

# 5.73

## Quiz 21 **ANSWERS**

$$[\mathbf{x}, \mathbf{p}] = i\hbar$$

$$\mathbf{H} = \mathbf{p}^2/2m + (1/2)k\mathbf{x}^2 \quad \text{Harmonic Oscillator}$$

$$\text{Force} = -\frac{dV(x)}{dx} \quad (\text{negative gradient of potential energy})$$

$$\left. \begin{aligned} F &= m \frac{d^2x}{dt^2} = \frac{dp}{dt} \\ p &= m \frac{dx}{dt} \end{aligned} \right\} \quad \text{Newton's laws}$$

$$\frac{d\langle \mathbf{A} \rangle}{dt} = \frac{i}{\hbar} \langle [\mathbf{H}, \mathbf{A}] \rangle + \left\langle \frac{\partial \mathbf{A}}{\partial t} \right\rangle \quad \text{Heisenberg's Equation of Motion}$$

A. Evaluate  $[\mathbf{H}, \mathbf{p}] = \frac{1}{2m} [p^2, p] + \frac{1}{2} k [x^2, p] = 0 + \frac{1}{2} k (x[x, p] + [x, p]x) = i\hbar kx$

B. Relate  $\frac{d}{dt} \langle \mathbf{p} \rangle$  to  $\langle \mathbf{x} \rangle$ .

$$\begin{aligned} \frac{d\langle p \rangle}{dt} &= \frac{1}{\hbar} \langle [H, p] \rangle + \left\langle \frac{\partial p}{\partial t} \right\rangle \\ &= \frac{i}{\hbar} i\hbar k \langle x \rangle + 0 = -k \langle x \rangle = \text{Force} \\ F &= ma \end{aligned}$$

C. Evaluate  $[\mathbf{H}, \mathbf{x}] = \frac{1}{2m} [p^2, x] + \frac{1}{2} k [x^2, x] = \frac{1}{2m} (p[p, x] + [p, x]p) + 0 = \frac{1}{2m} (-2i\hbar p) = -i\hbar p/m$

D. Relate  $\frac{d}{dt} \langle \mathbf{x} \rangle$  to  $\langle \mathbf{p} \rangle$

$$\begin{aligned} \frac{d\langle x \rangle}{dt} &= \frac{1}{\hbar} \langle [H, x] \rangle + \left\langle \frac{\partial x}{\partial t} \right\rangle \\ &= \frac{1}{\hbar} (-i\hbar \langle p \rangle / m) = \langle p \rangle / m = \text{velocity} \\ \langle v \rangle &= \langle p \rangle / m \end{aligned}$$

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