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Quiz 21 ANSWERS

$[\mathbf{x},\mathbf{p}] = i\hbar$

$\mathbf{H} = \mathbf{p}^2/2m + (1/2)k\mathbf{x}^2$ Harmonic Oscillator

Force $= -\dfrac{dV(x)}{dx}$ (negative gradient of potential energy)

$$\left.\begin{aligned} F &= m\frac{d^2x}{dt^2} = \frac{dp}{dt} \\ p &= m\frac{dx}{dt} \end{aligned}\right\}$$ Newton's laws

$$\frac{d\langle \mathbf{A}\rangle}{dt} = \frac{i}{\hbar}\langle[\mathbf{H},\mathbf{A}]\rangle + \left\langle \frac{\partial \mathbf{A}}{\partial t}\right\rangle$$ Heisenberg's Equation of Motion

A. Evaluate $[\mathbf{H},\mathbf{p}]$ $= \frac{1}{2m}\left[p^2,p\right] + \frac{1}{2}k\left[x^2,p\right] = 0 + \frac{1}{2}k\left(x[x,p]+[x,p]x\right) = i\hbar kx$

B. Relate $\dfrac{d}{dt}\langle \mathbf{p}\rangle$ to $\langle \mathbf{x}\rangle$.

$$\frac{d\langle p\rangle}{dt} = \frac{1}{\hbar}\langle[H,p]\rangle + \left\langle\frac{\partial p}{\partial t}\right\rangle$$

ma $\quad = \dfrac{i}{\hbar}i\hbar k\langle x\rangle + 0 = -k\langle x\rangle =$ Force

$F = ma$

C. Evaluate $[\mathbf{H},\mathbf{x}]$.

$$= \frac{1}{2m}[p^2,x] + \frac{1}{2}k[x^2,x] = \frac{1}{2m}\left(p[p,x]+[p,x]p\right) + 0$$
$$= \frac{1}{2m}(-2i\hbar p) = -i\hbar p/m$$

D. Relate $\dfrac{d}{dt}$ $\langle \mathbf{x}\rangle$ to $\langle \mathbf{p}\rangle$

$$\frac{d\langle x\rangle}{dt} = \frac{1}{\hbar}\langle[H,x]\rangle + \left\langle\frac{\partial x}{\partial t}\right\rangle$$
$$= \frac{1}{\hbar}\left(-i\hbar\langle p\rangle/m\right) = \langle p\rangle/m = \text{velocity}$$

$\langle v\rangle = \langle p\rangle/\mathrm{m}$