

5.73

Quiz 22 ANSWERS

$$[\mathbf{L}_i, \mathbf{p}_j] = i\hbar \sum_k \epsilon_{ijk} \mathbf{p}_k$$

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$\epsilon_{ijk} =$ +1 if ijk are in cyclic order (i.e. xyz , yzx , or zxy)
 -1 if ijk are in anti-cyclic order
 0 if any index is repeated.

$$\mathbf{L} = (\mathbf{q} \times \mathbf{p}) = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \mathbf{x} & \mathbf{y} & \mathbf{z} \\ \mathbf{p}_x & \mathbf{p}_y & \mathbf{p}_z \end{pmatrix}$$

- A. What are \mathbf{L}_y and \mathbf{L}_z in terms of $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ and $(\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)$?

$$\begin{aligned} \mathbf{L}_x &= \mathbf{y}p_z - \mathbf{z}p_y \\ \mathbf{L}_y &= -\mathbf{x}p_z + \mathbf{z}p_x \\ \mathbf{L}_z &= \mathbf{x}p_y - \mathbf{y}p_x \end{aligned}$$

- B. Use ϵ_{ijk} notation to evaluate $[\mathbf{L}_x, \mathbf{x}]$, $[\mathbf{L}_x, \mathbf{z}]$, $[\mathbf{L}_x, \mathbf{p}_x]$, and $[\mathbf{L}_x, \mathbf{p}_z]$.

$$[\mathbf{L}_x, \mathbf{x}] = \mathbf{0}$$

$$[\mathbf{L}_x, \mathbf{z}] = -i\hbar \mathbf{y}$$

$$[\mathbf{L}_x, \mathbf{p}_x] = \mathbf{0}$$

$$[\mathbf{L}_x, \mathbf{p}_z] = -i\hbar \mathbf{p}_y$$

- C. Use the results of part B to show that $[\mathbf{L}_x, \mathbf{L}_y] = i\hbar \mathbf{L}_z$. Recall that

$$[\mathbf{A}, \mathbf{BC}] = \mathbf{B}[\mathbf{A}, \mathbf{C}] + [\mathbf{A}, \mathbf{B}]\mathbf{C}.$$

$$\begin{aligned} [\mathbf{L}_x, -\mathbf{x}p_z + \mathbf{z}p_x] &= -[\mathbf{L}_x, \mathbf{x}p_z] + [\mathbf{L}_x, \mathbf{z}p_x] \\ [\mathbf{L}_x, \mathbf{x}p_z] &= \mathbf{x}[\mathbf{L}_x, p_z] + [\mathbf{L}_x, \mathbf{x}]p_z = -i\hbar \mathbf{x}p_y + 0 \\ [\mathbf{L}_x, \mathbf{z}p_x] &= \mathbf{z}[\mathbf{L}_x, p_x] + [\mathbf{L}_x, \mathbf{z}]p_x = -i\hbar \mathbf{y}p_x \\ [\mathbf{L}_x, \mathbf{L}_y] &= i\hbar [\mathbf{x}p_y - \mathbf{y}p_x] = i\hbar \mathbf{L}_z \end{aligned}$$

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