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Introduction

1.1 What's it about

This is a book about the Mechanics of Solids, Statics, the Strength of Materials, and Elasticity Theory. But that doesn't mean a thing unless you have had a course in the Mechanics of Solids, Statics, the Strength of Materials, or Elasticity Theory. I assume you have not; let us try again:

This is a book that builds upon what you were supposed to learn in your basic physics and mathematics courses last year. We will talk about forces – not political, but vector forces – about moments and torques, reactions, displacements, linear springs, and the requirements of static equilibrium of a particle or a rigid body. We will solve sets of linear algebraic equations and talk about when we can not find a unique solution to a set of linear algebraic equations. We will derive a whole raft of new equations that apply to particles, bodies, structures, and mechanisms; these will often contain the spatial derivatives of forces, moments, and displacements. You have seen a good bit of the basic stuff of this course before, but we will not assume you know the way to talk about, or work with, these concepts, principles, and methods so fundamental to our subject. So we will recast the basics in our own language, the language of engineering mechanics.

For the moment, think of this book as a language text; of yourself as a language student beginning the study of Engineering Mechanics, the Mechanics of Solids, the Strength of Materials, and Elasticity Theory. You must learn the language if you aspire to be an engineer. But this is a difficult language to learn, unlike any other foreign language you have learned. It is difficult because, on the surface, it appears to be a language you already know. That is deceptive: You will have to be on guard, careful not to presume the word you have heard before bears the same meaning. Words and phrases you have already encountered now take on a more special and, in most cases, narrower meaning; a *couple of forces* is more than just two forces.

An important part of learning the vocabulary, is the quick sketch. Along with learning to sketch in the engineering mechanics way, you will have to learn the meaning of certain icons; a small circle, for example, becomes a frictionless pin. So too, grammar and syntax will be crucial. Rigorous rules must be learned and obeyed. Some of these rules will at first seem pedantic; they may strike you as not only irrelevant to solving the problem, but wrong-headed or counter intuitive. But don't despair; with use they will become familiar and reliable friends.

When you become able to speak and respond in a foreign language without thinking of every word, you start to see the world around you from a new perspective. What was once a curiosity now is mundane and used everyday, often without thinking. So too, in this course, you will look at a tree and see its limbs as cantilever beams, you will look at a beam and see an internal bending moment, you will look at a bending moment and conjecture a stress distribution. You will also be asked to be creative in the use of this new language, to model, to estimate, to design.

That's the goal: To get you seeing the world from the perspective of an engineer responsible for making sure that the structure does not fail, that the mechanism doesn't make too much noise, that the bridge doesn't sway in the wind, that the latch latches firmly, the landing gear do not collapse upon touchdown, the drive-shaft does not fracture in fatigue... Ultimately, that is what this book is about. Along the way you will learn about stress, strain, the behavior of trusses, beams, of shafts that carry torsion, even columns that may buckle.

1.2 What you will be doing.

The best way to learn a foreign language is from birth; but then it's no longer a foreign language. The next best way to learn a new language is to use it – speak it, read it, listen to it on audio tapes, watch it on television; better yet, go to the land where it is the language in use and use it to buy a loaf of bread, get a hotel room for the night, ask to find the nearest post office, or if you are really proficient, make a telephone call. So too in the Mechanics of Solids, we insist you begin to use the language.

Doing problems and exercises, taking quizzes and the final, is using the language. This book contains mostly exercises explained as well as exercises for you to tackle. There are different kinds of exercises, different kinds for the different contexts of language use.

Sometimes an engineer will be asked a question and a response will be expected in five minutes. You will not have time to go to the library, access a database, check this textbook. You must estimate, conjure up a rational response on the spot. "....How many piano tuners in the city of Chicago?" (Try it)! Some of the exercises that follow will be labeled *estimate*.

Often practicing engineers must ask what they need to know in order to tackle the task they have been assigned. So too we will ask you to step back from a problem and pose a new problem that will help you address the original problem. We will label these exercises *need to know.*

A good bit of engineering work is variation on a theme, changing things around, recasting a story line, and putting it into your own language for productive and profitable use. Doing this requires experimentation, not just with hardware, but with concepts and existing designs. One poses "what if we make this strut out of aluminum... go to a cantilever support... pick up the load in bending?" We will label this kind of exercise *what if?*

Engineering analysis (as well as prototype testing and market studies) is what justifies engineering designs. As an engineer you will be asked to show that your design will actually work. More specifically, in the terms of this subject, you will be asked to show the requirements of static equilibrium ensure your proposed structure will bear the anticipated loading, that the maximum deflection of simply supported beam at midspan does not exceed the value specified in the contract, that the lowest resonant frequency of the payload is above 100 Hz. The *show that.* label indicates a problem where a full analysis is demanded. Most often this kind of problem will admit of a single solution – in contrast to the *estimate*, *need to know*, or even *what if* exercise. This is the form of the traditional textbook problem set.

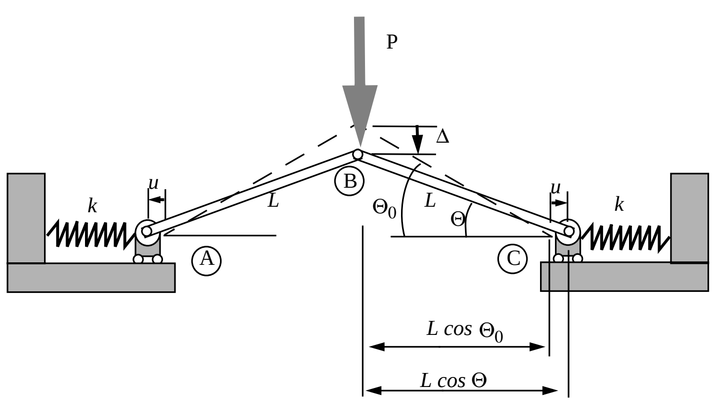
Closely related to *show that* exercises, you will be asked to *construct* a response to questions, as in "*construct* an explanation explaining why the beam failed", or "*construct* an expression for the force in member ab". We could have used the ordinary language, "explain" or "derive" in these instances but I want to emphasize the initiative you, as learner, must take in explaining or deriving. Here, too, *construct* better reflects what engineers actually do at work.

Finally, what engineers do most of the time is design, design in the broadest sense of the term; they play out scenarios of things working, construct stories and plans that inform others how to make things that will work according to their plans. These *design exercises* are the most open-ended and unconstrained exercises you will find in this text. We will have more to say about them later.

In working all of these different kinds of exercises, we want you to use the language with others. Your one-on-one confrontation with a problem set or an exam question can be an intense dialogue but it's not full use of the language. Your ability to speak and think in the language of engineering mechanics is best developed through dialogue with your peers, your tutors, and your teacher. We encourage you to learn from your classmates, to collectively learn from each other's mistakes and questions as well as problem-solving abilities.

Exercise 1.1 An Introductory Exercise

Analyse the behavior of the mechanism shown in the figure. That is, determine how the deflection, Δ, of point B varies as the load, P, varies.



This exercise is intended to introduce the essential concepts and principles of the engineering mechanics of solids. It is meant as an overview; do not be disturbed by the variety of concepts or range of vocabulary. We will try to grasp the essential workings of the device and begin to see the relevance of the concepts and principles of engineering mechanics to an understanding of how it functions and how it might be made to work better

We will apply *the requirements of static equilibrium.* We will analyze the *displacements* of different points of the structure, e.g., the vertical displacement of point B and the horizontal displacements of points A and C, and make sure these are *compatible.* We will consider the *deformation* of the springs which connect points A and C to ground and posit a relationship between the force each one bears and the *relative displacement* of one end with respect to the other end. We will assume this *force/deformation relationship* is *linear.*

We will learn to read the figure; k is the constant of proportionality in the equation relating the force in the spring to its deformation; the little circles are *frictionless pins*, members AB and BC are *two-force members* — as straight members they carry only *tension and compression*. The grey shading represents *rigid ground*. The arrow represents a vertically *applied load* whose *magnitude* is "P".

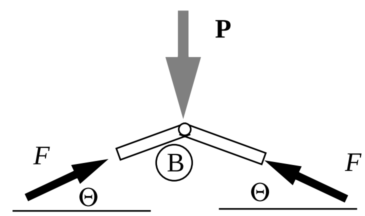
Our aim is to determine the *behavior of the structure* as the applied load increases from zero to some or any finite value. In particular, we want to determine how Δ varies as P changes. We will use a spread sheet to make a graph of this relationship — but only after setting up the problem in terms of *non-dimensional expressions* for the applied load, P, the vertical displacement, Δ, and the horizontal displacement, u. We will allow for relatively *large displacements and rotations.* We will investigate the possibility of *snap-through,* a type of *instability,* if P gets too large. In sum, our objective is to determine how the applied load P varies with Δ, or, alternatively, for any prescribed Δ, what P need be applied?

We will discuss how this funny looking linkage of impossible parts (frictionless pins, rollers, rigid grey matter, point loads, ever linear springs) can be a useful model of real-world structures. There is much to be said; all of this italicized language important.

We start by reasoning thus:

Clearly the vertical displacement, Δ, is related to the horizontal displacements of points A and C; as these points move outward, point B moves downward. We assume our system is *symmetric;* the figure suggests this; if A moves out a distance u, C displaces to the right the same distance. Note: Both u and Δ are measured from the *undeformed* or *unloaded configuration,* $P, \ \Delta = 0$. (This undeformend configuration is indicated by the dashed lines and the angle Θ_0). As P increases, Δ increases and so too u which causes the springs to shorten. This engenders a compressive force in the springs and in the members AB and BC, albeit of a different magnitude, which in turn, ensures static equilibrium of the system and every point within it including the node B where the load P is applied. But enough talk; enough story telling. We formulate some equations and try to solve them.

Static Equilibrium of Node B.

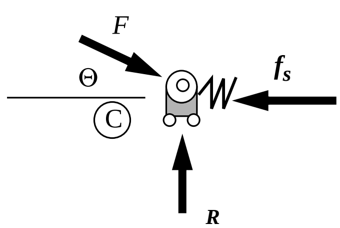


The figure at the right shows an *isolation* of node B. It is a *free body diagram*; i.e., the node has been cut free of all that surrounds it and the influence of those surroundings have been represented by forces. The node is compressed by the force, F, carried by the members AB and BC. We defer a proof that the force must act along the member to a later date.

Equilibrium requires that *the resultant force* on the node vanish; symmetry, with respect to a vertical plane containing P and perpendicular to the page, assures this requirement is satisfied in the horizontal direction; equilibrium in the vertical direction gives:

$$2F \cdot sin\theta - P = 0 \qquad \text{or} \qquad F = P/(2sin\theta)$$

Static Equilibrium of Node C.



The figure at the right shows an isolation of node C. Note how I have drawn F in this isolation acting opposite to the direction of F shown in the isolation of node B. This is because member BC is in compression. The member is compressing node C as it is compressing node B. R is the *reaction force* on the node *due to the ground*. It is vertical since the rollers signify that there is no *resistance to motion* in the horizontal direction; there is no *friction*. f_s is the compressive force in the spring, again, pressing on the node C.

Equilibrium requires that the *resultant* of the three forces vanish. Requiring that the sum of the horizontal components and that the sum of the vertical components vanish independently will ensure that the vector sum, which is the resultant, will vanish. This yields two scalar equations:

$$F \cdot cos\theta - f_s = 0 \qquad \text{and} \qquad R - F \cdot sin\theta = 0$$

The first ensures equilibrium in the horizontal direction, the second, in the vertical direction.

Force/Deformation of the spring:

For our *linear spring*, we can write:

$$f_s = k \cdot u$$

The force is proportional to deformation.

Compatibility of Deformation:

Δ and Θ are not *independent*; you can not choose one arbitrarily, then the other arbitrarily. The first figure indicates that, if members AB and BC remain *continuous*and rigid[1], we have

$$u = L(\cos\theta - \cos\theta_0)$$

But we want Δ, ultimately. Another *compatibility relationship* is seen to be

$$\Delta = L(\sin\theta_0 - \sin\theta)$$

Solution: The relation between *P* and Δ.

From equilibrium of node B we have an equation for *F,* the compressive force in members AB and BC in terms of *P* and θ; Using our result obtained from equilibrium of node C, we can express the force in the spring, f_s in terms of *P* and θ.

$$f_s = (P/2)(\cos\theta/\sin\theta)$$

From the force/deformation relationship for the spring we express, *u*, the displacement of the node C (and A) in terms of f_s, and, using the immediately above, in terms of *P* and θ.

$$u = f_s/k = (P/2k)(\cos\theta/\sin\theta)$$

The first compatibility relationship then allows us to write

$$\boxed{(P/2kL) = \sin\theta \cdot (1 - \cos\theta_0/\cos\theta)}$$

while the second may be written in *non-dimensional form* as

$$\boxed{\Delta/L = \sin\theta_0 - \sin\theta}$$

We could at this point try to eliminate theta; but this is unnecessary. This parametric form of the relationship between *P* and Δ will suffice. Theta serves as an intermediary - a parameter whose value we can choose - guided by our sketch of the geometry of our structure. For each value of theta, the above two equations then fix the value of the vertical displacement and the applied load. A spread sheet is an appropriate tool for carrying out a sequence of such calculations and for constructing a graph of the way *P* varies with Δ, which is our objective. This is left as an exercise for the reader.

1. They neither lengthen nor contract when loaded.

1.3 Resources you may use.

A textbook is only one resource available to you in learning a new language. The exercises are another, pehaps the most important other resource you have available. Still others are the interactive short simulations – computer representations of specific problems or phenomena – made available to you over the web. You will find there as well more sophisticated and generally applicable tools which will enable you to model truss and frame structures - structures which have many members. Another more standard and commonly available tool is the spreadsheet. You will find all these modeling tools to be essential and powerful aids when confronted with an open-ended design exercise where the emphasis is on *what if* and *show that*.

Another resource to you is your peers. We expect you to learn from your classmates, to collaborate with them in figuring out how to set up a problem, how to use a spreadsheet, where on the web to find a useful reference. Often you will be asked to work in groups of two or three, in class - especially when a design exercise is on the table - to help formulate a specification and flesh out the context of the exercise. Yet your work is to be your own.

1.4 Problems

1.1 Without evaluating specific numbers, sketch what you think a plot of the load P - in nondimensional form - $(P/2kL)$ versus the displacement, (Δ/L) will look like for the Introductory Exercise above. Consider θ to vary over the interval $\Theta_0 \geq \Theta \geq -\Theta_0$.

Now compute, using a spread sheet, values for load and displacement and plot over the same range of the parameter Θ.