

Homework 5

5.1 Uniform Attachment Model

[50 points] In this problem, we study a dynamic variation of the Erdős-Renyi model. Similar to the preferential attachment model, nodes are born over time and form edges to existing nodes at the time of their birth. The network formation process is as follows:

- Nodes are born over time and indexed by their date of birth, i.e., node i is born at date i , $i = 1, \dots$
- We start the network with m nodes (born at times $1, \dots, m$), all connected to one another. The first newborn node is thus the one born at time $m + 1$.
- Each newborn node uniformly randomly selects m nodes from the existing set of nodes and links to them.

Let $k_i(t)$ be the degree of node i at time t . We will use a *continuous-time mean-field analysis* (similar to what we did for the preferential attachment model in class) to track the evolution of the “expected degrees of nodes”.

1. What is $k_i(i)$ for $i > m$?
2. Find the expected number of new edges that node i ($1 \leq i \leq t$) receives at time $t + 1$.
3. Using $\frac{dk_i(t)}{dt}$ to approximate the change in the expected degree of node i from time t to time $t + 1$, write down the expression for $\frac{dk_i(t)}{dt}$.
4. Verify that $k_i(t) = m + m \log(\frac{t}{i})$ ($t \geq i$) is a solution to the above differential equation.
5. Let $i(d)$ be the node that has degree d at time t , that is $k_{i(d)}(t) = d$. Find $i(d)$ as a function of m , d and t .
6. Use the approximation $\mathbb{P}(k_i(t) \geq d) = \frac{i(d)}{t}$ to show that $\mathbb{P}(k_i(t) \geq d) = e^{1 - \frac{d}{m}}$.
7. Let p_d for $d \geq m$ be the probability of a node having expected degree d in the uniform attachment model as $t \rightarrow \infty$. Use $p_d \approx -\frac{\partial \mathbb{P}(k_i \geq d)}{\partial d}$ to find p_d .

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