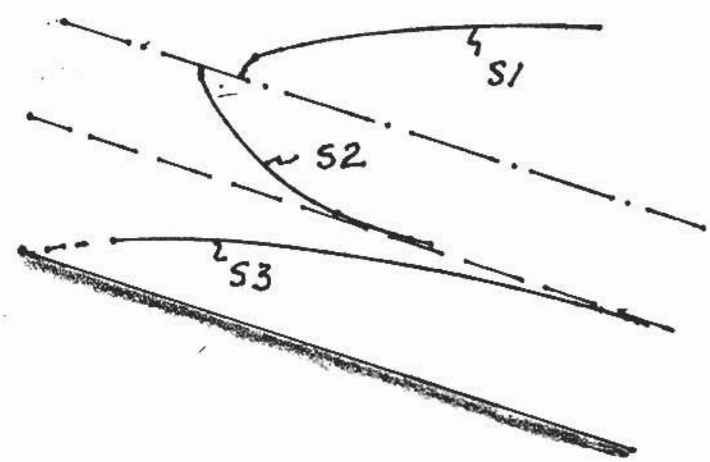
Darcy-Weisbach Chezy Manning

GRADUALLY VARIED FLOW PROFILES

Mild Slope



Steep Slope



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Problem No: 1

a)

Normal depth, h_n, corresponds to uniform steady flow and is given by

$$Q = V_n A_n = \frac{1}{n} \frac{A_n^{5/3}}{P_n^{2/3}} \sqrt{S_o} = \frac{1}{n} \frac{b^{5/3} h_n^{5/3}}{b^{2/3}\left(1+\frac{2h_n}{b}\right)^{2/3}} \sqrt{S_o}$$

or

$$h_n = \left(\frac{nQ}{b\sqrt{S_o}}\right)^{0.6} \left(1+\frac{2h_n}{b}\right)^{0.4}$$

$n = 0.038\, \varepsilon^{1/6} = 0.038\,(0.003)^{1/6} = 0.0144;\ S_o = \sin\beta = 8.73 \cdot 10^{-4}$

$Q = 100\ m^3/s;\ b = 20\ m$ gives

$$h_n = 1.706\,(1+0.1h_n)^{0.4} \Rightarrow \underline{h_n = 1.82\,m}$$

b)

$$Fr_n = \frac{V_n}{\sqrt{gh_n}} = \frac{Q/(bh_n)}{\sqrt{gh_n}} = \frac{2.75}{4.22} = 0.65 < 1$$

<u>Normal flow is subcritical</u>

c)

For critical flow we have $Fr = \frac{V_c}{\sqrt{gh_c}} = 1$, so

$$Q = V_c h_c b = \sqrt{gh_c}\, h_c b \Rightarrow h_c = \sqrt[3]{(Q/b)^2/g} = 1.37m$$

So,

$$V_c = \sqrt{gh_c} = 3.66 = \frac{1}{n}\left(\frac{h_c}{1+2h_c/b}\right)^{2/3} \sqrt{S_{oc}} = \frac{1}{0.0144}\left(\frac{1.37}{1.137}\right)^{2/3} S_{oc}^{1/2}$$

$$\underline{S_{oc}} = \left(\frac{0.0144 \cdot 3.66}{1.13}\right)^2 = \underline{2.17 \cdot 10^{-3}}$$

Problem No: 2

a)

$h_2 = C_c h_g$ w. C_c = contraction coefficient ≈ 0.6

$$\underline{h_g} = h_2 / C_c \approx 0.6 / 0.6 = \underline{1.0\,m}$$

b)

Short transition of converging flow $\Rightarrow \Delta H = 0$

$$h_1 + \frac{Q^2}{2gb^2h_1^2} = h_1 + \frac{1.276}{h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2} = 0.6 + 3.543 = 4.143\,m$$

$$h_1 = 4.143 - \frac{1.276}{h_1^2} \Rightarrow \underline{h_1 = 4.07\,m}$$

c)

F_g = Force on gate (>0 if in downstream direction) =

$MP_1 - MP_2 = \frac{1}{2}\rho g h_1^2 b + \rho Q^2/(h_1 b) - \left(\frac{1}{2}\rho g h_2^2 b + \rho Q^2/(h_2 b)\right) =$

$1623.4 + 122.9 - (35.3 + 833.3) = \underline{877.7\,kN}$

d)

Taking depth at gate $= h_1 = 4.07\,m$

$\underline{F_{g,hy}} = \frac{1}{2}\rho g (h_1 - h_g)^2 b = \underline{923.6\,kN} > F_{g,\,in\,(c)}$

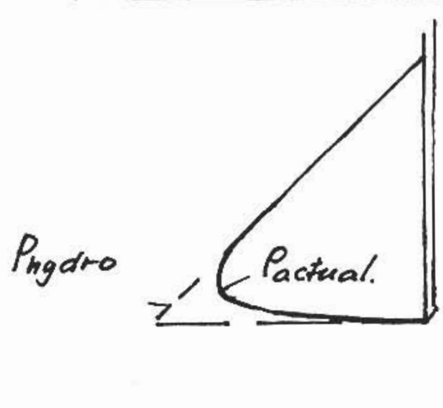
but since $V = 0$ in corner where surface meets gate, one could also take $h_{1,g} = h_1 + V_1^2/2g = 4.15\,m$

for which $\underline{F_{g,hy} = 972\ kN > F_{g,\,in\,(c)}}$

e)

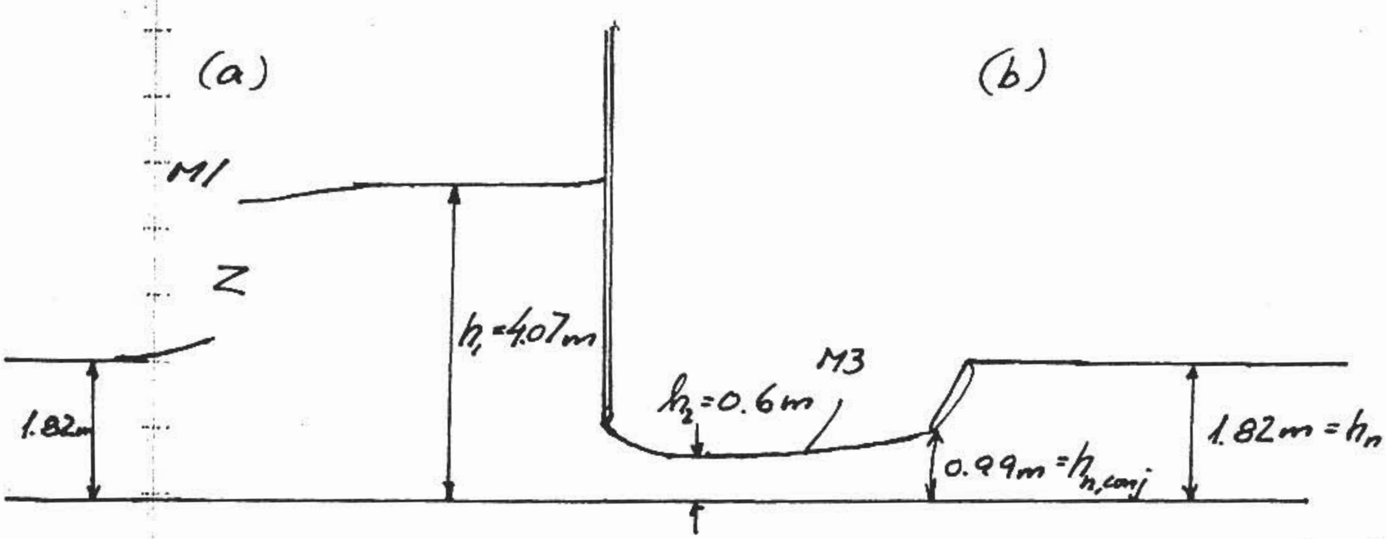
$\underline{F_{g,\,in\,(c)} < F_{g,hy}\ \text{since}\ p < p_{hydrostatic}\ \text{near gap}}$

$\underline{\text{since velocity here is not small}}$



Nature of pressure distribution on upstream face of gate

Problem No: 3



a)

Upstream of gate depth varies from $h_n = 1.82m$ (far upstream) to $h_1 = 4.07m$ (immediately upstream). Slope is "mild" since $Fr_n = 0.65 < 1$. M1-profile (backwater curve) will do the job.

"Far" upstream normal flow has $dh/dx = 0$. Just upstream of gate $(dh/dx)_1 = (S_0 - S_{f1})/(1 - Fr_1^2)$

Because $h_1 \gg h_n$: $S_{f1} \approx 0$ [actual value $= 7.6 \cdot 10^{-5}$] and $Fr_1^2 \approx 0$ [actual value $= 0.038$], so $(dh/dx)_1 \approx S_0 = \sin\beta = 8.7 \cdot 10^{-4} \Rightarrow$ Average $(dh/dx) \cong \frac{1}{2} S_0 = 4.35 \cdot 10^{-4} = \overline{dh/dx} \Rightarrow \Delta x = \Delta h / \overline{dh/dx} = (4.07 - 1.82)/(4.35 \cdot 10^{-4}) = \underline{5.2 km}$

b)

Far downstream of gate we have normal flow, $h_n = 1.82 m$, which is subcritical, whereas flow immediately downstream of gate, $h_2 = 0.6m$, is supercritical, $Fr_2^2 = 11.8$. Only way to get from a super- to a subcritical flow is through a jump from $h_{n,conjugate}$ to h_n. Jump condition (Cheat sheet)

$$h_{n,conj} = \frac{1}{2} h_n \left(-1 + \sqrt{1 + 8 Fr_n^2}\right) = \underline{0.99\,m}$$

So, downstream of gate the depth increases from $h_2 = 0.6m$ to $h_{n,conj} = 0.99m$ following an M3 profile. When $h_{n,conj}$ is reached there is a hydraulic jump up to normal depth, $h_n = 1.82m$ which is the depth from there on.

Immediately after gate $Fr_2^2 = 11.8$ and $S_{f2} = n^2 Q^2 P_2^{4/3} / A_2^{10/3} = 3.08 \cdot 10^{-2}$; i.e. $(dh/dx)_2 = (S_0 - S_{f2}) / (1 - Fr_2^2) = 2.77 \cdot 10^{-3}$.

Jump takes place (very) roughly a distance of $\Delta x = (h_{n,conj} - h_2) / (dh/dx)_2 = 140\,m$ downstream of the gate.

GRADING

Problem No:1 a) 15 b) 5 c) 10

Problem No:2 a) 5 b) 12 c) 12 d) 6 e) 5

Problem No: 3 a) 12 b) 18

GRADE DISTRIBUTION

High: 96 Low: 60 Median: 81 Mean: 80 St.Dev: 10

Score	60-69	70-74	75-79	80-84	85-89	90-94	95-100
# in range	𝍸 /	//	𝍸 /	𝍸 /	𝍸 /	///	//
	6	2	6	6	6	3	2