1.020 Ecology II: Engineering for Sustainability
Spring 2008

Problem Set 4 – MATLAB Practice
Due: 5PM Friday April 4, 2008

Introduction

This problem set is designed to give you some practice with MATLAB programming. You should develop your own programs rather than modify those handed out in class. The programming problems given here are based on simplified versions of the Exam 1 problems.

Write your programs as MATLAB scripts rather than functions and display as many of your calculations as possible (i.e. use semi-colons only when needed to suppress large amounts of output). Also, try using the debugger (see HELP) to check your program if it is working correctly.

For this Problem Set only, please hand in individual rather than team solutions. You are free to consult with others if you have difficulties but your final MATLAB codes should be your own. **You will learn more if you are able to find and correct mistakes on your own**. Please provide (to Stellar) electronic copies of the MATLAB (.m) and plot (.fig) files, where requested. Also, provide (to Stellar) an electronic text file with answers to the questions and be sure to include your name on all files.

Problem 1. Mass Balance -- Transportation
Description: The 3 link transportation network shown below has the following characteristics:
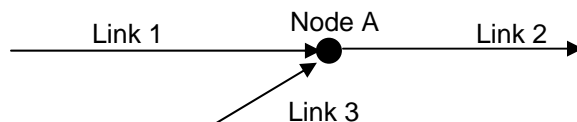   Link lengths (km): $L_2 = 10$
   Link flows (vehicles hr-1): $x_1 = 40$, $x_3 = 20$
   Travel time for Link 2 (hrs): $t_2 = .01\ x_2$

   Speed on Link 2: $S_2 = \dfrac{L_2}{t_2}$ km hr^{-1}.

Question:
1. Write a MATLAB script that specifies the values for all input variables and then computes the vehicle speed on Link 2

Problem 2. Mass Balance -- NAPL dissolution
Description: This question conerns the dissolution of a non-aqueous phase liquid (NAPL)
trichlorethane (TCA, $C_2H_3Cl_3$) to dichloroethane (DCA, $C_2H_4Cl_2$) in a **closed system**.

1 mole of *NAPL TCA* yields 1 mole of *dissolved TCA*.
Unknowns: M_{ntca} = NAPL TCA mass (mol), C_{dtca} = Dissolved TCA concentration (mol L^{-1})
Control volume $V_{cv} =$ = 2 L
Initial dissolved TCA concentration = 0 mol L^{-1}.
Initial NAPL TCA = 1 mol
The NAPL TCA mass dissolves in the water at a rate r (mol day^{-1}) given by:
$$r_{dissol} = k_{dissol} M_{ntca} \qquad k_{dissol} = 0.01 \quad day^{-1}$$

Questions:
1. Write differential equations for the unknown NAPL TCA mass and dissolved TCA
concentrations.
2. Write a MATLAB script that computes and plots (with labels) these two unknowns as functions
of time. **Do not use MATLAB ODE functions** (such as ODE45) to solve the differential
equations. Instead, discretize the equations in time by. approximating the derivatives as follows:
$$\frac{dM(t)}{dt} \approx \frac{M_{t+1} - M_t}{\Delta t}$$

Compute the unknowns by solving the discretized equations recursively using a **for** loop, as was
done in the Everglades example of Lectures 1 and 2. Select a time step and overall simulation time
interval that are reasonable for the problem inputs. Try checking your program by running only a
small number of time steps so you can examine each calculation to make sure it is doing what you
want.

Problem 3. Energy Balance
Description: An outdoor swimming pool has the following properties:
Pool surface area: $A = 100$ m^2 Pool volume: $V = 200$ m^3
Specific heat of pool water: $c = c_v = c_p = 4$ KJoules kg^{-1} $°C^{-1}$
Density of water: $\rho = 1.0$ kg L^{-1}
Longwave radiative heat fluxes:
$\dot{Q}_{out} = \sigma A T_{pool}^4$ watts
$\dot{Q}_{in} = \sigma A T_{sky}^4$ watts
Stefan Bolzman constant: $\sigma = 5.7$ E-8 ($Wm^{-2}K^{-4}$)
Pool-air surface convection coefficent: $h_{conv} = 5$ watts $°C^{-1}$ m^{-2}
Assume $T_{air}(t) = 15$ °C + a uniformly distributed random fluctuation between -4 and +4 °C
generated independently at each time t.
$T_{sky} = 5$ °C (constant)
The initial pool temperature = 15 °C .
Time step $\Delta t = 1$ hr.
Total simulation time = 8 hrs.

Night time conditions (no incident shortwave solar radiation)

Questions:

1. Write a differential equation for the unknown pool temperature (with $T_{air}(t)$ left as a symbolic variable).

2. Write a MATLAB script that computes and plots (with labels) the pool temperature as a function of time. **Do not use MATLAB ODE functions** (such as **ode45**) to solve the differential equation. Instead, discretize the equations in time as in Problem 2 above. Obtain the $T_{air}(t)$ values you need with the MATLAB function **rand**, which generates a random number between 0 and 1 each time it is called (you need to use the number you obtain from **rand** to compute a random $T_{air}(t)$ value for time t that ranges between 11 °C and 19 °C). The **rand** function will genearte different random numbers every time you run it. If you want to use the same numbers repeatedly for testing display them (no semicolon) the first time and enter them as an array in subsequent tests.

For this problem you should be able to display the results at each hour to make sure your program is doing what you want. After it is working correctly decrease the time step to see how your results change.

Problem 4. Mass Balance -- Closed Cycle Ecosystem Model
Description: A simple closed cycle ecosystem is composed of the 2 compartments of nitrogen shown in the figure below:
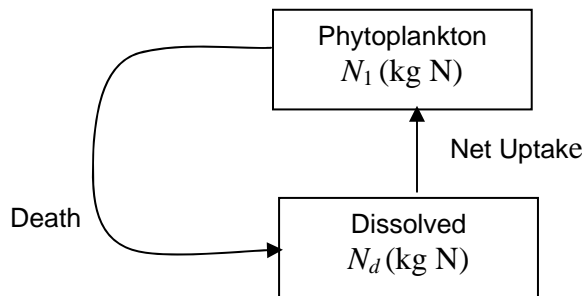
The mass rates that control the N mass transferred along each path in the diagram are:

Phytoplankton net uptake rate $= \left[\dfrac{N_d}{2 + N_d} \right]$ day^{-1}

Phytoplankton death rate $= k_1 = 0.2$ day^{-1}

Other inputs are:
    Initial dissolved nitrogen $= 3$ kg:
    Initial phytoplankton nitrogen $= 1$ kg:
    Increase the dissolved nitrogen to 5 kg at time $= 10$ days
    Total simulation time $= 20$ days
    Time step $= 0.02$ days

Questions:

1. Write differential equations for the two unknown Nitrogen masses in this problem.
2. Write a MATLAB script that computes and plots (with labels) these two masses as a function of time. **Do not use MATLAB ODE functions** (such as ODE45) to solve the differential equations. Instead, discretize the equations in time as in Problems 2 and 3 above. Be sure to **increase the dissolved nitrogen to 5 kg at time = 10 days, within your program.**

This program will produce too much output to display all the calculations at every time step. Instead, if you want to check your results print out only the first few time steps or temporarily decrease the simulation time to a small number so you can display each calculation.